Mixed-integer linear programming models to optimize residential demand response to dynamic tariffs

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INTRODUCTION – Consumption flexibility

- Retail companies procure electricity in wholesale markets and offer flat or (slightly variable) **tariffs** to their residential customers.
- These tariffs do not convey **price signals** reflecting generation costs and grid conditions → **consumers do not have sufficient incentives** to adopt consumption patterns different from habitual behaviors.
- **Flexibility** regarding **time of operation** of some **end-use loads**
  - improves the system overall efficiency,
  - lowers peak generation costs,
  - facilitates the penetration of renewable sources,
  - reduces network losses,

  while offering consumers economic benefits.

INTRODUCTION – Demand as a manageable resource

theguardian

Winter electricity blackouts risk recedes, says National Grid

Extra power will mean lights will not go out this winter, says firm that operates UK's electricity transmission network

Energy bills are set to rise Great Britain from April 2022. Illus.: Kyle Smar

Los Angeles Times

Why 'dynamic' pricing based on real-time supply and demand is rapidly spreading

INSIDE ENERGY

Timing Is Everything When It Comes To Your Future Electricity Bill

By Stephanie Joyce, Wyoming Public Radio | March 3, 2016

Historically, electricity pricing has been relatively straightforward: the more you use, the more you pay. But today, that simple equation is not so simple. Increasingly, the time of day when you use electricity factors into the cost as well.

INTRODUCTION – Time-of-Use (ToU) tariffs

- **Time-differentiated retail tariffs** are expected to become a common tariff scheme in **smart grids.**
- **Dynamic time-of-use (ToU) tariffs** will motivate consumers to engage in different consumption patterns → making the most of the flexibility in the operation of some appliances through demand response actions affecting the provision of energy services.

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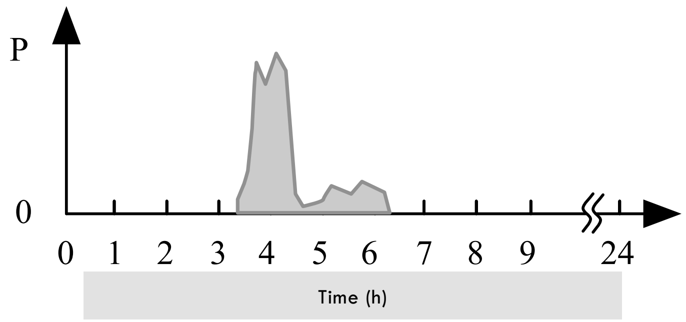
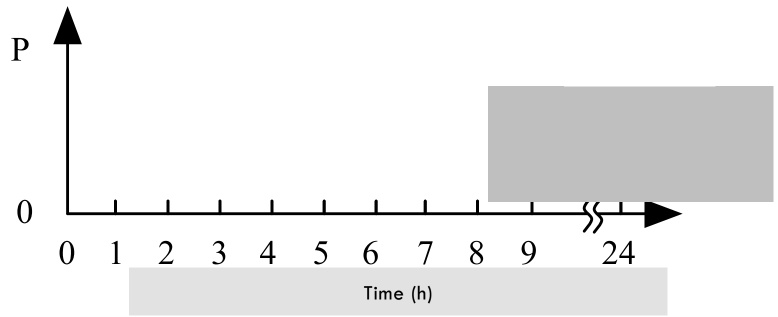
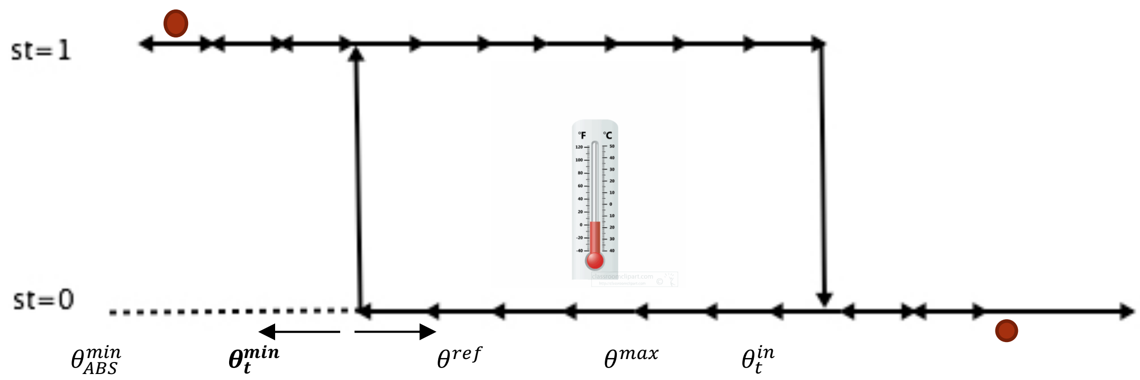
Períodos horários: P1 - Ponta alta; P2 - Ponta baixa; P3 - Cheias altas; P4 - Cheias baixas; P5 - Vazio normal; P6 - Super vazio

Épocas: 1 - Jan., Fev.; 2 - Mar., Nov., Dez.; 3 - Jul.; 4 - Abr., Mai., Jun., Ago., Set., Out.

INTRODUCTION – Appliance control for demand response





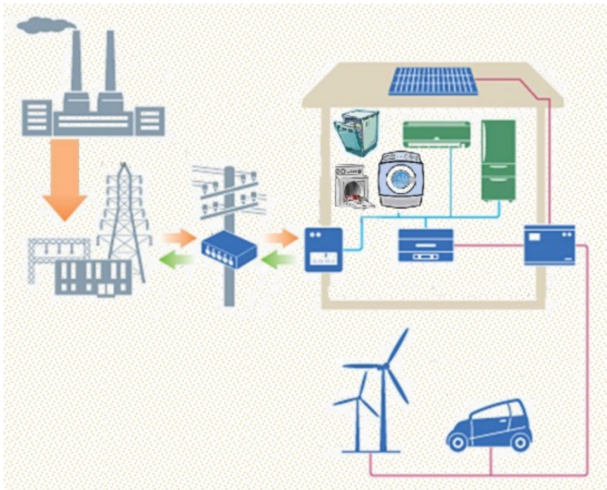


INTRODUCTION – EMS for demand response

- Consumers receive tariff information some time in advance (e.g., one day) and respond by **scheduling load operation** [shiftable and interruptible loads] and **changing thermostat settings** [air conditioning systems].

- Trading-off **electricity bill** (to profit from periods of low energy prices) and **comfort** (associated with appliance operation according to preferences and requirements).

Autonomous home **energy management systems** working behind the meter to make the integrated optimization of all energy resources

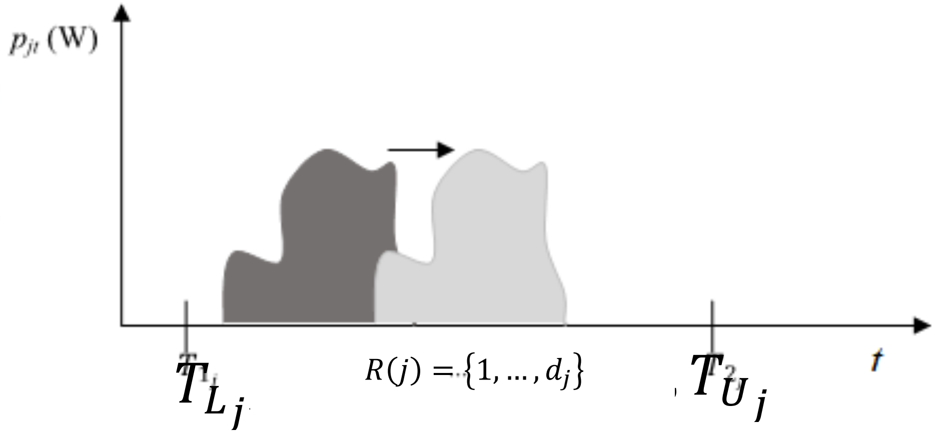


APPLIANCE CONTROL – SHIFTABLE LOADS

Dishwasher

Clothes dryer

Washing machine



$$P_{jt}^{Sh} = \sum_{r \in R(j): r \leq t \wedge r \leq t+1-T_{L_i}} g_{jr} \cdot s_{j(t-r+1)} \qquad j = 1, \dots, J\,, t = T_{L_j}, \dots, T_{U_j}$$

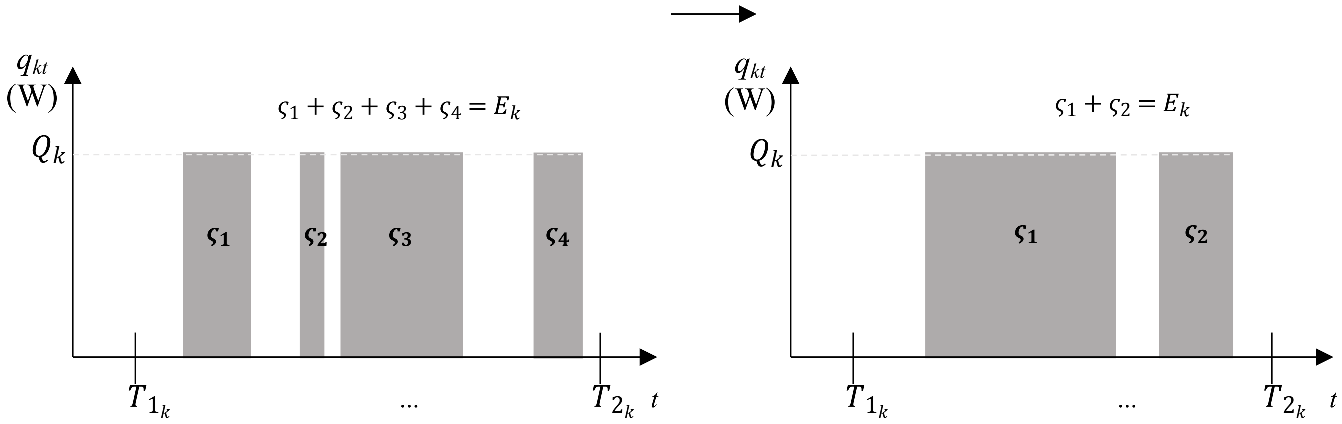
$$P_{jt}^{Sh} = 0 \qquad , \; j = 1, \dots, J\,, t < T_{L_j} \vee t > T_{U_j}$$

$$s_{jt} = \begin{cases} 1 & \text{if load } j \text{ begins its operation in time } t \\ 0 & \text{otherwise} \end{cases} \qquad j = 1, \cdots, J; t = T_{L_j}, \dots, T_{U_j} - d_j + 1$$

APPLIANCE CONTROL – INTERRUPTIBLE LOADS (EWH, EV)

Electric water heater (EWH)

Electric vehicle (EV)



$$q_{kt} = v_{kt} Q_k \,,\; k = 1,\dots,K \,,\, t = T_{1_k},\dots,T_{2_k}$$

$$q_{kt} = 0 \,,\; k = 1,\dots,K \,,\, t < T_{1_k} \vee t > T_{2_k}$$

$$\sum_{t=T_{1_k}}^{T_{2_k}} q_{kt} = E_k \,,\; k = 1,\dots,K$$

$$v_{kt} \in \{0,1\} \qquad k = 1,\dots,K,\, t = T_{1_k},\dots,T_{2_k}$$

$$q_{kt} \geq 0 \,,\; k = 1,\dots,K \,,\, t = 1,\dots,T$$

v_{kt} =1 if load *k* is operating in instant *t*

APPLIANCE CONTROL – EWH AS A THERMOSTATIC LOAD

EWH as a thermostatic-controlled load (instead of interruptible load)

$$P_t^{losses} = A.U\left(\tau_t - \tau_t^{amb}\right) \quad , \quad t = 1, \dots, T$$

$$\tau_{t+1} = \left(\frac{M - m_t}{M} \cdot \tau_t + \frac{m_t}{M} \cdot \tau^{net}\right) + \frac{P^R v_t - P_t^{losses}}{M\, c^p} \cdot \Delta t \quad , \quad t = 0, \dots, T-1$$

$$\tau_t \geq \tau^{min} - \mathcal{M} v_t \qquad , \qquad t = 1, \dots, T$$

$$\tau_t \leq \tau^{max} + \mathcal{M}(1 - v_t) \quad , \qquad t = 1, \dots, T$$

$$\sum_{t=1}^{T - t^{req} + 1} n_t = 1$$

$$\tau_t \geq \sum_{\substack{t'=1 \\ (t' \leq t)}}^{t^{req}} \tau^{req} . n_{t-t'+1} \qquad , \qquad t = 1, \dots, T$$

$$v_t \in \{0,1\}, \qquad n_t \in \{0,1\} \quad , \quad t = 1, \dots, T$$

$$P_t^{losses} \geq 0 \,, \tau_t \geq 0 \quad , \qquad t = 1, \dots, T$$

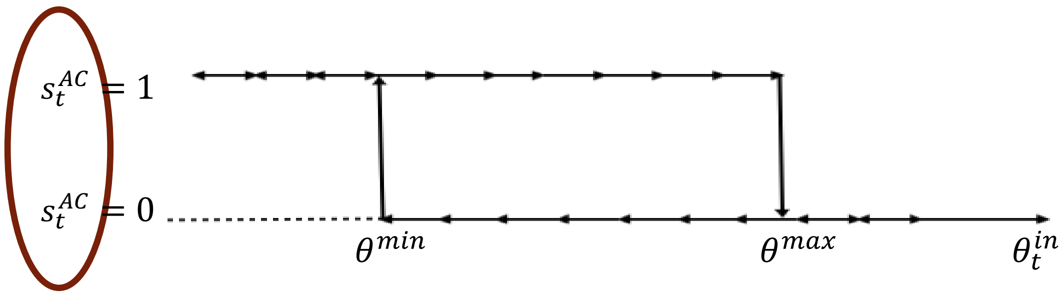
v_t= on/off control of the heating element in time *t*

τ_t= hot water temperature inside the tank in time t (°C)

n_t= binary variable equal to 1 in the first *t* in which $\tau_t > \tau_{req}$ for t_{req}

P_t^{losses}= power losses through the envelope in time t (kW)

APPLIANCE CONTROL – AIR CONDITIONER SYSTEM (1)



Air conditioner in heating mode

MILP implementation of a rule-based system

$$\theta_t^{in} = (1-\beta)\,\theta_{t-1}^{in} + \beta\,\theta_{t-1}^{ext} + \gamma\,P^{AC}\,s_{t-1}^{AC}\,,\quad t = 1,\dots,T$$

$$\theta_t^{in} \geq \theta^{min} - \mathcal{M}s_t^{AC}\,,\qquad t = 1,\dots,T$$

$$\theta_t^{in} \leq \theta^{min} + \mathcal{M}z_t\,,\qquad t = 1,\dots,T$$

$$\theta_t^{in} \geq \theta^{max} - \mathcal{M}y_t\,,\qquad t = 1,\dots,T$$

$$z_t + y_t - s_{t-1}^{AC} + s_t^{AC} \leq 2\,,\qquad t = 1,\dots,T$$

$$z_t + y_t + s_{t-1}^{AC} - s_t^{AC} \leq 2\,,\qquad t = 1,\dots,T$$

$$\theta_t^{in} \leq \theta^{max} + \mathcal{M}(1 - s_t^{AC}),\qquad t = 1,\dots,T$$

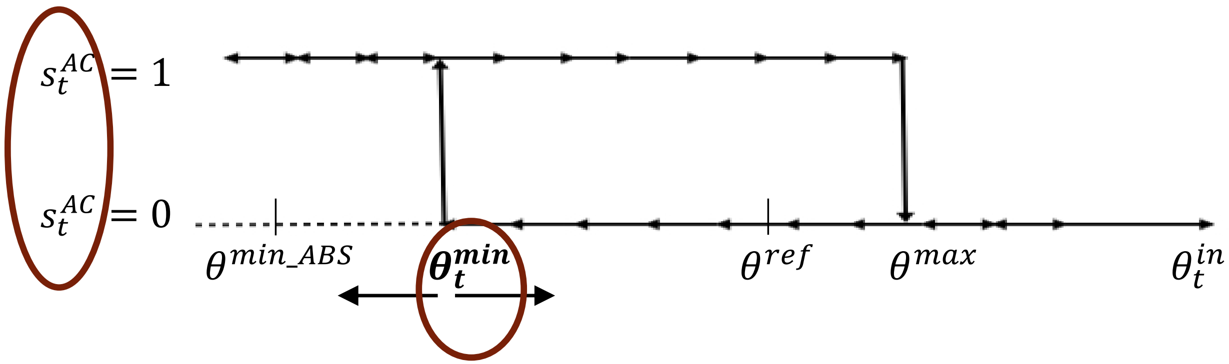
$$s_t^{AC}, z_t, y_t \in \{0,1\}\,,\qquad t = 1,\dots,T$$

s_t^{AC} =1 if the AC is operating in instant *t*

The AC is *on* when the indoor temp. is below the minimum temp., and it is *off* when the indoor temp. is above the maximum allowed temp.

The AC keeps the state *on* or *off* when the indoor temp. is between the lower and the upper bound of the thermostat deadband

APPLIANCE CONTROL – AIR CONDITIONER SYSTEM (2)



Exploiting the consumer's willingness to trade-off some discomfort with profiting from lower price periods → the minimum indoor temp. for which the system should turn *on* is a decision variable

$$\theta_t^{min} - \theta^{ref} = \delta_t^+ - \delta_t^- , \qquad t = 1, \ldots, T$$

$$\theta^{min\_ABS} \leq \theta_t^{min} \leq \theta^{max}, \qquad t = 1, \ldots, T$$

$$\delta_t^-, \delta_t^+ \geq 0 , \qquad t = 1, \ldots, T$$

θ_t^{min} = thermostat lower bound (°C)

δ_t^-, δ_t^+ = temp. deviations of the minimum temp. below/above the reference temp. (°C)

→ higher computation effort

APPLIANCE CONTROL – STATIONARY AND EV BATTERIES

$$E_{x,t} = E_{x,t-1} + \left(\eta_x^{ch}\, P_t^{H2x}\, \Delta t\right) - \left(\frac{P_t^{x2H}\, \Delta t}{\eta_x^{dch}}\right) \;,\; t \in T_x \;, x \in \{B,V\}$$

$$E_x^{min} \le E_{x,t} \le E_x^{max} \qquad ,\; t \in T_x \;, x \in \{B,V\}$$

$$0 \le P_t^{H2x} \le P_x^{ch\_max}\, s_t^{H2x} \qquad ,\; t \in T_x \;, x \in \{B,V\}$$

$$0 \le P_t^{x2H} \le P_x^{dch\_max}\, s_t^{x2H} \qquad ,\; t \in T_x \;, x \in \{B,V\}$$

$$s_t^{H2x} + s_t^{x2H} \le 1 \qquad , t \in T_x \;, x \in \{B,V\}$$

$$E_{B,T} \ge E_B^{req} \;;\; E_{V,t_d} \ge E_V^{req}$$

$$s_t^{H2x}, s_t^{x2H} \in \{0,1\} \qquad ,\; t \in T_x \;, x \in \{B,V\}$$

$x \in \{B,V\}$

$P_t^{x2H}\Delta t$ = energy transferred from the battery x to home (B2H or V2H) in time t (battery discharge)

$P_t^{H2x}\Delta t$ = energy transferred from the home to the battery x (H2B or H2V) in time t (battery charge)

$E_{x,t}$ = energy (kWh) in the battery x in time t

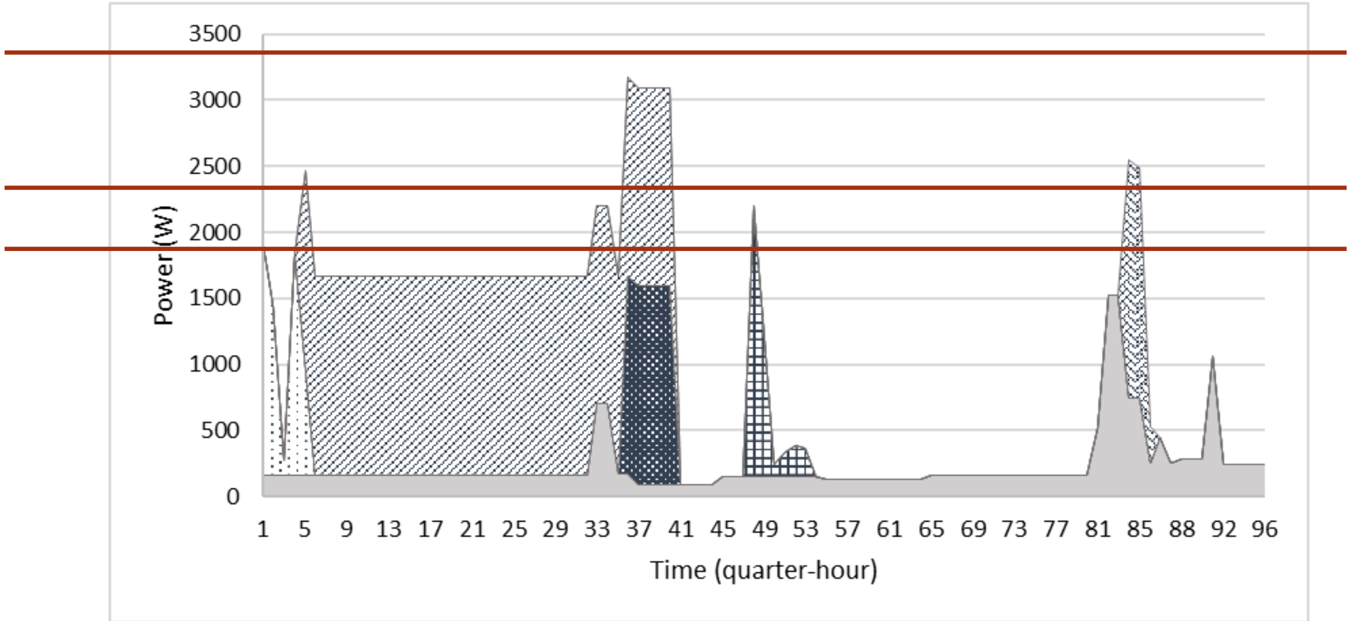
s_t^{H2x} = binary variables equal to 1 when the battery x is charging in time t

s_t^{x2H} = binary variables equal to 1 when the battery x is discharging in time t

POWER COST COMPONENT

L price levels

the consumer is charged by the peak power taken



$$P_t^{G2H} \leq \sum_{l=1}^{L} P_l^{Cont} . u_l^{Cont} \quad , \ t = 1, \dots, T$$

$$\sum_{l=1}^{L} u_l^{Cont} = 1$$

$$u_l^{Cont} \in \{0,1\} \ , \ l = 1, \dots, L$$

u_l^{Cont}=1 if power level l is required

OVERALL MODEL
OBJECTIVE FUNCTIONS: MIN COST, MIN DISCOMFORT

Cost: of the energy consumed by all types of loads + power cost – revenue of selling back to the grid:

$$\min_{P^{G2H},\ P^{H2G}} f = \sum_{t=1}^{T} \left[\left(C_t^{buy}\, P_t^{G2H}\, \Delta t \right) - \left(C_t^{sell}\, P_t^{H2G}\, \Delta t \right) \right] + \sum_{l=1}^{L} \left(C_l^{Cont}\, u_l^{Cont} \right)$$

energy cost - revenue + power cost

Discomfort: Penalizing positive and negative deviations (δ_t^+ , δ_t^-) of the thermostat lower bound, θ_t^{min}, to the reference temperature θ^{ref}:

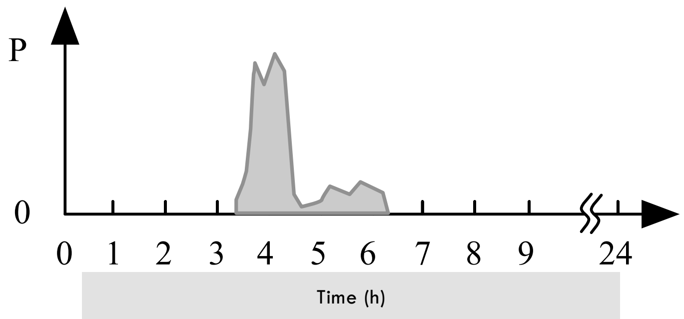
$$\min \sum_{t \in T} \left(c^+ \delta_t^+ + c^- \delta_t^- \right)$$

[heating mode: $c^- > c^+$]

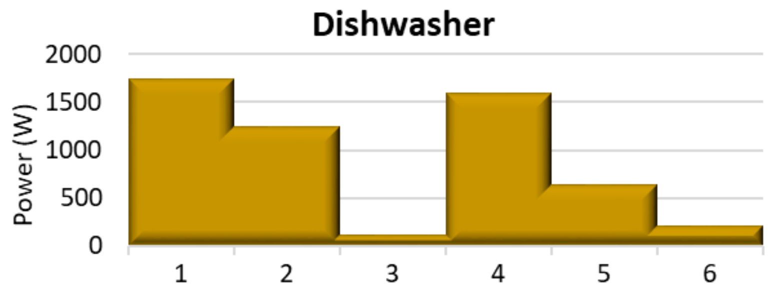
Additional constraints:

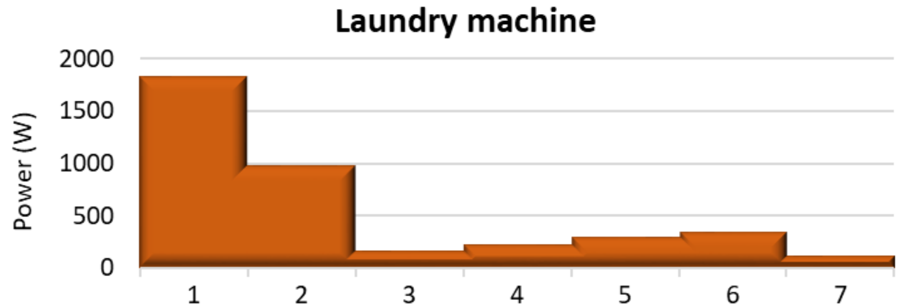
- balance of exchanges grid to home (*G2H*) and home to grid (*H2G*)
- battery charge / discharge

SHIFTABLE APPLIANCES

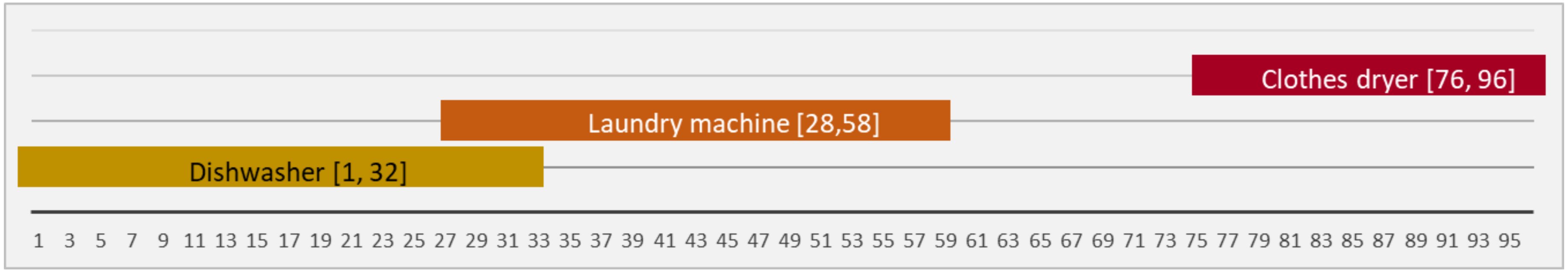


Power required:

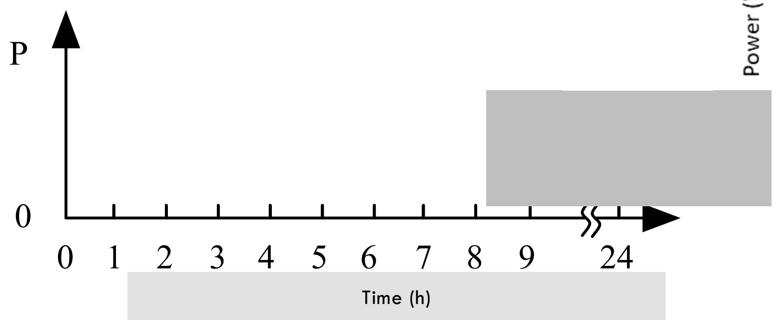




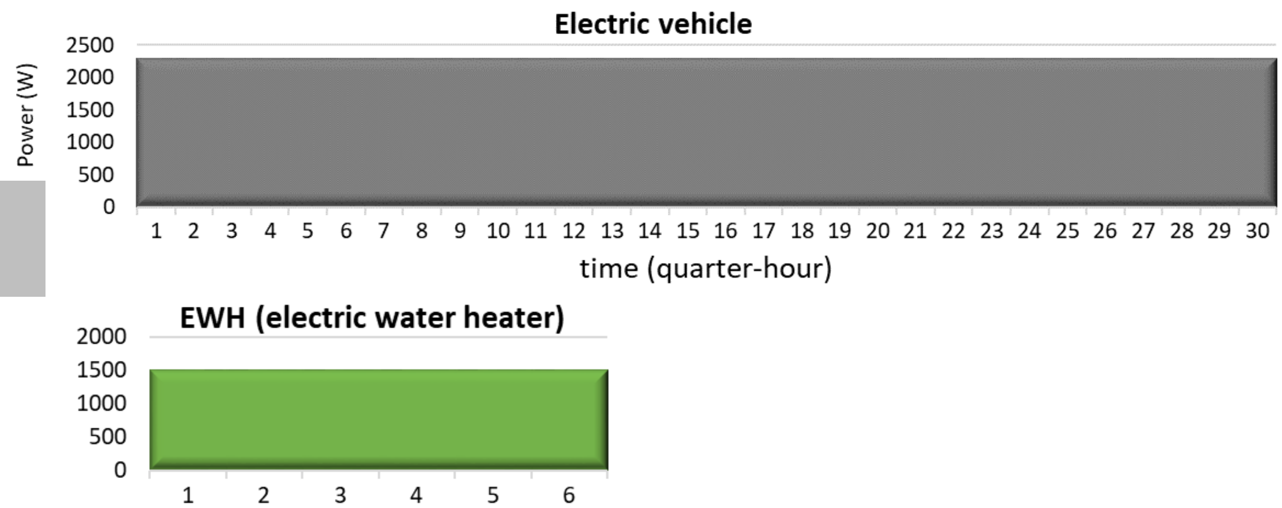
Comfort time slots allowed for the operation of shiftable appliances:



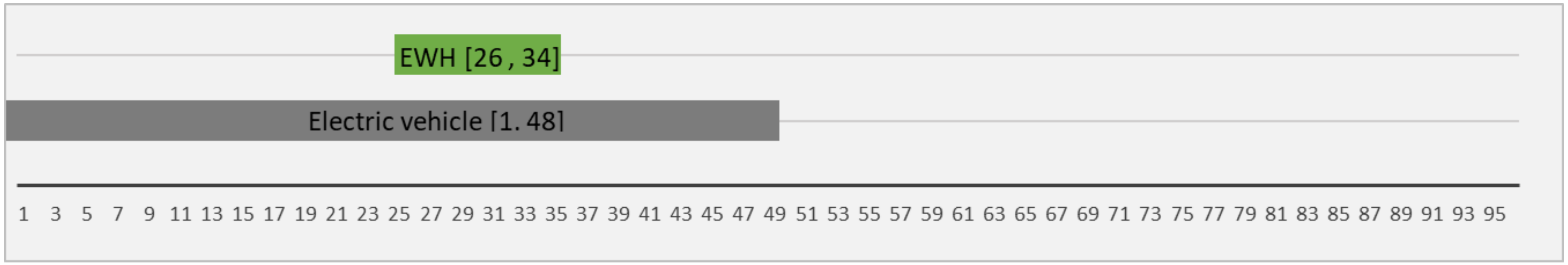
INTERRUPTIBLE APPLIANCES



Energy required to provide the service:



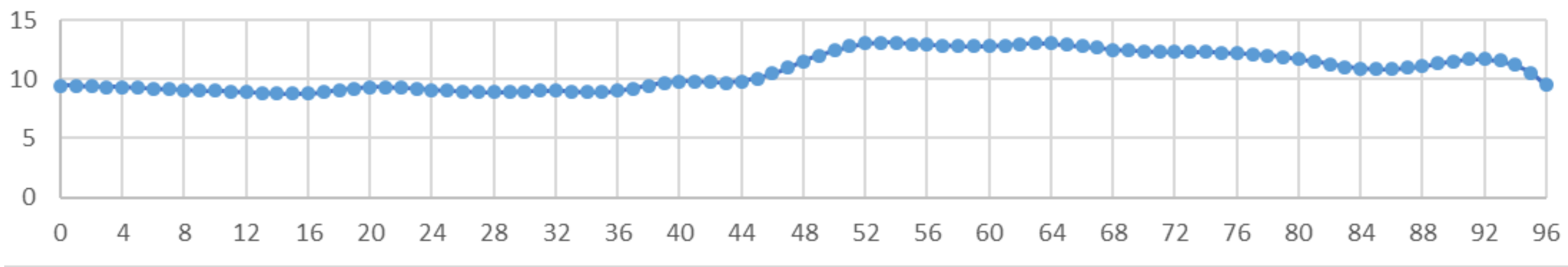
Comfort time slots for the operation of interruptible appliances: :



PARAMETERS OF THE THERMOSTATIC LOAD (AC)

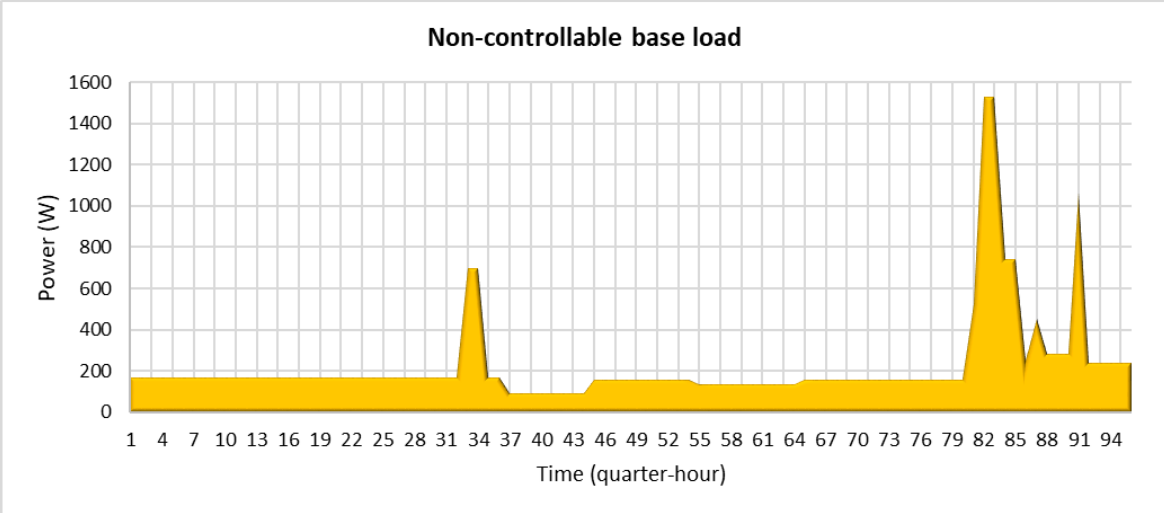
θ^{max}	θ_{Abs}^{min}	θ^{ref}	θ_0^{in}	P_{AC}^{nom}	s_0
24ºC	18ºC	20ºC	12ºC	1400W	0

Outdoor temperature:

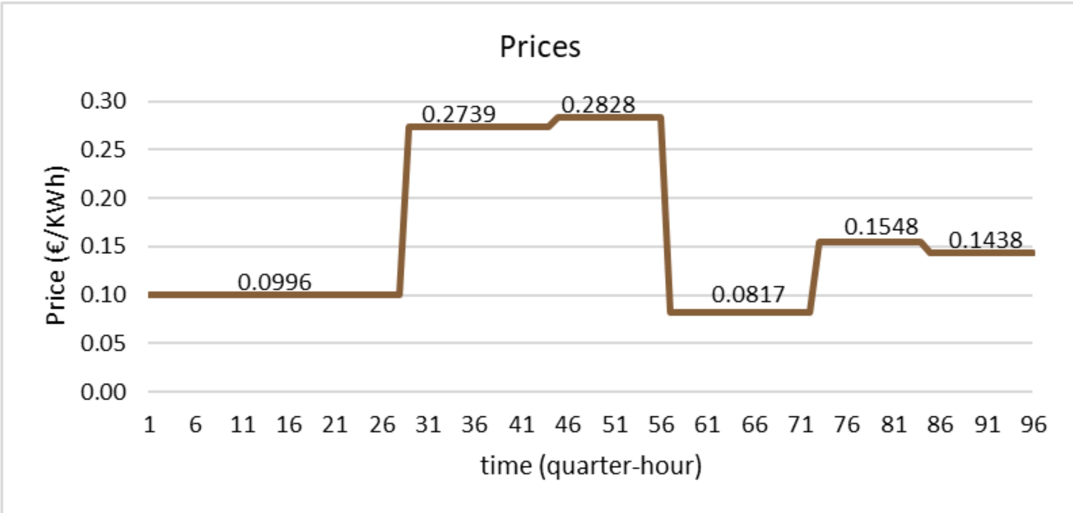


BASE LOAD AND TOU TARIFFS

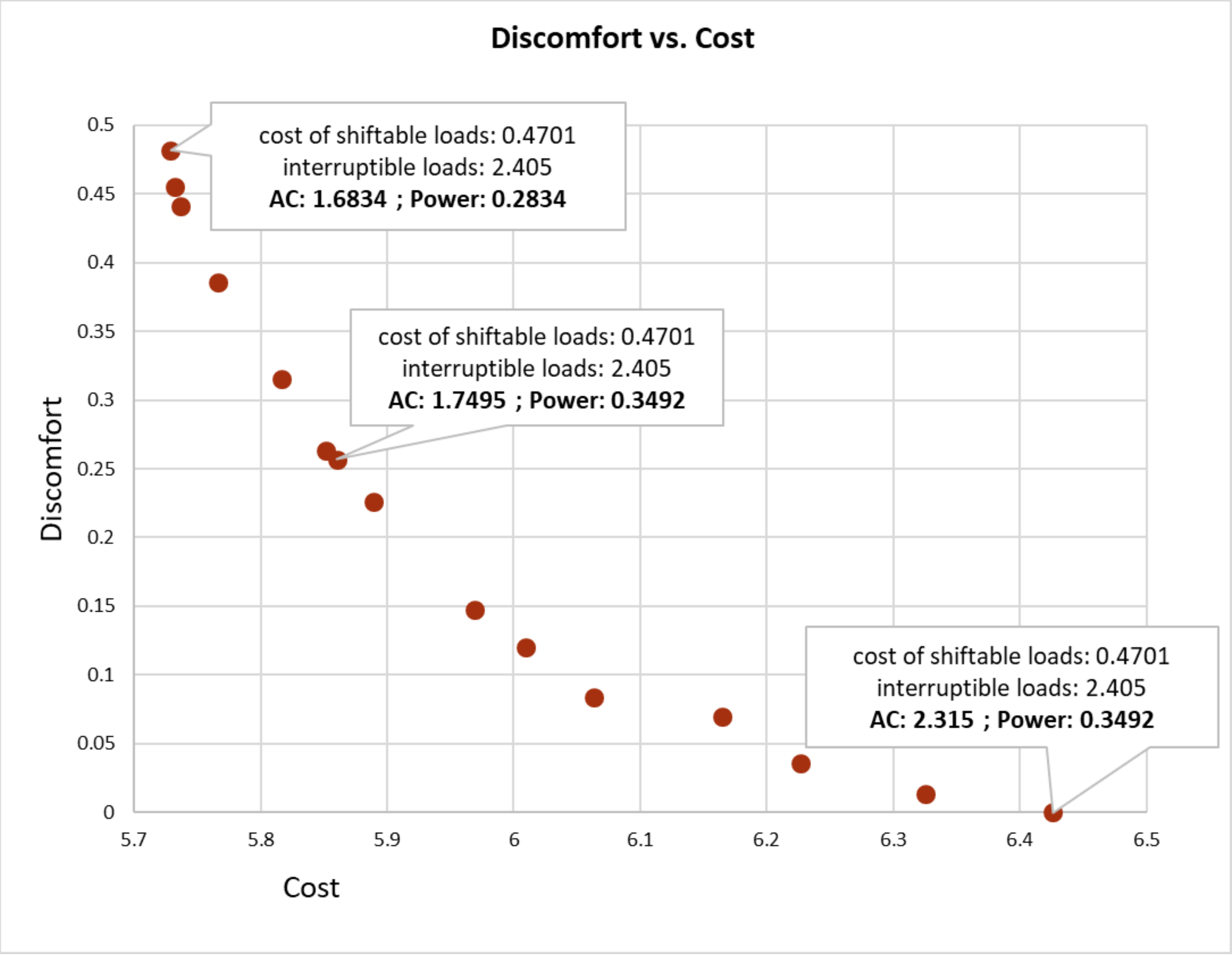
Base load:



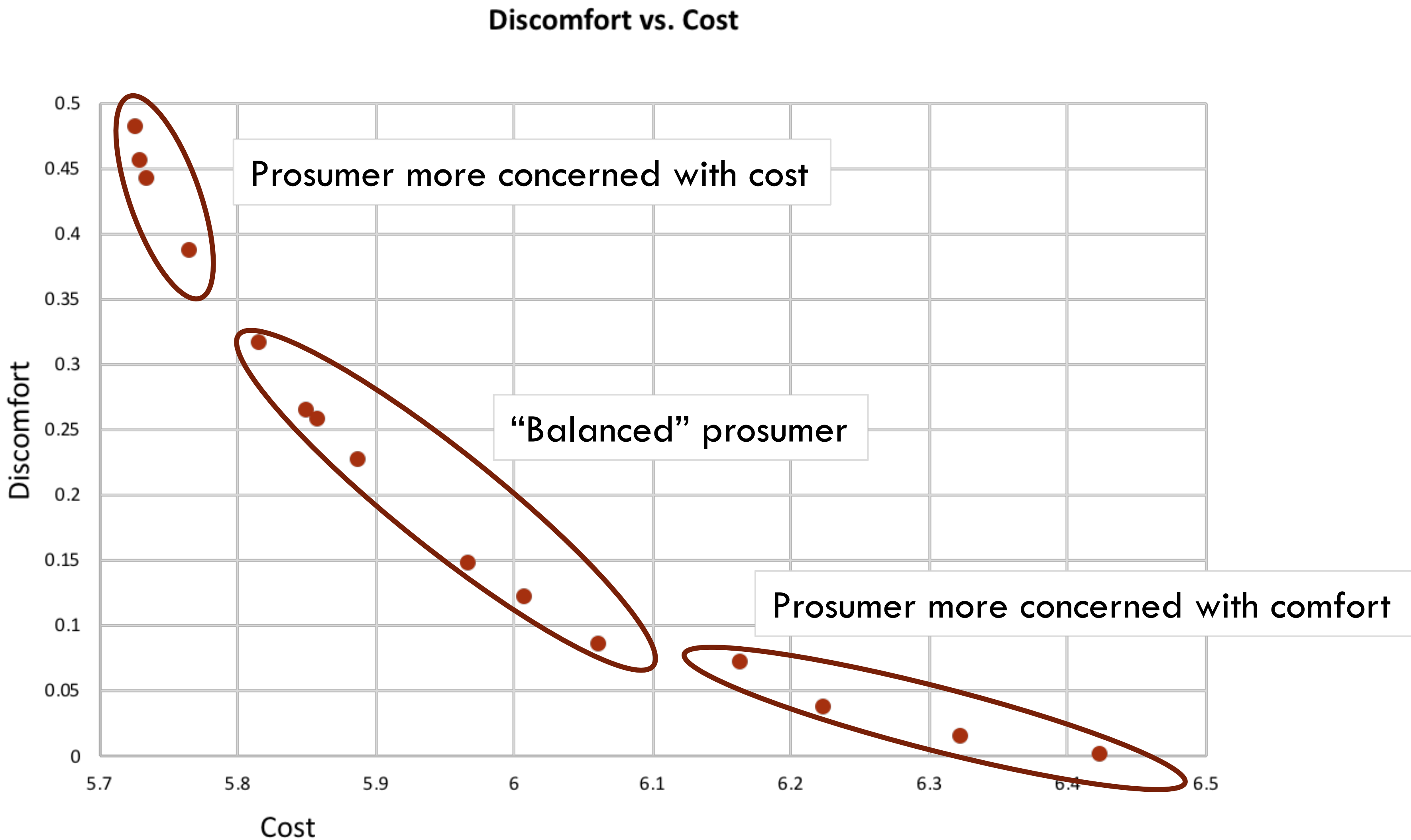
Electricity Prices:



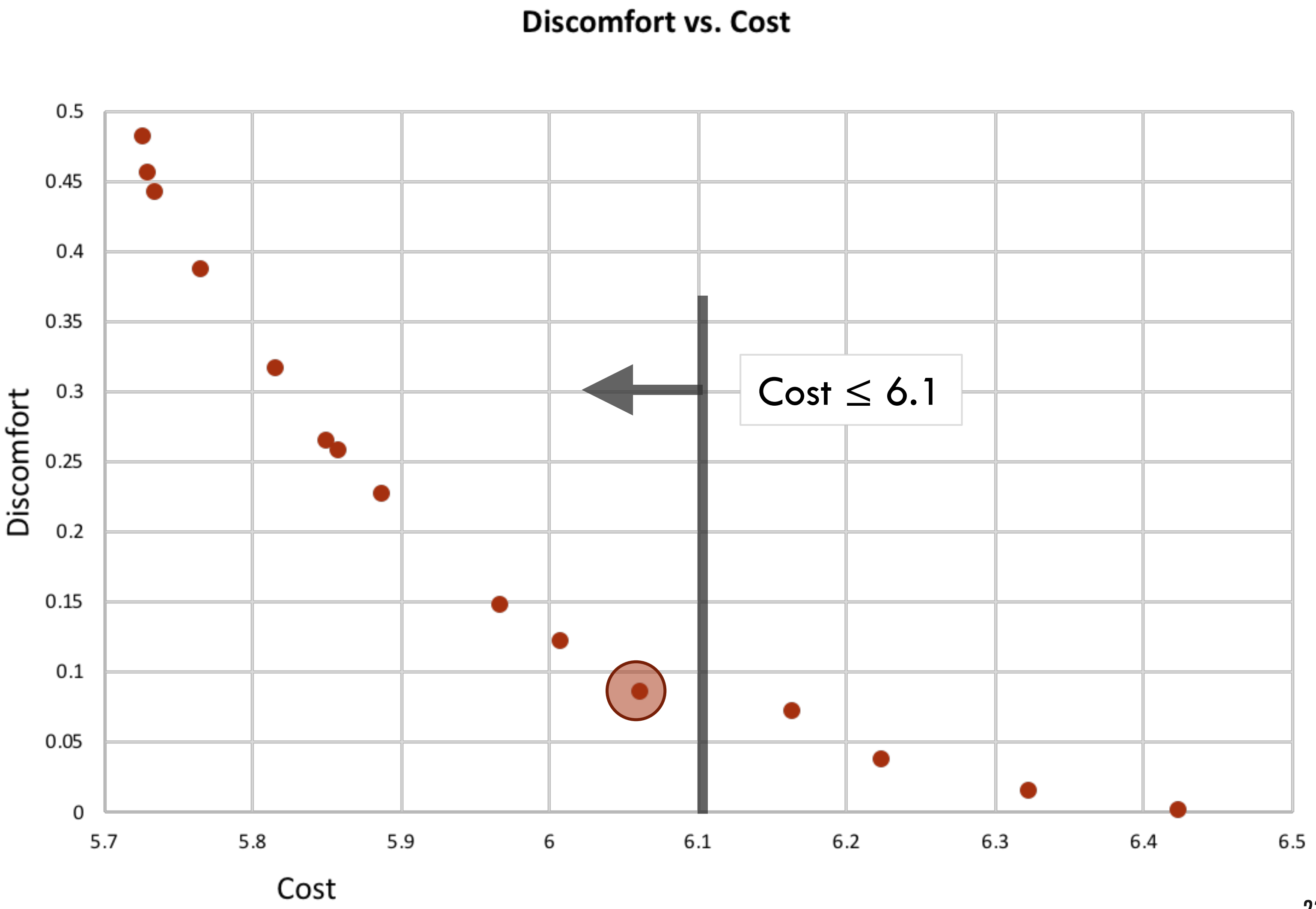
RESULTS MODEL 1 (bi-objective, without batteries and not selling back to the grid)



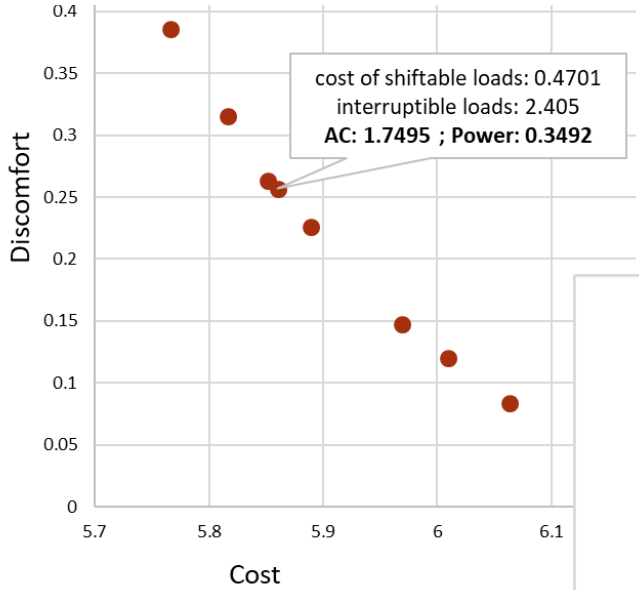
RESULTS MODEL 1



RESULTS MODEL 1

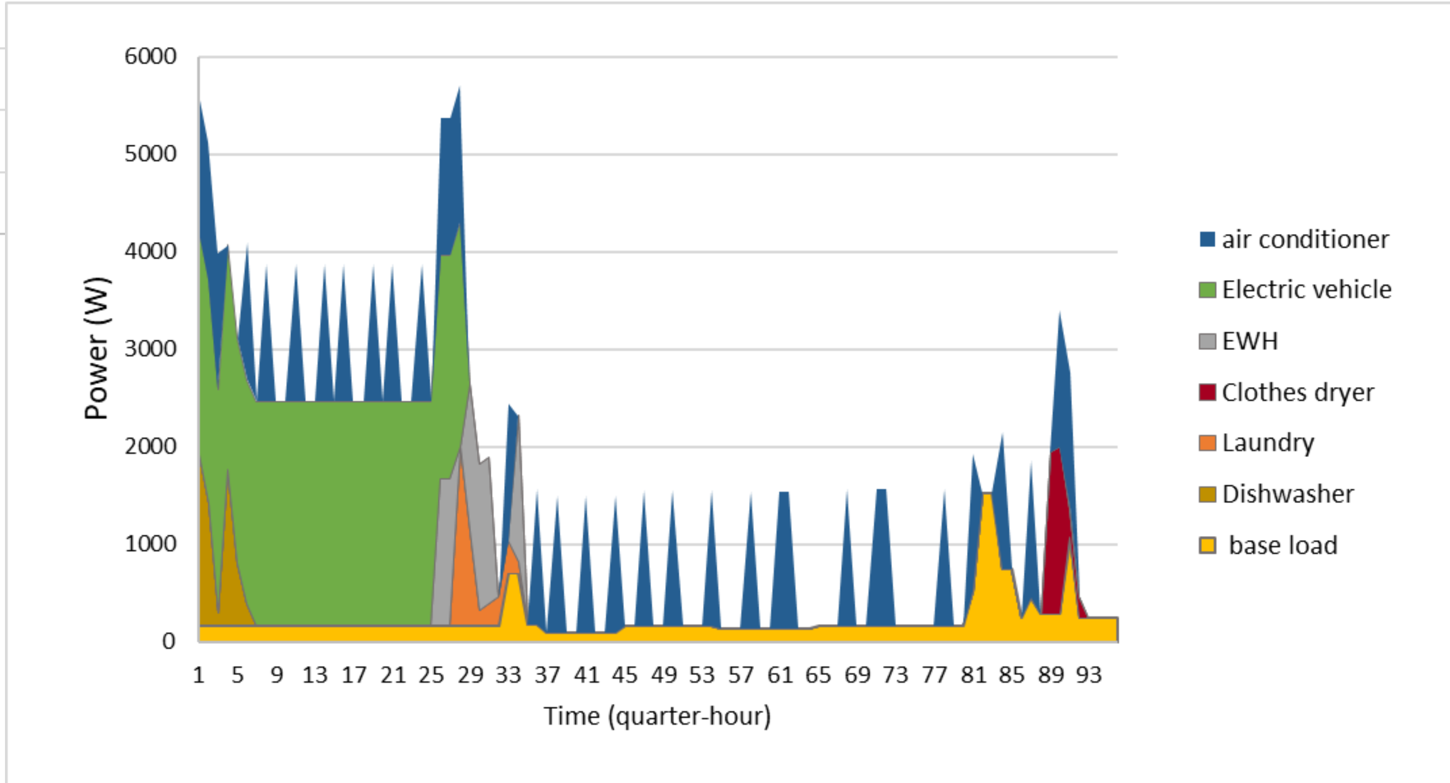


RESULTS MODEL 1

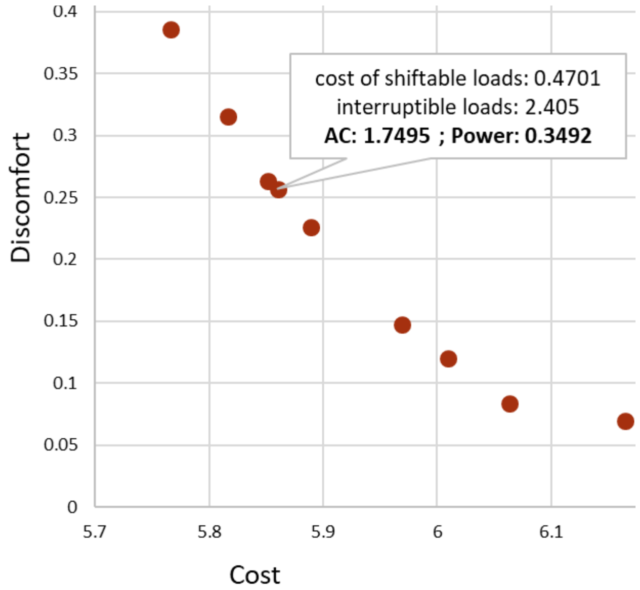


← weighted-sum with equal weights

Load diagram:

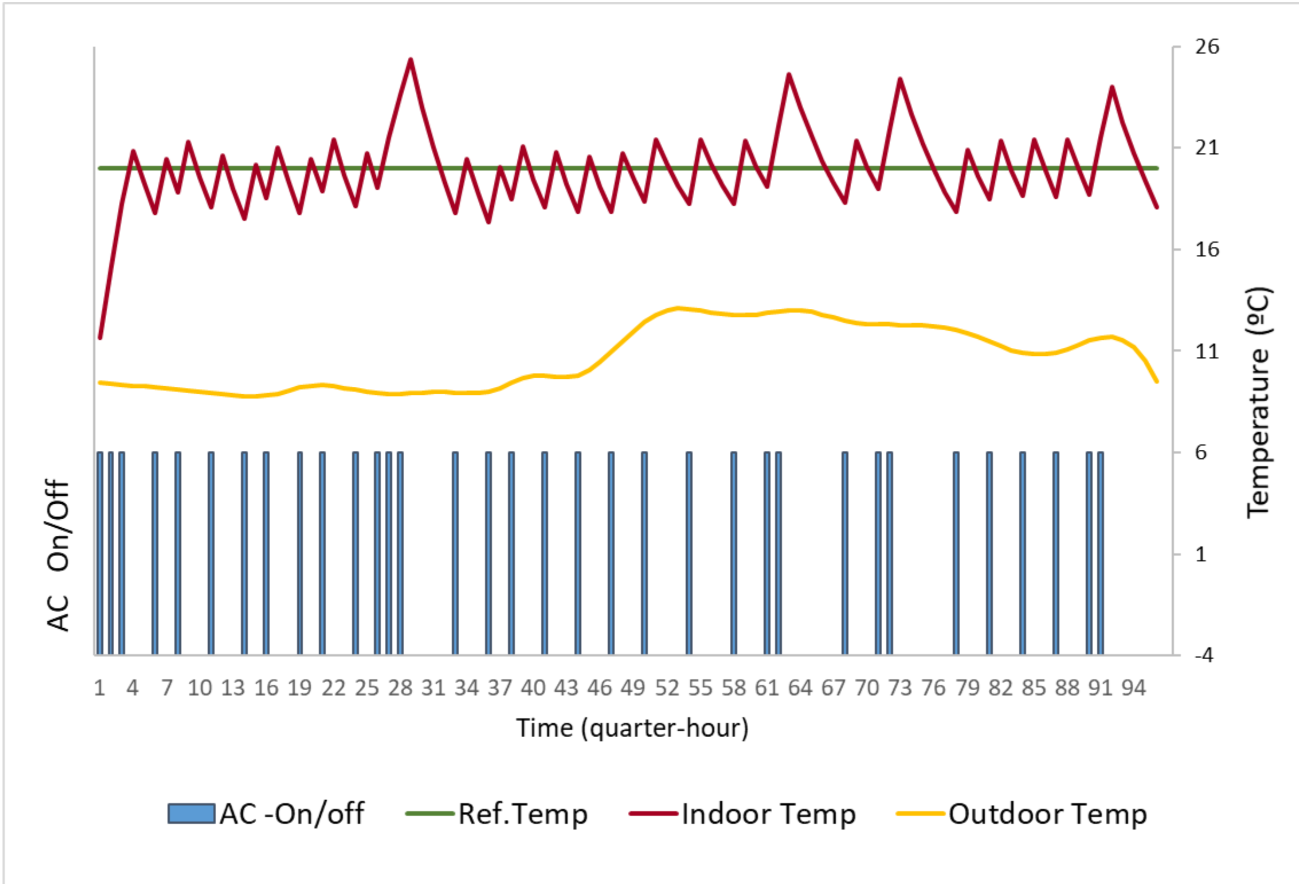


RESULTS MODEL 1



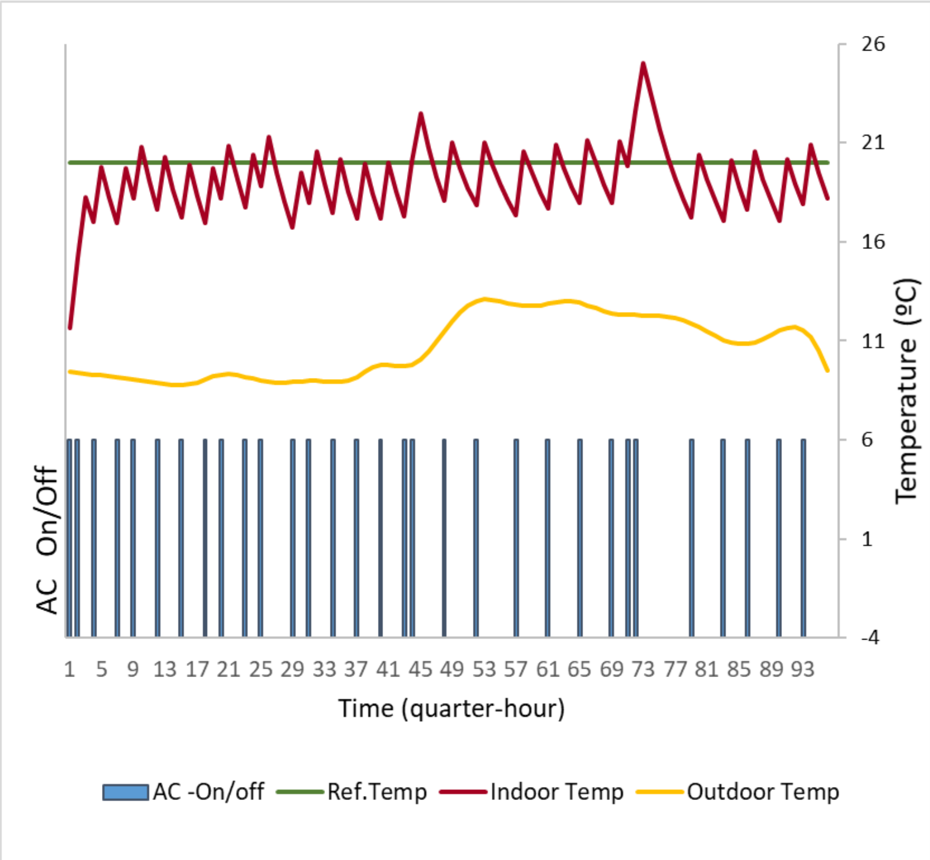
← weighted-sum with equal weights

Air conditioner operation and temperatures:

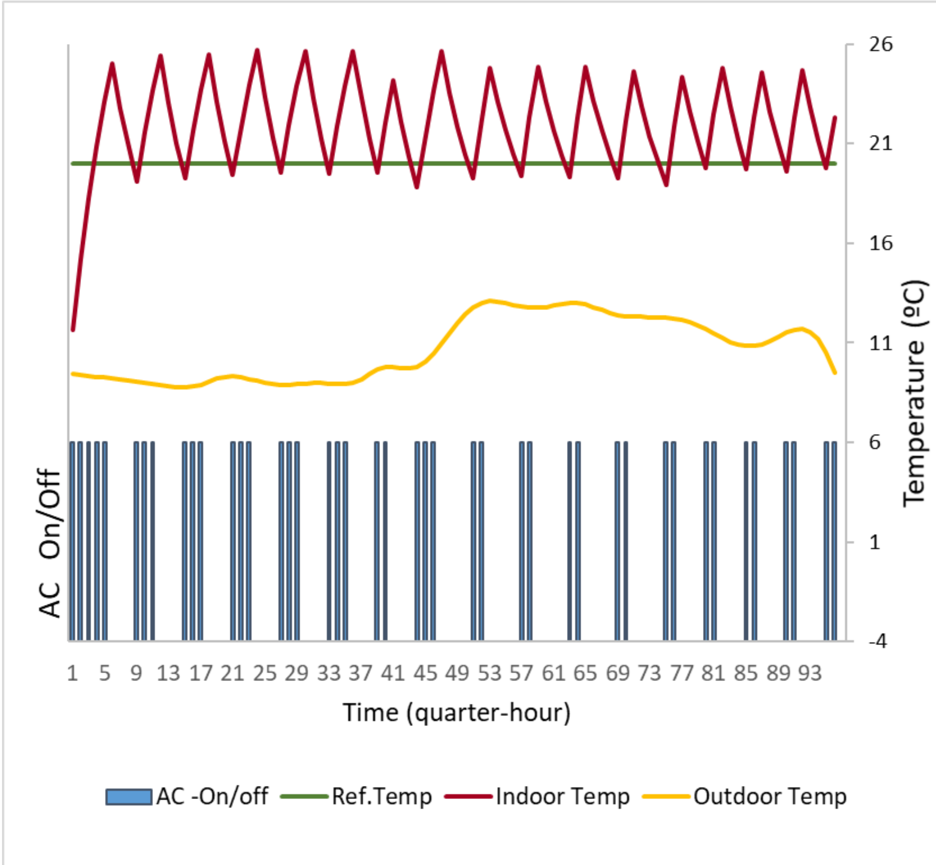


EXTREME NONDOMINATED SOLUTIONS: AIR CONDITIONER AND TEMPERATURES

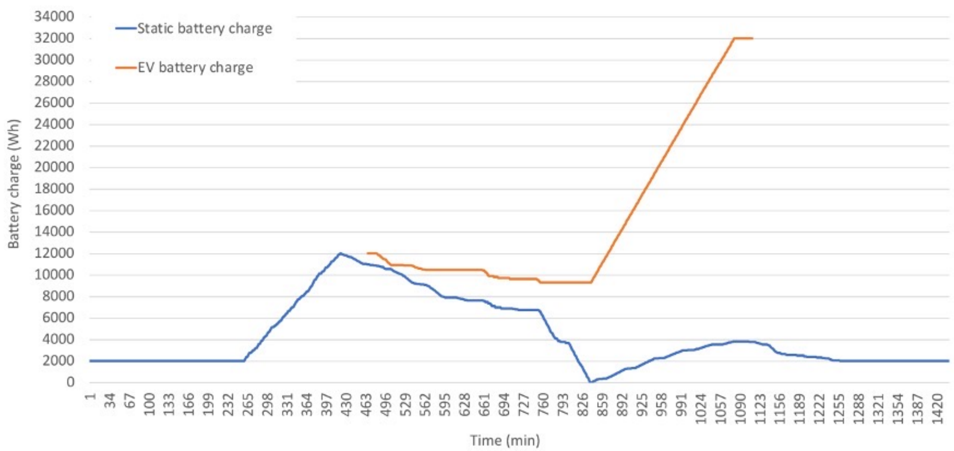
Minimum cost solution:



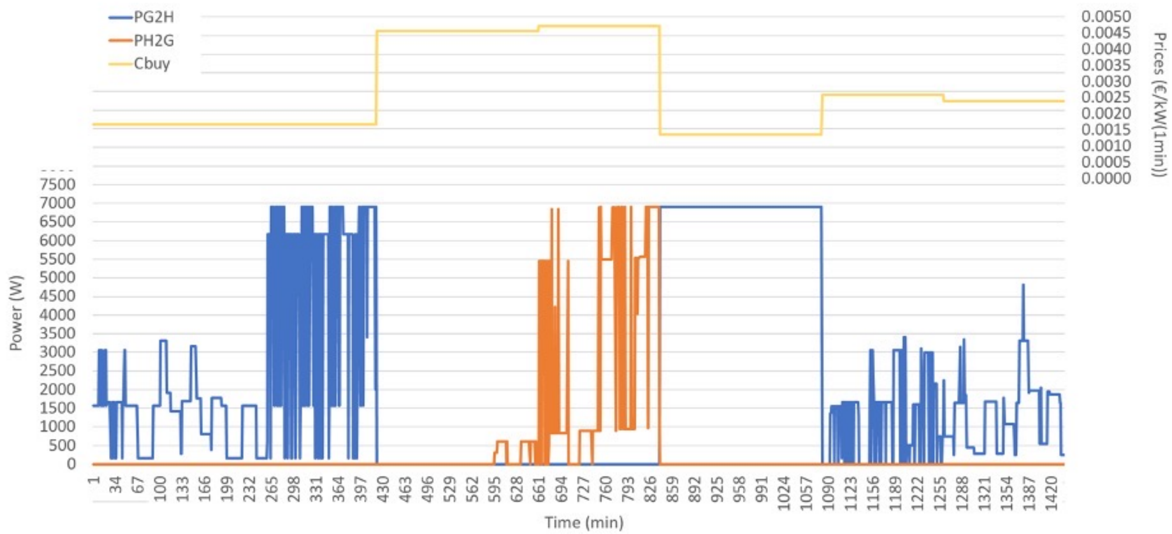
Minimum discomfort solution:



MODEL 2 (min cost with batteries and changes with the grid) EVOLUTION OF BATTERY CHARGE AND G2H / H2G FLOWS



Evolution of static ($E_{B,t}$) and EV ($E_{V,t}$) battery charge



Grid to home power (P_t^{G2H}) and home to grid (P_t^{H2G}) powers

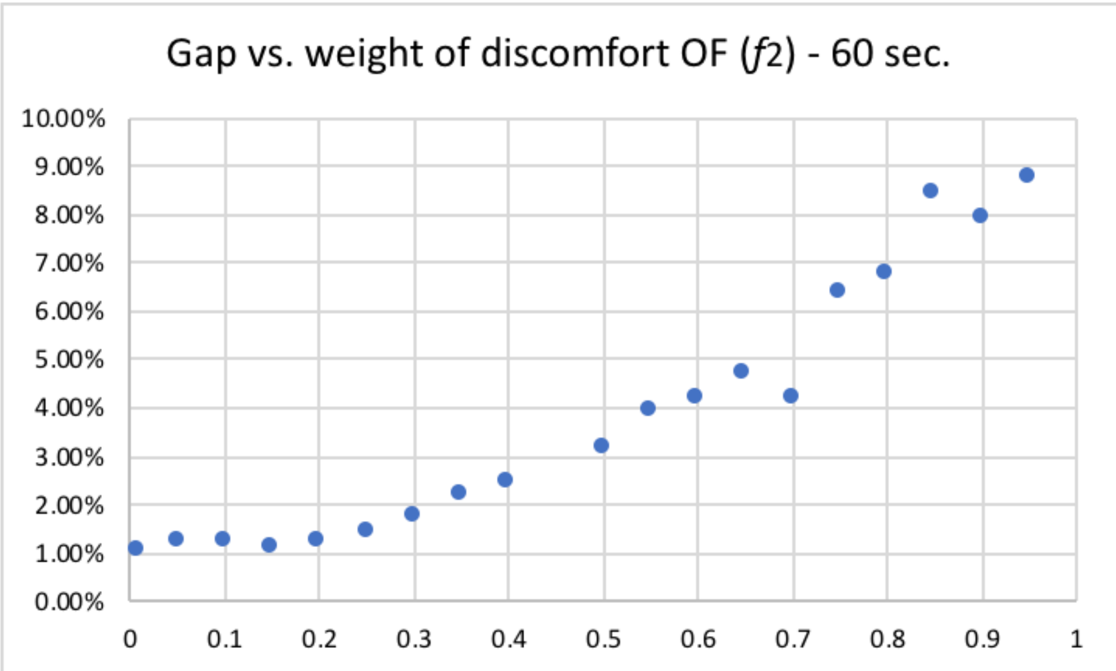
COMPUTATIONAL DIFFICULTIES

Shiftable and interruptible loads are easily dealt with by the solver.

The control of the AC system imposes a significant computational effort.

The MIP gap increases with the weight assigned to the discomfort objective in the bi-objective problem → 1% ~ 9% with a computational budget of 60 sec.

Computational budget of 5 min.: gap ~4-5%



PRACTICAL IMPLEMENTATION OF HEMS

H**ome energy management system (HEMS)** parameterized with the consumer's preferences, with communication capabilities to receive grid information (prices and other signals) and control appliances.

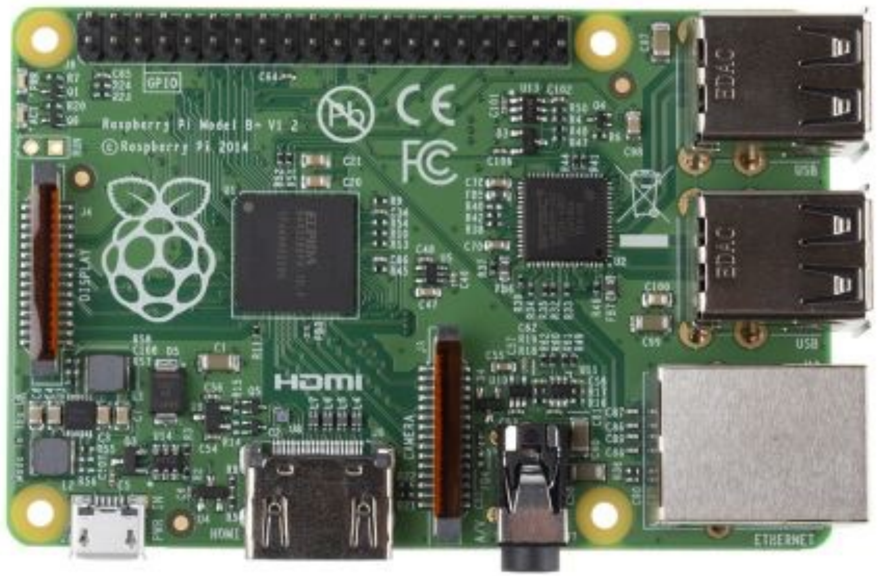
Raspberry Pi 3+ : Broadcom BCM2837B0 SoC 1.4 GHz 64-bit quad-core ARM Cortex-A53 processor, 512 KB shared L2 cache

Several OS can be installed in a MicroSD, MiniSD or SD card, depending on the motherboard and adapters available

Raspbian, Debian-based Linux distribution, and third-party Ubuntu, Windows 10 IoT Core, RISC OS

Size: 85.60 mm × 56.5 mm × 17[90] mm

$35: low-cost solution



CONCLUSIONS

- Dynamic (ToU) tariffs provide **price signal incentives** for consumers to engage in **demand response** by means of HEMS.
- **Bi-objective MILP model** to optimize demand response in face of dynamic tariffs → minimization of energy **costs** and minimization of the **discomfort** associated with changes regarding most preferred settings or time slots.
- For finer time discretization of the planning period the **model may not be solved to optimality** with a solver due to its combinatorial nature.
- Feasibility to **implement HEMS** on a low-cost computer.

Thank you

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