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Mi ZHANG

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Hub location routing problem for less than truckload shipments of freight transport providers

JURY

Président :	**M. Frédéric SEMET**, Professeur des Universités, Ecole Centrale de Lille
Rapporteurs :	**M. Dominique FEILLET**, Professeur, Ecole des Mines de Saint-Etienne **Mme Caroline PRODHON**, Maître de Conférence HDR, Université de Technologie de Troyes
Examinateurs :	**Mme Sibel ALUMUR ALEV**, Assistant Professor, University of Waterloo, Canada **M. Alain BERNARD**, Professeur des Universités, Ecole Centrale de Nantes-IRCCyN
Directrice de thèse :	**Mme Nathalie BOSTEL**, Maître de Conférence HDR, IUT de Saint-Nazaire
Co-directeur de thèse :	**M. Pierre DEJAX**, Professeur des Universités, Ecole des Mines de Nantes

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Résumé en français

L'objectif général de cette thèse est de proposer de nouveaux modèles pour la distribution des marchandises dans une organisation de type messagerie transportant des petits lots, inférieurs à un camion complet, (less than truckload shipments, LTL). Les collectes et les livraisons sont réalisées par des tournées de véhicules et sont consolidées dans des plateformes de transfert (ou hubs). En effet, avec la pression pour augmenter les performances des systèmes logistiques en termes de réduction des coûts et d'amélioration du niveau de service, le transport de marchandises par messagerie (B to B ou B to C) reçoit de plus en plus d'attention. Le transport des marchandises, en particulier le transport routier, est un élément essentiel dans l'environnement économique. Selon les statistiques du transport de marchandises de la Commission Européenne [1], 75.1 % du transport intérieur total de marchandises dans les États membres de l'EU (EU-28) a été réalisé sur les routes en 2012, estimé à près de 1575 milliards tonne-kilomètres (tkm). Dans le système logistique et le cadre de gestion de la chaîne d'approvisionnement (supply chain management, SCM), le transport routier de marchandises prend en charge les activités d'approvisionnement, de production et de distribution en déplaçant matières premières, produits semi-finis et finis d'une façon efficace et opportune. Comme tous les autres secteurs économiques, le transport routier de marchandises doit atteindre des niveaux de haute performance en termes d'efficacité économique et de qualité de service.

Normalement, le transport routier de marchandises peut être divisé en deux types basés sur les quantités de produits que les expéditeurs individuels peuvent charger dans un camion: transport par camions complets (full truckload shipments, FTL) et transport par camions incomplets (LTL) [5]. Le FTL implique le transport de grandes quantités de cargaison homogène d'un expéditeur donné dans un seul camion, tandis que le transport LTL concerne des petites quantités de cargaison collectées auprès des différents expéditeurs et consolidés sur des remorques

1. http://epp.eurostat.ec.europa.eu/statistics_explained/index.php/Freight_transport_statistics

à un terminal par l'entreprise de transport. Habituellement, un transporteur FTL est spécialisé dans le déplacement d'un type spécifique de fret et propose un transport personnalisé à partir d'une origine vers une destination unique directement sans aucun transbordement. Ainsi, il est souvent utilisé entre une origine et une destination avec une grande demande.

Toutefois, un transporteur LTL peut traiter des demandes dont les expéditions ne rempliraient pas un camion complet en termes de poids ou de volume, mais qui peuvent être consolidées avec d'autres marchandises dans un terminal, pour réduire les coûts et la pollution. De plus, dans le transport LTL, au lieu de traiter chaque demande entre origine-destination (O-D) directement, un transporteur collecte des cargaisons de diverses origines (i.e. des expéditeurs, des producteurs) pour l'expédition vers des destinations différentes (par exemple, les clients, les magasins de détail) via un ou plusieurs hubs. Les collectes peuvent être des allers et retours directs ou une tournée avec de multiples arrêts, en fonction de la demande de chaque origine. Après la collecte, la marchandise est triée et consolidée dans un hub soit pour un transfert vers un autre hub ou pour l'expédition vers les destinations soit directement, soit par tournées [23]. Dans certains cas comme les services postaux, les collectes et livraisons peuvent être effectuées simultanément dans un même camion. Dans d'autres cas par exemple les biens de consommation, les collectes et les livraisons sont organisées séparément. Cela correspond au le cas courant où les collectes de différents expéditeurs sont faites dans l'après-midi, alors que les livraisons vers des destinations différentes sont effectuées le lendemain matin, pour permettre le transport inter-hub pendant la nuit avec des camions pleins.

Pour les cas mentionnés ci-dessus, avec du transport LTL, le réseau appelé "hub et spoke", est plus complexes et difficile à organiser que les opérations de FTL, parce que la performance du système de LTL est non seulement liée à la distance entre les origines et les destinations mais est également dépend de la conception du réseau de hubs et l'efficacité des opérations de transport. Ainsi afin de concevoir un réseau de transport LTL efficace avec l'objectif de minimiser le coût total et satisfaisant aux exigences requises, les entreprises devront simultanément déterminer la localisation des hubs, la répartition des origines (chargeurs) et les destinations (récepteurs) sur les hubs, le routage des flux entre l'origine et la destination, ainsi que les tournées de collecte et livraison optimales au sein du réseau. Dans certains cas, la collecte et la livraison sont uniquement considérés comme des allers-retours directs entre spokes satellites ou nœuds non-hub et hubs. Les spokes correspondent aux origines et aux destinations des flux. Ceci est connu comme le problème de localisation de hub (hub location problem, HLP), qui se concentre sur le choix des emplacements de hubs et d'affectation des spokes. Toutefois, dans d'autres cas tels que le transport de biens de consommation, afin de réduire les coûts de transport entre spokes et hubs, des tournées avec multiples arrêts sont réalisées pour la collecte et la livraison dans ce réseau. Le problème de tournées de véhicules (vehicle routing problem, VRP) est combiné à l'optimisation des transports LTL. Ceci conduit à un nouveau problème d'optimisation connu comme le problème de localisation de hubs et tournées combinées (hub location-routing problem, HLRP), qui est le problème abordé de cette thèse. Son but est de minimiser le coût total du système LTL, y compris les coûts fixes pour établir les hubs, les coûts de transport inter-hubs , les coûts fixes des véhicules et les coûts des tournées pour les collectes et livraisons.

Pour illustrer le système de HLRP pour le transport LTL, un exemple pour le transport de marchandises générales est représenté sur la Figure 1, où les carrés, les cercles et les triangles représentent les hubs candidats, les origines des marchandises (fournisseurs) et les destinations (clients), respectivement. Les flèches en gras représentent les arcs de transfert connectant les hubs. Les lignes en pointillés représentent les arcs de collecte entre fournisseurs et hubs (routes de collecte), tandis que les lignes pleines simples représentent les arcs de livraison. Pour les tournées de collecte ou de livraison, il y a deux modes de routage possibles : la tournée avec un seul nœud telles que les routes $R1$ ou $R2$ et la tournée avec multiples nœuds comme les routes $R3$ ou $R4$.

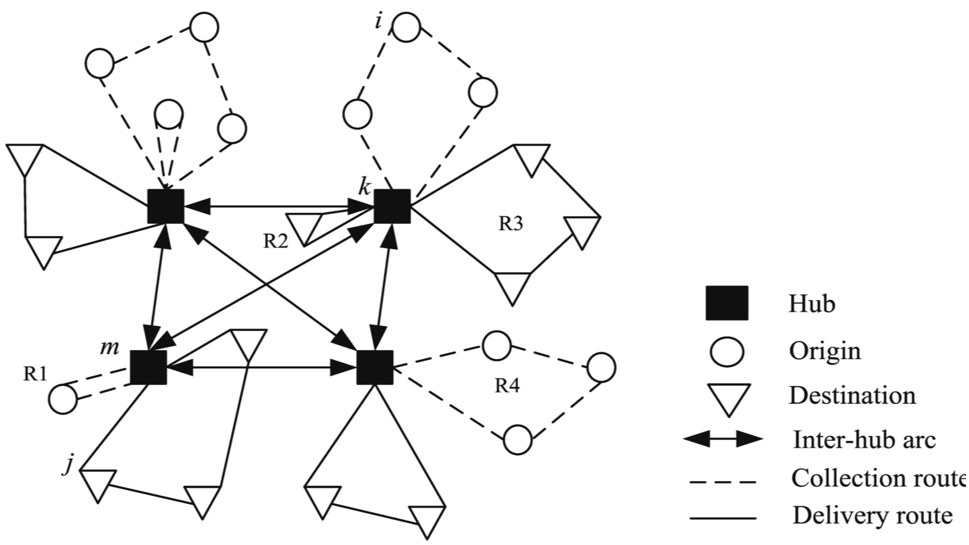

Figure 1: Le réseau de la HLRP pour le transport de LTL

Extension du HLP et du VRP, le HLRP est étroitement lié au problème de localisation et routage (LRP), et peut aussi être appelé le problème de localisation et routage entre plusieurs origines et plusieurs destinations (many-to-many location-routing problem, MMLRP) dans certaines recherches [66, 149, 179]. Le LRP classique ne considère que la partie livraison et optimise les coûts totaux de distribution et routages à partir des dépôts sélectionnés, en satisfaisant les demandes des clients. Les objectifs du LRP sont de déterminer l'emplacement des dépôts, l'affectation des clients aux dépôts, et la conception des routes de distribution (ou le collecte) associées aux dépôts. Contrairement au LRP, qui ne considère qu'un type de route, le HLRP traite les routes à la fois de collecte et de livraison séparément ou non. En outre, le HLRP prend en compte des échanges de flux entre paires d'origines - destinations et les connexions entre les hubs. Les principales différences du HLRP et des problèmes connexes susmentionnés peuvent être vues dans le Tableau 1. Le HLRP considère la plupart des facteurs de décision pour les transports LTL et peut être adapté aux autres problèmes connexes facilement. Par exemple, si le transport inter-hub n'est pas considéré, le HLRP peut être considéré comme deux LRPs.

Tableau 1: Les différences entre HLP, VRP, LRP et HLRP

Décision / Problème	Localiser le hub	Affecter les nœuds non-hub	Concevoir les tournées des véhicules	Considérer les flux entre origines et destinations
HLP	×	×		×
VRP			×	
MDVRP		×	×	
LRP	×	×	×	
HLRP	×	×	×	×

Dans la littérature, contrairement aux HLP et LRP, qui ont été très étudiés par la communauté de la recherche depuis plusieurs décennies (voir les articles de synthèse comme [176, 183]) , très peu de travaux ont abordé le HLRP directement. En outre, la plupart des études sur ce problème mettent l'accent sur les systèmes de services postaux dans lesquels collectes et livraisons peuvent être effectuées simultanément dans le même camion [40, 66, 180]. Il y a un manque de modèles et de méthodes de résolution traitant du HLRP pour les transports LTL de marchandises générales, où la collecte et la livraison se produisent à différents moments [149, 179]. Sur le développement de méthodes de résolution, la plupart d'entre eux ont proposé des heuristiques hiérarchiques et résolvent certains cas réels ou des instances de taille moyenne. Pour ces raisons, il semble plutôt prometteur pour développer des modèles et des méthodes efficaces pour résoudre ce cas général de HLRPs.

Comme mentionné ci-dessus, cette thèse se concentre principalement sur le problème de localisation de hubs et tournées combinées (HLRP) pour le transport LTL des marchandises, impliquant la localisation des hubs, l'affectation des nœuds satellites aux hubs, le routage des flux entre chaque origine et destination, et des tournées de collecte et de livraison optimales. L'objectif est de minimiser le coût total de transport du réseau de hub et satellites en développant des modèles et algorithmes spécifiques. Pour atteindre cet objectif, les problèmes suivants sont pris en compte et doivent être résolus:

(1) détermination du nombre et de la localisation des hubs parmi les candidats potentiels, ainsi que l'allocation de chaque nœud non-hub à un hub, et les routages des flux de toutes les origines vers les destinations.

(2) détermination des routes de service entre chaque entité de collecte ou livraison affectée à un hub donné. Les routes peuvent consister en un transport direct (entre le hub et un fournisseur ou un client donné) ou en une tournée locale avec plusieurs arrêts pour laquelle on doit décider de l'ordre de visite.

Pour résoudre les problèmes de recherche définis ci-dessus simultanément, nous développons un modèle stratégique, comprenant les décisions de localisation-affectation du HLP ainsi que la conception des tournées types qui pourraient être adaptées au niveau opérationnel. Des modèles mathématiques et des méthodes de résolution efficaces sont proposées pour résoudre le HLRP et sont évalués sur des instances inspirés de la littérature. Basée sur cette thématique de recherche, cette thèse est organisé comme suit:

Après une introduction, le *Chapitre 2* est consacré à un état de l'art sur le HLRP et à une revue de la littérature macroscopique sur les problèmes connexes, comme le HLP, le VRP et le LRP. Tout d'abord, nous avons introduit la définition, les caractéristiques, les variantes et les applications pour chaque problème connexe (HLP, VRP et LRP). Puis les formulations mathématiques et méthodes de résolution ont été résumées pour chaque problème, en particulier le HLP avec l'affectation unique, le VRP avec capacité (capacitated VRP, CVRP) et le LRP avec capacité (capacitated LRP, CLRP). Enfin, une comparaison entre les problèmes connexes et le HLRP est proposée, suivie d'une revue des travaux publiés sur le HLRP. Contrairement aux nombreux travaux relatifs aux problèmes connexes, le HLRP a reçu peu d'attention jusqu'à maintenant en particulier pour les applications au transport LTL de biens de consommation. À notre connaissance, seulement 7 travaux publiés (voir Tableau 2) ont abordé le HLRP et la plupart d'entre eux surviennent après l'année 2010. En outre, la plupart des papiers sur le HLRP traite des applications particulières dans lesquelles la capacité du véhicule n'est pas prise en compte, facilitant la collecte et la livraison combinées des produits dans la même tournée, comme c'est le cas dans le domaine des services postaux. Même si des heuristiques ou des méthodes exactes ont été proposées, seulement les petites et moyennes instances peuvent être résolues. De cette revue de la littérature, nous remarquons qu'aucune recherche n'a été menée sur le HLRP avec des collectes et des livraisons séparées, correspondant néanmoins à des applications réelles que pour des entreprises de transport spécialisée dans le transport de charges de moins de 3 tonnes, nécessitant ce type d'organisation pour réduire les coûts et respecter les délais de livraison (moins de 24h ou moins de 48h).

Tableau 2: Comparaison détaillée entre les travaux connexes sur le HLRP

Article	Characteristiques							
	Location	Allocation	Number of hubs	Hub constraint	Routing constraint	Solution	Application area	Size
Nagy et al. (1998)[149]	yes	single	not fixed	capacitated	length	Hierarchical heuristic	One instance	249
Liu et al. (2003)[130]	no	single+direct shipment	one hub	uncapacitated	length	Heuristic	Random instances	25
Wasner et al. (2004)[203]	yes	multiple+direct shipment	not fixed	capacitated	capacitated	Heuristic	Austria postal	10
Çetiner et al. (2010) [40]	yes	multiple	p hubs	uncapacitated	length	two-stage heuristic	Turkish postal	81
Camargo et al. (2013) [66]	yes	single	not fixed	uncapacitated	length	Benders decomposition	AP	100
Rodriguez-Martin et al. (2014) [180]	yes	single	p hubs	uncapacitated	number of nodes	B&C	CAB+AP	50
Rieck et al. (2014) [179]	yes	single+direct shipment	p hubs	uncapacitated	capacitated	Multi-start procedure +GA	timber-trade industry	140
Our research	yes	single	not fixed	capacitated	capacitated	MA and B&C	freight and postal	100

Ainsi, le développement d'un modèle général pour le HLRP avec les collectes et les livraisons distinctes et des méthodes exacte et approximative efficaces pour ce problème semblent être une avenue intéressante de recherche. Nous nous sommes inspirés de la littérature sur le HLP, LRP et VRP pour la modélisation de ce problème et le développement de méthodes de résolution. La revue de la littérature nous indique aussi la nécessité de développer des méthodes de résolution spécifiques et des métaheuristiques particulières pour résoudre ce problème.

Le *Chapitre 3* se concentre sur le développement de modèles pour le HLRP avec capacité et affectation unique (capacitated single allocation HLRP, CSAHLRP), qui est le problème central de recherche de cette thèse. Sur ce problème, chaque satellite doit être affecté à un seul hub et une seule route. Les transports directs entre les fournisseurs et les clients ne sont pas considérés; chaque flux peut être transporté à travers deux hubs au plus, et les véhicules utilisés sont homogènes et avec capacité. De plus,chaque hub potentiel est limité par une capacité. Suite à la description détaillée de ce problème, nous présentons deux modèles mathématiques pour le CSAHLRP comprenant une formulation avec une variable à 4 indices (Modèle 1) et l'autre avec une variable à 3 indices (Modèle 2). Dans le premier modèle, la variable Y_{ijkl} est utilisée pour désigner la fraction de flux du fournisseur i au client j via les hubs k et l. Le second modèle utilise une variable avec 3 indices Y_{ikl} représentant la fraction du flux total provenant du fournisseur i acheminée à travers les hubs k et l. Le Modèle 2 réduit évidemment le nombre de variables et de contraintes par rapport au Modèle 1 (comme représenté sur la Figure 2).

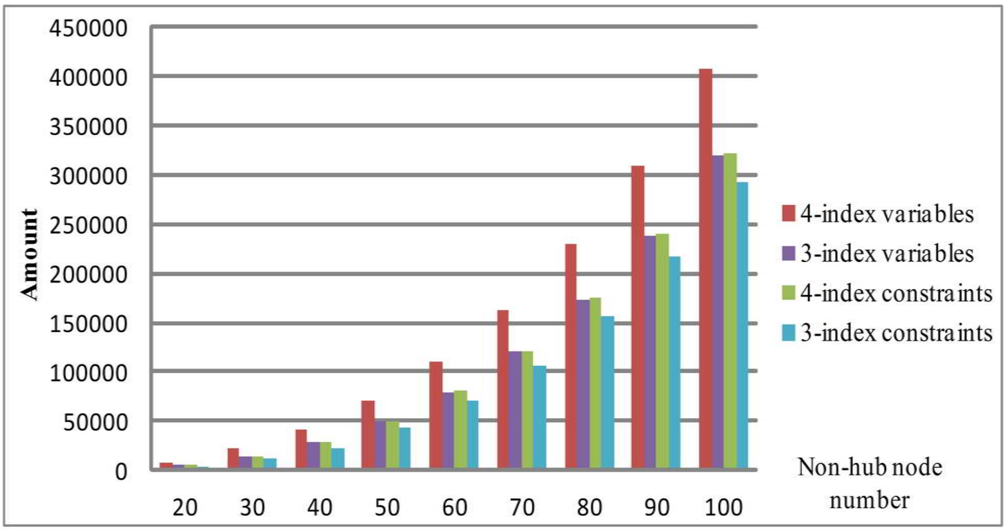

Figure 2: La comparaison de la taille du Modèle 1 et Modèle 2 avec 6 hubs potentiels

En outre, afin d'évaluer les deux modèles proposés à travers des expérimentations numériques, nous avons généré des jeux de données. Les valeurs des paramètres de coût sont générées sur la base des données des coûts de transport proposés par le Comité National Routier français (CNR) [2]. Pour les coordonnées des nœuds de hubs potentiels, fournisseurs et clients du réseau, nous avons utilisé les données de Australie Poste (AP) issues de la littérature [78] ((obtenus de *OR-Library* [3]). Toutes les instances produites sont divisées en trois groupes en fonction du nombre des hubs potentiels. Pour chaque groupe, nous avons distingué trois types d'instances

2. http://www.cnr.fr/en
3. http://people.brunel.ac.uk/ mastjjb/jeb/orlib/phubinfo.html

en fonction du niveau de capacité des hubs (petite, moyenne, grande).

Pour les petites et moyennes instances générées, nous comparons la complexité et la performance des deux modèles à partir des résultats de calcul obtenus par CPLEX dans une limite de 3 heures. Le Tableau 3 évalue les forces et les faiblesses de chaque modèle pour l'obtention de bornes supérieures et de bornes inférieures, la possibilité de trouver des solutions optimales et le temps de calcul pour atteindre la borne supérieure. Dans ce tableau, la colonne 1 définit chaque groupe d'instances. Les colonnes 2 et 3 montrent l'écart moyen avec la borne supérieure $Gap_{UB}\% = (UB_M - UB_{best})/UB_{best} * 100\%$ et l'écart moyen avec la borne inférieure $Gap_{LB}\% = (UB_{best} - LB_M)/UB_{best} * 100\%$, respectivement, où UB_M, LB_M représentent la borne supérieure, la borne inférieure obtenue par CPLEX sur chaque modèle, et UB_{best} représente la meilleure borne supérieure fournie par n'importe quel modèle. Puis la capacité à trouver des solutions à partir de chaque modèle pour toutes les instances de test est indiquée dans les colonnes 4 et 5. En outre, la dernière colonne T_{UB} indique le temps moyen pour atteindre les bornes supérieures correspondant à chaque groupe d'instances.

Tableau 3: Comparaison des résultats des deux modèles (Modèle 1 et Modèle 2)

Instance groupe	4-index (Modèle 1)					3-index (Modèle 2)				
	$Gap_{UB}\%$	$Gap_{LB}\%$	Optimal /all	No solution /all	T_{UB}	$Gap_{UB}\%$	$Gap_{LB}\%$	Optimal /all	No solution /all	T_{UB}
3 hubs potentiel	1.45	14.71	3/15	0/15	3730.10	0.02	**13.35**	5/15	0/15	4633.85
6 hubs potentiel	2.36	**18.05**	0/12	3/12	7502.80	0.00	18.57	0/12	3/12	6765.29
10 hubs potentiel	4.31	**25.15**	0/12	3/12	6172.18	0.00	27.91	0/12	3/12	8453.24
Moyenne totale	2.48	**19.31**	3/39	6/39	5801.69	**0.01**	19.94	**5/39**	6/39	6617.46

Tout d'abord, à partir de l'écart moyen avec la borne inférieure et la borne supérieure pour chaque groupe d'instances, on peut voir que le modèle avec 3 indices peut trouver de meilleures bornes supérieures que le modèle à 4 indices en trois heures de calcul par CPLEX, tandis que le modèle avec 4 indices peut fournir de meilleures bornes inférieures. A partir du nombre de solutions optimales trouvées par CPLEX sur la base de chaque modèle, le modèle à 3 indices a trouvé cinq solutions optimales contre trois solutions optimales pour le modèle à 4 indices. Il convient également de noter que CPLEX n'a pas trouvé de solutions réalisables aux problèmes de taille moyenne avec 6 et 10 hubs potentiels quelque soit le modèle. Au regard du temps nécessaire pour atteindre la borne supérieure avec CPLEX, les modèles montrent tous que ce problème est difficile à résoudre par un solveur commercial.

En outre, une analyse de la solution est proposée sur la base de différentes valeurs des paramètres permettant d'illustrer la conception du réseau de CSAHLRP pour le transport LTL. Un exemple des meilleures solutions obtenues avec l'instance 6-10-10 est illustré sur la Figure 3. Dans cette figure, les cercles, les triangles et les carrés représentent les fournisseurs, les clients et les hubs sélectionnés, respectivement. Les lignes pointillées, lignes solides et les lignes continues avec des doubles flèches représentent des arcs de collecte, des arcs de livraison et des arcs de inter-hub, respectivement.

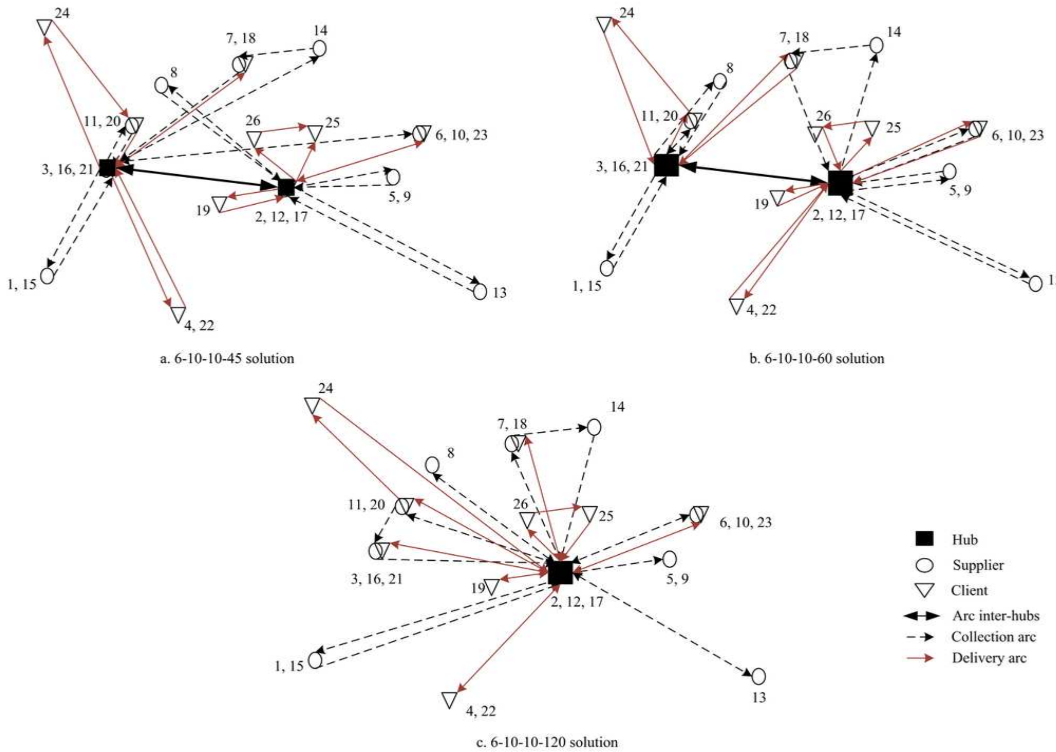

Figure 3: Illustration de la solution d'instances 6-10-10

De la Figure 3, nous pouvons voir la variété des décisions sur la localisation et le routage en fonction des capacités différentes des hubs. Par exemple, la meilleure solution pour l'instance 6-10-10-120 sur Fig.3 c ouvrira le hub numéro 2. Tous les fournisseurs/clients sont affectés à ce hub pour échanger le flux des marchandises par 7 circuits de collecte et 8 tournées de livraison. Cependant, pour l'instance 6-10-10-45 (voir Fig.3 a), avec deux hubs ouverts, il y a 8 tournées de collecte locales et 8 tournées de livraison locales depuis les hubs, y compris les tournées de nœuds uniques $2 \leftrightarrow 13$ et $3 \leftrightarrow 22$. Les détails sur les meilleures solutions pour l'instance 6-10-10 peuvent être vus dans le Tableau 4.

Ce chapitre fournit non seulement une base de calcul pour les chapitres suivants, mais montre également la difficulté de résoudre le HLRP avec CPLEX, et la nécessité de développer une

Tableau 4: Détails des meilleures solutions pour instance 6-10-10

H-I-J	Hub-Cap	Hub sélectionné	Collection tours	Livraison tours	Coût
6-10-10	45	2, 3	2-8-2, 2-9-2	2-17-2, 2-26-25-2	7613.94
			2-12-2, 2-13-2	2-23-2, 2-19-2	
			3-10-3, 3-14-7-3	3-21-3, 3-22-3,	
			3-11-16-3, 3-15-3	3-24-20-3, 3-18-3	
	60	2, 3	2-12-10-2, 2-9-2	2-23-2, 2-17-2	6828.25
			2-13-2, 2-14-7-2	2-19-2, 2-22-2	
			3-16-11-3, 3-8-3	2-25-26-2, 3-21-3	
			3-15-3	3-24-20-3, 3-18-3	
	120	2	2-10-2, 2-13-2	2-17-2, 2-22-2	6249.60
			2-11-16-2, 2-8-2	2-20-24-2, 2-25-26-2	
			2-9-2, 2-12-15-2	2-19-2, 2-18-2	
			2-7-14-2	2-23-2, 2-21-2	

métaheuristique ou un autre algorithme spécifique pour résoudre ce problème, en particulier pour les instances de grande taille.

Le *Chapitre 4* est consacré au développement d'une telle métaheuristique pour résoudre le CSAHLRP efficacement. Un algorithme mémétique (memetic algorithm, MA) est proposé. C'est une heuristique évolutionnaire hybride basée sur un algorithme génétique combiné à une procédure de recherche locale. Il s'est avéré très efficace pour résoudre les problèmes connexes et fournit des solutions fiables dans des applications réelles. Par exemple, dans le HLR, le VRP et le LRP, il a été appliqué avec succès. Cependant, il n'a pas été utilisé pour le HLRP. C'est pourquoi, nous avons choisi cette méthode pour résoudre le CSAHLRP. A chaque itération de cet algorithme mémétique (Figure 4), après avoir trouvé des solutions globales à travers les opérateurs génétiques (opérateurs de sélection, croisement et de mutation), une recherche locale itérative est exécutée pour améliorer les décisions de localisation et de routage pour la collecte/livraison jusqu'à ce qu'aucune amélioration ne soit plus trouvée. Toutes les étapes sont répétées jusqu'à ce que le critère d'arrêt soit satisfait. Après le test des paramètres, les critères d'arrêt dans ce MA sont définis par le nombre maximal de génération fixé à 200 ou si la meilleure solution n'est pas améliorée pendant 100 générations successives.

Pour le codage du chromosome dans cet algorithme mémétique, il est composé de deux sections : la section de localisation (location section) et la section de routage (routing section) (représentées sur la Figure 5). La première section montre les localisations des hubs sélectionnés et la seconde est composée d'une permutation des fournisseurs et des clients. Ensuite, chaque fournisseur ou client dans la section de routage est affecté au hub à la position corre-

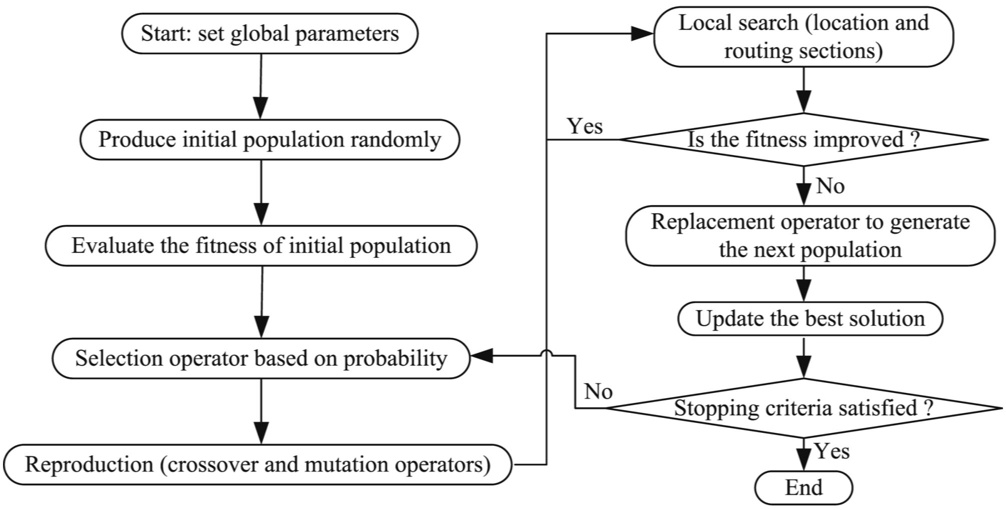

Figure 4: Le cadre de l'algorithme mémétique (MA)

spondante dans la section de localisation. Par exemple dans Figure 5, le fournisseur 4 est affecté au hub 1 et le client 11 est affecté au hub 2. Puis les tournées de collecte ou de livraison sont déterminées par l'ordre des fournisseurs ou des clients dans la section de routage et la capacité du véhicule. Il convient de noter que les fournisseurs et clients ne peuvent pas être attribués à une même tournée.

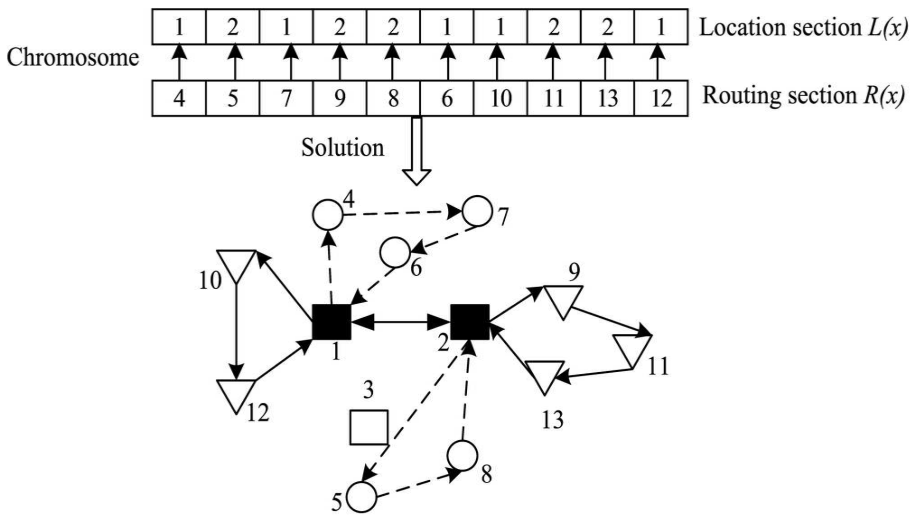

Figure 5: Un exemple du codage du chromosome pour le HLRP

La fonction d'évaluation de l'algorithme comprend non seulement les coûts totaux de solutions, mais également une pénalité si la capacité des hubs est dépassée pour la collecte et la livraison. Pour l'opérateur de sélection, la sélection avec roulette (the roulette wheel selection) est utilisée sur la base de l'équation de la probabilité suivante:

$$P(|k|) = \frac{2k}{PopSize(PopSize+1)} \tag{1}$$

où k est le $kime$ chromosome ordonné dans l'ordre décroissant des valeurs de la fonction d'évaluation dans la population, et $PopSize$ est la taille de la population. Cette probabilité est inversement proportionnelle à la valeur de la fonction d'évaluation. Ainsi, un individu avec une meilleure valeur de fitness dispose d'une plus grande probabilité d'être sélectionné en tant que parents ou d'entrer dans la prochaine génération.

Pour l'opérateur de croisement (voir Figure 6), un point de croisement (one-point crossover) classique est utilisé sur la section de localisation. Pour la section de routage, après avoir généré un point, la première partie de l'enfant est identique au parent 1 avant ce point. Pour le reste, l'enfant prend successivement le code du parent 2, sauf les parties de code qui existent déjà dans la première partie. A propos de l'opérateur de mutation (Figure 7), un hub de la section de localisation peut muter aléatoirement ; pour la section de routage, deux points sont générés tout d'abord, puis le point 1 (nœud 7) est retiré de sa position actuelle et inséré après le point 2 (nœud 10).

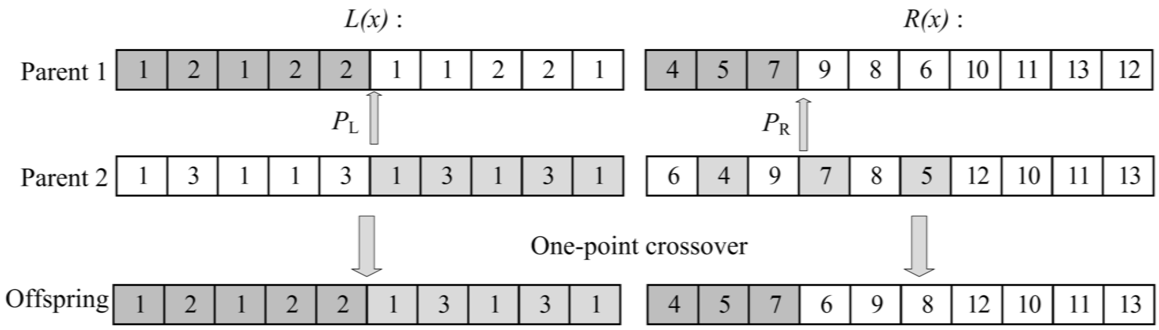

Figure 6: L'opérateur de croisement pour les sections de localisation et routage

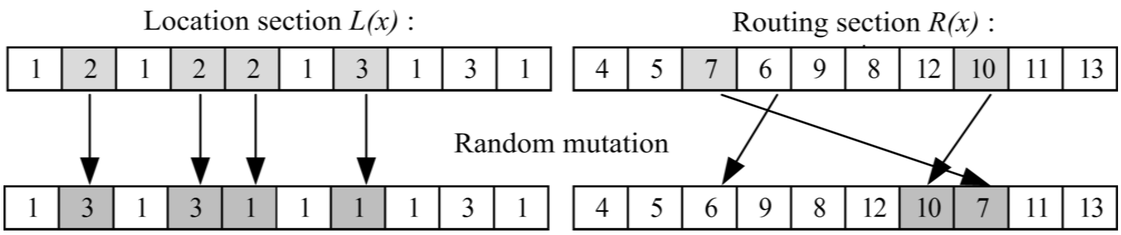

Figure 7: L'opérateur de mutation pour les sections de localisation et routage

Pour la recherche locale itérative dans ce MA, deux types d'opérateurs sont utilisés pour améliorer les décisions de routage de collecte et livraison simultanément. Ils incluent des opérateurs intra-routes et des opérateurs inter-routes. Pour les opérateurs intra-routes, les opérateurs de swap et 1-insertion sont utilisés. Puis le 2-opt* et le déplacement entre les différentes routes sont utilisés. Des opérateurs de perturbation sont utilisés pour améliorer les décisions de localisation-affectation à savoir les opérateurs de fermeture de hub, d'ouverture de hub et d'échange de hubs .

Pour évaluer le performance du MA proposé, de nombreuses expérimentations numériques sont effectuées sur la base des instances générées. Tout d'abord, avec les instances de petites et moyennes tailles générées, les meilleures valeurs objectif trouvées par le MA sont comparées avec la meilleure borne supérieure obtenue par CPLEX pour chaque taille d'instance. A partir de l'écart moyen de la borne supérieure $UB\%$ pour chaque groupe d'instances (Tableau 5), nous constatons que le MA peut atteindre toutes les solutions optimales trouvées par CPLEX. De plus, quand CPLEX ne peut pas trouver de solutions optimales, le MA peut trouver une meilleure solution que CPLEX (voir l'écart de la borne inférieure moyenne $LB\%$). Le MA améliore 18 meilleures solutions sur 33 instances de test avec une amélioration maximale de 10%. En outre, les temps de calcul du MA et de CPLEX pour arriver à la meilleure borne supérieure sont comparés dans la colonne 5 (T_{UB}) et dans la colonne 9 (T_{aver}). On remarque que, pour toutes les instances de test, le MA peut atteindre toutes les meilleures solutions dans un temps plus court que CPLEX. On constate que le MA peut résoudre les instances de petite et moyenne taille en une minute. En outre, dans les 3 heures, CPLEX n'a pas trouvé les solutions optimales pour les instances moyennes (vu de la colonne $Ttotal$). De plus, CPLEX n'a pas trouvé de solutions réalisables pour les instances de grandes tailles avec plus de 50 satellites. Ainsi, avec les expériences sur les instances de grande tailles résolues par le MA, nous constatons que le MA proposé résout efficacement toutes les instances et peut fournir des solutions réalisables pour les instances jusqu'à 100 satellites et 10 hubs potentiels. La figure 8 montre l'écart moyen de tous les tests avec le MA pour les différentes tailles d'instances. Les petits écarts moyens trouvés démontrent une bonne stabilité du MA proposé et illustre que cette méthode peut fournir des solutions prometteuses pour le CSAHLRP dans les transports LTL.

Récemment, une question a été posée dans [32], concernant la validation des économies d'échelle du fait de la concentration des flux dans les hubs: Les flux inter-hubs sont-ils supérieurs aux flux provenant des satellites? Par conséquent, sur la base des meilleures solutions obtenues par MA pour les instances avec grande taille, le Tableau 6 compare les flux inter-hubs et les flux entre satellites et hubs et valide le rôle de consolidation des hubs dans les transports LTL. Les deux premières colonnes présentent le nom de l'instance et la capacité des hubs. Les

Tableau 5: La comparaison des résultats entre CPLEX et le MA proposé

Instance groupe		CPLEX				MA			
Hub potentiel	Spokes	$UB\%$	$LB\%$	T_{UB}(s)	T_{total}(s)	$UB\%$	$LB\%$	T_{aver}(s)	T_{total}(s)
3	10	0.00	0.00	1.64	5.81	0.00	0.00	0.19	1.92
	20	0.00	1.46	415.69	6770.23	0.00	1.46	1.22	12.19
	30	0.00	13.52	6557.94	10800.00	-0.10	13.44	3.17	31.71
	40	0.00	24.20	6867.86	10800.00	-1.14	23.41	9.49	94.94
	50	0.00	27.68	9646.76	10800.00	-4.67	24.50	24.47	244.71
Moyenne		0.00	13.37	4697.98	7835.43	**-1.18**	**12.56**	**7.71**	**77.10**
6	20	0.00	6.97	4179.32	10800.00	0.00	6.97	2.49	24.89
	30	0.00	16.95	7025.19	10800.00	-0.09	16.87	9.94	99.45
	40	0.00	29.58	9164.35	10800.00	-3.45	22.45	22.45	236.49
Moyenne		0.00	17.83	6789.62	10800.00	**-1.18**	**15.43**	**11.63**	**120.27**
10	20	0.00	12.96	6723.72	10800.00	-1.14	12.01	3.37	33.73
	30	0.00	26.86	9258.68	10800.00	-1.55	25.71	15.76	157.61
	40	0.00	38.49	9377.31	10800.00	-9.50	31.98	43.27	432.66
Moyenne		0.00	26.11	8453.24	10800.00	**-4.06**	**23.24**	**20.80**	**208.00**
Moyenne totale		0.00	19.68	6292.59	10434.51	**-1.97**	**16.25**	**12.35**	**124.57**

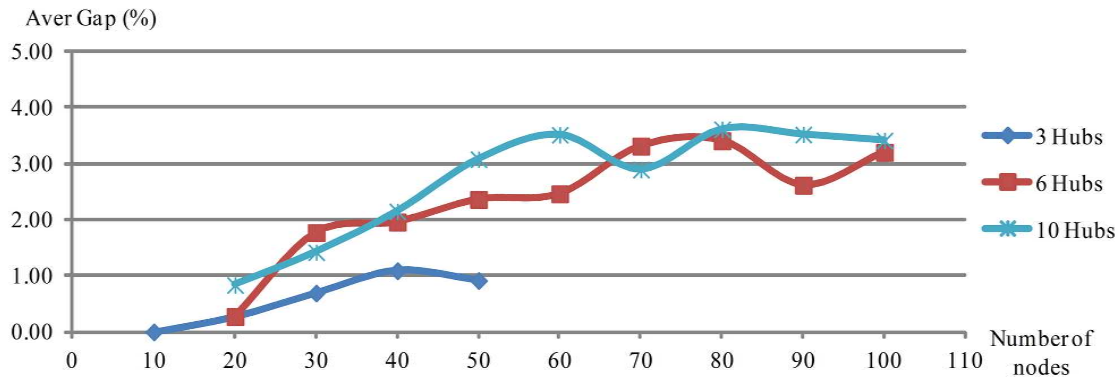

Figure 8: L'écart moyen de le MA en fonction de le taille du problème (nombre de nœud)

colonnes 3 et 4 fournissent respectivement le nombre d'arcs inter-hubs et de routes locales dans la meilleure solution pour chaque instance. En outre, il indique également le flux minimal entre hubs $Minflow_{hub}$ et le flux maximal des routes locales $Maxflow_{route}$ dans les colonnes 7 et 8, respectivement. La dernière colonne donne le pourcentage moyen $(\%)$de nœuds non-hubs avec un flux plus grand que le flux inter-hub. Il montre que, dans les meilleures solutions, le flux des arcs inter-hubs est toujours plus grand que le flux des routes locales, et qu'il est également plus grand que chaque flux provenant de satellite. Ces tests démontrent l'intérêt des hubs pour consolider les flux et l'efficacité du transport inter-hubs pour les compagnies de transport LTL.

Bien que le MA améliore la borne supérieure obtenue par CPLEX, il faut améliorer également la borne inférieure. Par conséquent, une méthode exacte basée sur l'algorithme de branchement et coupes (branch-and-cut algorithm, B&C) est proposé dans le chapitre 5 pour résoudre

Tableau 6: La comparaison des flux sur les arcs inter-hubs et les routes locales

Instance	Γ_k	Hub arcs	Routes	$Flow_{hub}$	$Flow_{route}$	$Minflow_{hub}$	$Maxflow_{route}$	Average %
6-40-40	90	6	36	26.58	13.03	22.36	14.97	0%
	135	2	36	53.23	13.06	42.21	14.97	0%
6-50-50	105	6	42	25.93	12.88	19.08	14.91	0%
	150	2	42	65.54	12.87	56.37	14.98	0%
10-40-40	90	6	36	26.58	13.03	22.36	14.97	0%
	135	2	36	53.23	13.06	42.21	14.97	0%
10-50-50	105	6	43	29.07	12.60	21.71	14.96	0%
	150	2	41	60.98	13.19	55.03	14.95	0%

optimalement le CSAHLRP et améliorer les bornes inférieures. Nous proposons tout d'abord une nouvelle formulation adaptée du Modèle 1 pour le problème traité. Dans cette nouvelle formulation, la variable de décision pour les parties de routage local est modifiée et devient une variable entière x_{kij}. Elle représente le nombre de fois où l'arc bord (i, j) est visité par une route à partir du hub k. Elle peut être égale à 0, 1, 2 en fonction du type d'arc. La fonction objectif minimise toujours le coût total. Ensuite, pour les contraintes, les principaux changements concernent les parties de routage pour la collecte et la livraison à savoir les contraintes de capacité des véhicules, les contraintes de conservation de flux et les contraintes de cohérence pour chaque hub.

Puis, afin de renforcer la borne inférieure de la nouvelle formulation, certaines inégalités valides sont proposées et utilisées dans cet algorithme. La première famille d'inégalités valides est appelées inégalités valides simples (*Sim*). Ils renforcent les liens entre les variables de routage et les variables d'affectation. Elles montrent que n'importe quel nœud doit être visité par une route à partir d'un hub sélectionné. De toute évidence, ces inégalités sont de taille polynomiale. En plus des inégalités valides simples, plusieurs inégalités valides de taille exponentielle dérivées des problèmes connexes sont utilisées dans la méthode exacte. Le premier grand groupe correspond à celles dérivées du VRP. Si on réduit tous les hubs à un grand hub fictif, la partie de routage pour la collecte ou la livraison dans le HLRP correspond au CVRP. Ensuite, les inégalités valides pour le VRP peuvent être adaptées au HLRP. La première famille de ce groupe concerne les contraintes d'arrondi de capacité des tournées (rounded route capacity constraints, *RRC*). Elles sont linéaires et garantissent les contraintes de capacité des véhicules et évitent les sous-circuits de collecte et de livraison. Le deuxième groupe est appelé inégalités généralisées Multistar (generalized large multistar inequalities, *GLM*). Elles sont valables pour des sous-ensembles de fournisseurs et de clients dans le HLRP, lorsque leur taille n'est pas inférieure à 2. D'autres inégalités sont inspirées du LRP. La première famille correspond aux inégalités valides de capacité de hub (hub capacity valid inequalities, *HC*). Elles renforcent les contraintes de capacité de hubs dans les processus de collecte et de livraison, et sont valables pour les sous-ensembles de fournisseurs et de clients lorsque le flux total est supérieur à la capacité du hub. Elles indiquent que les nœuds dans ce type de sous-ensemble ne peuvent pas être totalement servis par les routes à partir d'un seul hub. Un deuxième groupe issu du LRP est appelé inégalités valides de degré de hub renforcé (strengthened hub degree valid inequalities, *SHD*). Elles renforcent les contraintes de degré de hub lorsque le flux total d'un sous-ensemble est inférieur à la capacité du véhicule. Elles sont valables lorsque les distances respectent les inégalités triangulaires. La dernière famille appelée inégalités valides de co-circuit désagrégées (disaggregated co-circuit valid inequalities, *DCoC*) est valable pour un sous-ensemble de fournisseurs et de clients et un ensemble d'arcs F impair et sans tournées d'aller-retour.

Sur la base du cadre classique de l'algorithme de B&C, une méthode exacte pour le CSAHLRP est présentée en Figure 9. A l'étape d'initialisation, deux cas sont possibles. Le premier initialise l'algorithme sans solution initiale et la borne supérieure est réglée sur l'infini, nous l'appellerons pure B&C. Dans l'autre cas une solution initiale du MA initialise l'algorithme et la valeur objectif de cette solution est utilisée comme borne supérieure. Nous l'appelons B&C+MA. Les autres parties de la méthode sont traitées par CPLEX. Lors de l'identification des inégalités violées, l'approche de séparation pour les inégalités valides simples est appelée en premier, puis l'algorithme de séparation pour les contraintes de *RRC*, les inégalités *SHD* et *GLM* est appliqué séquentiellement. Si aucune inégalité violée pour les types *SHD* et *GLM* n'est trouvée, la routine de séparation pour les *HC* est appelée. Enfin, le procédé de séparation pour les contraintes *DCoCC* est appelé si aucune inégalité ci-dessus ne se trouve violée. Dans cette stratégie de séparation, le problème de la séparation des inégalités valides simples peut être résolu d'une manière simple. Toutefois, pour d'autres inégalités valides de taille exponentielle, un algorithme randomisé glouton (a greedy randomized algorithm [94]) est appliqué pour générer les contraintes de *RRC*, ainsi que les inégalités valides *SHD*, *GLM* et *HC*.

Des expérimentations numériques ont été menées sur les instances de petites et moyennes

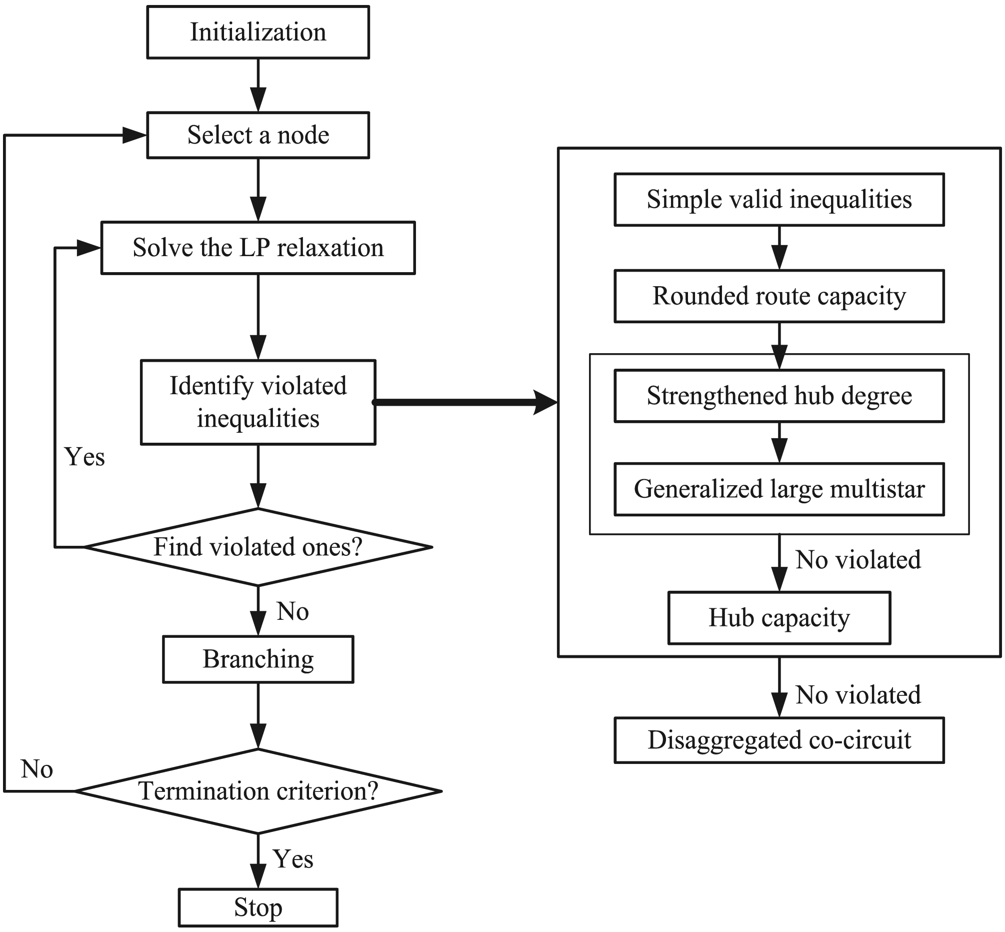

Figure 9: Le cadre de B&C proposé pour la CSAHLRP

tailles pour évaluer les effets de chaque famille d'inégalités valides sur le renforcement de la borne inférieure de la nouvelle formulation. Les résultats sont comparées en utilisant l'écart de la borne inférieure au nœud racine obtenu en ajoutant chaque famille d' inégalités valides à la fois pour la nouvelle formulation (voir Tableau 7). La colonne 3 donne l'écart moyen de la borne inférieure obtenue sur la formulation originale sans inégalités valides. Les autres colonnes indiquent les valeurs pour chaque famille d' inégalités valides et pour chaque taille d'instances. A partir de ce tableau, nous constatons que les contraintes *RRC* sont les inégalités les plus efficaces, suivies par les inégalités *GLM* et *SHD*. En outre, la formulation initiale peut être améliorée par l'ajout des inégalités valides, à l'exception des inégalités *DCoCC*. Donc, cette famille d'inégalités valides ne sera pas utilisée dans l'expérience finale.

Ensuite, nous évaluons la performance de B&C+MA en comparant les résultats avec ceux obtenus par le B&C sans la solution initiale du MA et CPLEX pour résoudre des instances de petite et moyenne tailles (Tableau 8). Ils sont comparés à l'aide de l'écart moyen de la borne inférieure $LB\%$, la possibilité de trouver des solutions (Optimal/all et No solution/all) et le

Tableau 7: La comparaison de l'effet de chaque famille des inégalités valides

Instance groupe	Spokes	Original	Sim	RRC	HC	SHD	DCoCC	GLM
3 hubs potentiel	10	10.45	9.19	2.67	9.97	5.99	10.45	9.50
	20	19.71	19.59	9.81	19.71	18.75	19.71	15.84
	30	20.09	20.01	14.46	20.09	16.61	20.09	18.80
	40	27.45	27.42	17.25	27.11	27.40	27.45	24.27
	50	27.13	27.09	20.24	27.13	26.86	27.13	22.84
Moyenne		20.97	20.66	**12.88**	20.80	**19.12**	20.97	**18.25**
6 hubs potentiel	20	20.02	19.53	11.63	20.02	18.87	20.02	17.31
	30	19.48	19.44	16.18	19.48	16.94	19.48	18.11
	40	26.01	25.94	22.07	25.19	24.77	26.01	24.98
	50	26.59	26.56	21.69	26.59	26.46	26.59	25.13
Moyenne		23.03	22.87	**17.89**	22.82	**21.76**	23.03	**21.38**
10 hubs potentiel	20	20.11	20.07	15.24	20.11	19.39	20.11	19.59
	30	21.10	21.07	16.95	21.10	19.78	21.10	20.51
	40	26.16	25.37	22.05	26.01	25.79	26.16	23.76
	50	25.82	25.21	21.98	25.82	25.30	25.82	25.47
Moyenne		23.30	22.93	**19.05**	23.26	**22.57**	23.30	**22.33**
Moyenne totale		22.32	22.04	**16.32**	22.18	**20.99**	22.32	**20.47**

temps T_{UB} pour atteindre la borne supérieure par chaque méthode et chaque groupe d'instances. Tout d'abord, à partir des valeurs de $LB\%$, on peut voir que le B&C+MA améliore la borne inférieure par rapport au pure B&C et CPLEX pour tous les groupes d'instances. De plus, le B&C trouve 17 solutions optimales alors que CPLEX ne trouve que trois solutions optimales sur les 39 instances de test. Nous avons également trouvé que le pure B&C n'a pas trouvé de solutions réalisables pour les instances avec 50 spokes et 10 hubs potentiels. Enfin, en analysant le temps nécessaire pour atteindre la borne supérieure par chaque méthode, on remarque que le B&C+MA est plus rapide.

Tableau 8: Comparaison des résultats de B&C+MA, pure B&C et CPLEX

Instance groupe	B&C+MA				Pure B&C				CPLEX			
	$LB\%$	Optimal /all	No solution /all	T_{UB}(s)	$LB\%$	Optimal /all	No solution /all	T_{UB}	$LB\%$	Optimal /all	No solution /all	T_{UB}(s)
3 hubs potentiel	1.15	8/15	0/15	710.15	1.95	8/15	0/15	3631.62	13.84	3/15	0/15	3730.10
6 hubs potentiel	3.02	5/12	0/12	40.20	4.49	5/12	0/12	6049.94	17.93	0/12	3/12	8470.31
10 hubs potentiel	3.38	4/12	0/12	1597.40	3.69	4/12	3/12	5916.44	20.70	0/12	3/12	6172.18
Moyenne totale	**2.52**	17/39	0/39	**782.58**	3.37	17/39	3/39	5199.33	17.49	3/39	6/39	6124.20

Pour les grandes instances résolues par le B&C+MA, les meilleurs résultats obtenus sont comparés avec les meilleures valeurs objectif fournies par le MA pour tous les groupes d'instances.

Après avoir comparé l'écart moyen de la borne supérieure, on constate que bien que l'algorithme B&C proposé n'a pas résolu toutes les instances optimalement, il trouve 20 nouvelles meilleures solutions sur 69 instances par rapport au MA De plus, l'écart moyen de la borne supérieure pour toutes les instances est négatif. Cela indique une amélioration de l'objectif avec le B&C algorithme proposé, et reflète également que la solution initiale du MA est très proche de la meilleure solution trouvée par toute méthode utilisée dans cette thèse.

Le *Chapitre 6* traite de l'application du HLRP dans les systèmes de services postaux. Tout d'abord, nous avons analysé les caractéristiques individuelles de cette application et avons présenté une formulation mathématique du HLRP dans les systèmes de services postaux où la collecte et la livraison à un nœud sont opérées simultanément. Un exemple de réseau pour cette application est représenté en Figure 10, où les carrés et les cercles représentent les nœuds de hubs établis et les nœuds non-hub, respectivement. Les lignes pleines sont les arcs inter-hubs, les lignes pointillées représentent les arcs des tournées locales. Ainsi, afin d'optimiser ce réseau avec le coût total minimum, on doit déterminer la localisation des nœuds de hubs, la répartition des nœuds non-hub, ainsi que les tournées locales visitant chaque localisation de nœuds satellites pour la collecte et la livraison. En outre, les hypothèses suivantes sont prises en compte dans le modèle:

(1) Chaque nœud non-hub doit être affecté à un hub et doit être servi par une seule tournée locale (affectation unique);

(2) La quantité totale de flux affecté à un hub ne peut pas dépasser sa capacité, à la fois pour le flux de collecte et également le flux de livraison (problème avec capacité);

(3) Chaque tournée locale est limitée à un nombre maximal de nœuds visité q, incluant le nœud hub;

(4) Chaque tournée locale doit commencer à un hub et revenir au même hub;

(5) Le flux entre deux nœuds origine-destination peut passer à travers deux hubs au plus.

Afin de résoudre des instances de grandes tailles dans les systèmes de services postaux, l'algorithme mémétique proposé dans chapitre 4 est adapté à ce problème. Après un grand nombre d'expérimentations sur la base des instances inspirées du jeu de données de AP, nous avons analysé les effets des différents paramètres sur les solutions, comme le niveau de capacité

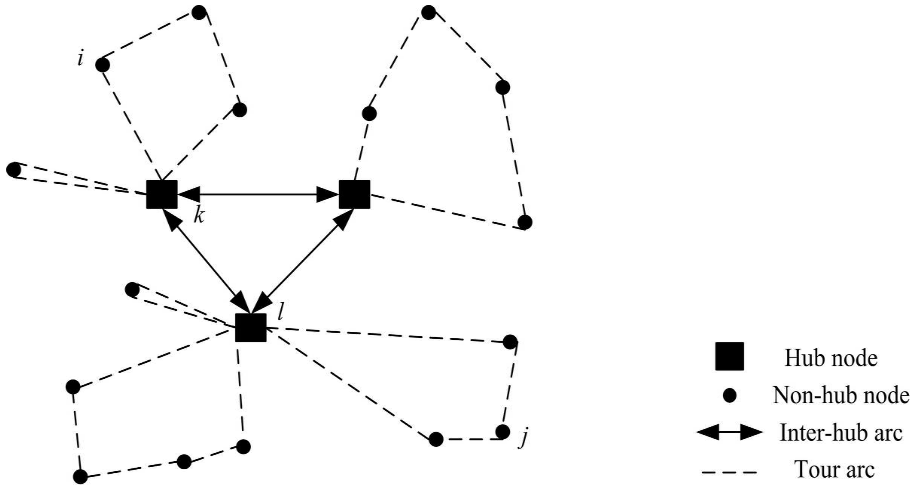

Figure 10: Le réseau de HLRP dans le système de service postal

des hubs, le nombre de hubs potentiels et la proportion du coût de routage dans le coût total. En particulier, la structure des coûts de chaque solution avec différentes valeurs de paramètres est analysée. La Figure 11 montre les changements de chaque élément de coût pour l'instance 6-10 quand la proportion de coût de routage (λ) augmente. On observe que le coût d'affectation (coût de manutention) a une plus grande proportion dans le coût total même quand λ augmente. En outre, les solutions ne présentent pas de changements évidents quand λ est égal à 1 et 100 d'une part, ainsi que $\lambda = 500$ et $\lambda = 1000$ d'autre part.

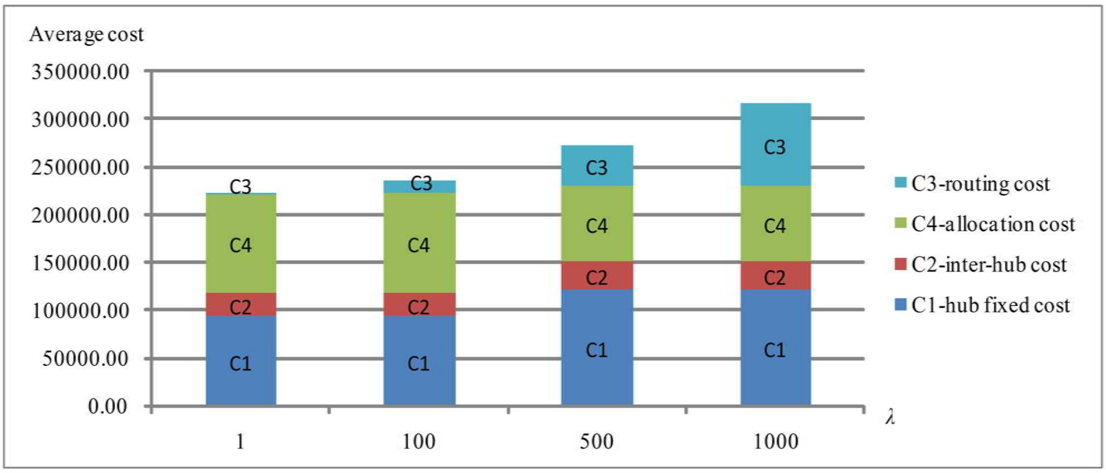

Figure 11: La structure des coûts pour instance 6-10 avec différentes valeurs de λ

Enfin, les résultats obtenus sur des instances de test pour $\lambda = 100$ et $\lambda = 500$ prouvent que le MA surpasse CPLEX en termes de qualité de la solution et de temps de résolution. Le Tableau 9 compare, pour différentes valeurs de λ , l'écart moyen de la borne supérieure $UB\%$, l'écart moyen de la borne inférieure $LB\%$ et le temps moyen T_{UB} (s) pour trouver la meilleure borne supérieure pour chaque groupe d'instances. Il se trouve que le MA peut obtenir des solutions optimales pour les instances de petite taille et améliorer la plupart des meilleures solutions de CPLEX dans un temps le plus court quand les solutions optimales sont introuvables.

Tableau 9: La comparaison des résultats entre CPLEX et le MA adapté pour les systèmes de services postaux

λ	Instance groupe	CPLEX			MA		
		$UB\%$	$LB\%$	T_{UB} (s)	$UB\%$	$LB\%$	T_{UB} (s)
100	3 potential hubs	0.00	1.05	8117.69	-0.18	0.87	3.11
	6 hubs potentiel	0.00	7.44	9955.60	-4.25	4.34	503.45
	10 hubs potentiel	0.00	10.85	9930.51	-7.72	4.79	1063.18
	Moyenne	0.00	6.45	9334.60	**-4.05**	**3.33**	**523.25**
500	3 hubs potentiel	0.00	2.27	7586.77	-0.07	2.20	3.32
	6 hubs potentiel	0.00	3.27	7213.88	-1.09	2.24	9.38
	10 hubs potentiel	0.00	4.84	7267.63	-2.45	2.55	22.38
	Moyenne	0.00	3.46	7356.09	**-1.20**	**2.33**	**11.69**

Enfin, l'efficacité et la stabilité du MA est vérifiée par de nombreuses expérimentations sur toutes les instances produites et avec $\lambda = 100$. On voit sur la Figure 12 que l'écart moyen entre le meilleur résultat obtenu et le résultat moyen du MA est inférieur à 5.00%. En outre, pour toutes les instances, lorsque le nombre de nœuds est supérieur à 50, l'écart moyen est relativement stable. Cela démontre également que le MA peut fournir des solutions prometteuses pour le HLRP dans les systèmes de services postaux avec jusqu'à 100 nœuds satellites et 10 hubs potentiels.

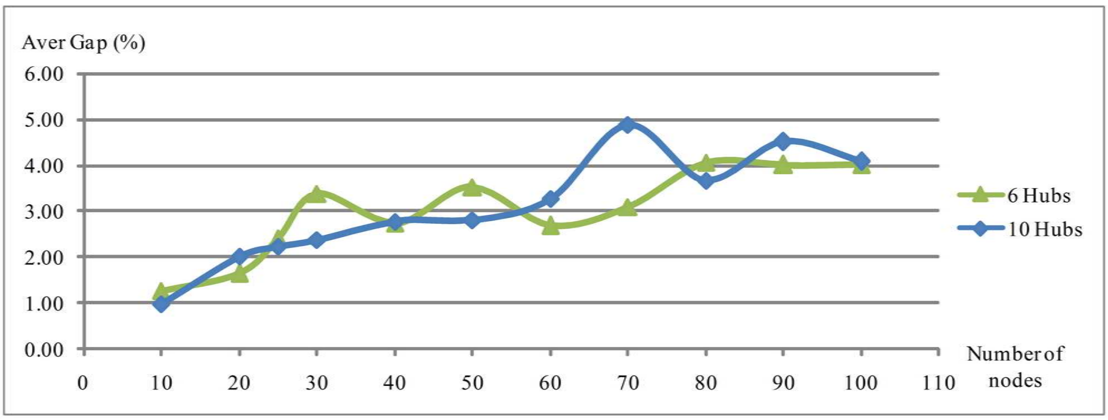

Figure 12: La tendance de l'écart moyen selon la taille du problème pour les instances postales

Enfin, la conclusion résume les principales contributions de la thèse et définit quelques perspectives pour les futurs travaux de recherche. Le travail de recherche principal et les contributions théoriques de cette thèse sont résumés comme suit:

(1) Un état de l'art sur HLRP est donné sur la base des travaux publiés, ainsi qu'une revue de la littérature des problèmes connexes, i.e. le HLP, le LRP et le VRP. Les caractéristiques individuelles, les modèles mathématiques classiques et les principales méthodes de résolution de chaque problème sont résumés. La principale contribution est d'analyser les relations et les différences entre les problèmes connexes et de suggérer des directions pour l'analyse de HLRP par rapport aux contraintes du problème, méthodes de résolution et domaines d'application.

(2) De nouveaux modèles sont proposés pour le HLRP avec affectation unique et capacité avec des processus de collecte et de livraison séparés ou non, respectivement. Deux formulations de programmation linéaires générales sont consacrées à une résolution par CPLEX pour les instances de petite et moyenne tailles et pour donner un aperçu des résultats du CSAHLRP avec différentes valeurs de paramètres. Une formulation avec une variable de flux à 3 indices a été proposé pour mettre en œuvre un algorithme de branchement et coupes pour résoudre le CSAHLRP. Un autre modèle mathématique est dédié au HLRP dans les systèmes de services postaux où la collecte et la livraison sont opérées simultanément dans une même tournée de véhicule. Ce chapitre est principalement consacré à l'analyse des solutions et à l'évaluation de différentes valeurs de paramètres.

(3) Un algorithme mémétique (MA) est proposé pour résoudre le CSAHLRP pour le transport de biens de consommation et pour les systèmes de services postaux. Il combine une procédure de recherche locale itérative (ILS) dans le cadre d'un algorithme génétique (GA) pour trouver des solutions réalisables de bonne qualité dans un temps compétitif. Les résultats obtenus sur des instances de petites et moyennes tailles, par rapport aux résultats de CPLEX prouvent que le MA proposé surpasse CPLEX en termes de meilleures solutions et de temps de calcul. En outre, cette approche nous permet de trouver efficacement de bonnes solutions réalisables pour des instances de grande taille avec un maximum de 100 nœuds satellite.

(4) Un algorithme de branchement et coupes (B&C) est développé pour résoudre le CSAHLRP pour le transport de marchandises générales s'appuyant sur certaines familles d'inégalités valides, qui renforcent le programme linéaire relaxé (LP). En outre, la meilleure solution de l'algorithme mémétique est importé sous forme de la solution initiale au niveau du nœud racine du B&C proposé. Les résultats montrent une bonne performance de notre B&C pour résoudre les petites et moyennes instances de manière optimale jusqu'à 30 nœuds satellites et 10 hubs potentiels. En outre, il donne d'importantes améliorations sur les bornes inférieures obtenues par CPLEX. Les résultats sur des instances de grande taille montrent que le B&C proposé peut trouver quelques nouvelles meilleures solutions par rapport à celles obtenus par le MA.

(5) De nouvelles instances pour le CSAHLRP sont générées à partir du jeu de données de Australie Poste (AP), ainsi qu'une véritable base de données des coûts issus du Comité National Routier français (CNR). De plus, pour chaque méthode proposée, un grand nombre d'expérimentations ont été menées permettant le réglage des paramètres, les comparaisons des résultats et des évaluations de performance.

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1

Introduction

1.1 Background and motivation

The overall goal of this thesis is to propose new models for goods distribution within the framework of less than truckload (LTL) shipments, when collections and deliveries are organized through vehicle routing and consolidated at freight terminals. Indeed with the pressure to increase the performance of logistics systems in terms of reducing costs and improving service levels, LTL freight transportation (for B to B or B to C shipments) has been receiving more and more attention. Freight transportation, especially road freight transport, is an essential element of the economic environment. According to freight transport statistics from the European Commission [1], 75.1% of the total inland freight transport in the EU Member States (EU-28) was transported over roads in 2012, which was estimated to be close to 1575 billion tonne-kilometer (tkm). In the logistics system and the supply chain management framework, road freight transport supports procurement, production and distribution activities by moving raw materials, semi-finished and finished products in an efficient and timely way. Like all other economic sectors, road freight transportation must achieve high performance levels in terms of economic efficiency and service quality [59].

Normally, road freight transportation can be divided into two types based on the amounts of cargoes that individual shippers can load in a truck: full truckload shipping (FTL) and less than truckload (LTL) shipping [5]. FTL shipping involves the transportation of large amounts of homogeneous cargo from a given shipper in a single truck, while LTL shipping relates to the transportation of small freight collected from various shippers and consolidated onto trailers at a terminal by the transport firm. Usually, a FTL carrier specializes in moving a specific kind of freight and offers customized transportation from an origin to a single destination directly without any transshipment. Thus, it is often used between origins and destination with a large demand.

However, an LTL carrier can handle demands which shipments would not fill one full truck in terms of weight or volume capacity but that can be consolidated with other goods at a terminal, to reduce costs and pollution. Then, in the LTL industry, instead of handling each demand between origin-destination (O-D) pairs directly, a carrier collects cargoes from various origins

1. http://epp.eurostat.ec.europa.eu/statistics_explained/index.php/Freight_transport_statistics

(i.e. shippers, producers) for shipment to different destinations (i.e. clients, retail stores) via one or several hubs. The collections can be operated in a straight-and-back way or a multiple-stop tour, depending on the demand of each origin. After collection, the freight is sorted and consolidated in a hub terminal either for additional line-hauls to another hub terminal or for shipment to the destinations either directly or through delivery tours [23]. In some cases such as postal services, collection and deliveries may be done simultaneously in one truck. In other cases such as consumer goods, collections and deliveries are organized separately. This corresponds for example to the common case where collections from various shippers are made in the afternoons and while deliveries to different destinations are made in the next morning, to allow inter-hub transportation overnight with fully loaded long-haul trucks.

For the cases mentioned above, both of the LTL networks, called hub-and-spoke networks, are more complex and difficult to organize than FTL operations, because the performance of the LTL system is not only related to the distance between origins and destinations but is also dependent on the design of the network of hub terminals and the efficiency of transportation routing operations. So in order to design an efficient LTL shipment network with the objective of minimizing the total cost and meeting the required demands, companies need to simultaneously determine the location of the hubs, the allocation of origins (shippers) and destinations (receivers) to the hubs, the routing of flow between origins and destinations, as well as the optimal collection and delivery routes within the network. In the past, the design of collection and delivery routes through this kind of network has often been considered by means of straight-and-back modes between spokes and hubs. This is knows as the hub location problem (HLP), which focuses on the location of hubs while it may lead to the use many local transportation links. Nowadays, in order to deal with more accurate models and obtain more realistic solutions in a hub-and-spoke network, some managers and researchers take into account the local tour planning between spokes and hubs, especially for the postal service cases [40, 180, 203]. This problem is known as the Hub Location-Routing Problem (HLRP), combining the hub location and vehicle routing problems, which are interdependent in real life. It has become a new and popular topic in the area of transportation network design. For this reason, this dissertation addresses the HLRP, which goal is to minimize the total cost of the LTL system, including fixed costs to establish hubs, inter-hub transportation costs, and collection/delivery routing costs.

Besides being an extension of the hub location problem, the HLRP is closely related to the location-routing problem (LRP), which it is also called the many-to-many location-routing problem (MMLRP) in some researches [66, 149, 179]. The classical LRP assumes that customers have only delivery demand and it optimizes the distribution through routes from the depots. The goals of the LRP are to determine the location of the depots, the allocation of customers to depots, and the design of the distribution (or collection) routes associated with the depots, and which corresponds to the Vehicle Routing Problem (VRP). Differently from the LRP, which only considers one type of route, the HLRP deals with both collection and delivery routes separately or not. In addition, the HLRP takes into account exchanges of flows between O-D pairs and connections between hubs.

In the literature, in contrast to the HLP and the LRP, which have been the focus of the research community for several decades (see review papers in [176, 183]), only very few works have directly addressed the HLRP. Moreover, most of the studies on this problems arise after 2010 and focused on postal service systems in which collection and delivery may be done simultaneously in the same truck [40, 66, 180]. However there is a lack of models and solution methods addressing the HLRP for LTL shipments of general freight transport providers, where collections and deliveries occur separately [149, 179]. Regarding the development of solution methods, most of them proposed hierarchical heuristics and solved some real cases or medium size instances. For these reasons, it looks rather promising to develop models and efficient methods for solving this general case of HLRPs.

1.2 Research problem and outline of the thesis

As mentioned above, this dissertation focuses on the hub location-routing problem for less-than-truckload shipments, involving the location of hubs, the allocation of non-hub nodes, the routing of flows between each origin and destination and the optimal collection and delivery routes. The objective is to optimize the total operating cost of the hub-and-spoke network by means of models and algorithms development. To achieve this goal, the following problems are addressed:

(1) determination of the number and location of the hubs among potential candidates, as well as the allocation of each non-hub node to a hub and the itineraries of flows from all origins to destinations.

(2) determination of the service routes between each collection or delivery entities allocated to a given hub. The routes can consist in a direct shipment (between the hub and a given supplier or client) or in a multi-stop local tour for which we have to decide the visiting order.

To address the above defined research problems simultaneously, we develop a strategic model comprising the location-allocation decisions of the HLP as well as the design of representative tours that could be adapted at the operational level. Mathematical models and efficient solution methods are proposed to solve the HLRP and are evaluated on instances inspired from the literature. Based on this research road-map, this dissertation is organized as follows, following this first introductory Chapter:

Chapter 2 is devoted to a state of the art on the hub location-routing problem and a macroscopic literature review on the related problems including the hub location problem, the vehicle routing problem and the location-routing problem. A particular attentions is paid to the features, mathematical models, exact and heuristic solutions methods for each related problem. Finally, a review of the HLRP is provided, based on the current published researches and discusses different model, constraints, solving methods and application areas. The main purpose of this chapter is to show the limitation of existing researches on hub location-routing problems and the necessity of developing new models and methods for solving various situation of this problem, which allows us to define the goal and the research problems of the thesis.

Chapter 3 focuses on the development of models for the capacitated single allocation hub location-routing problem (CSAHLRP), which is the central research problem of this thesis. Following a detailed description of this problem, we present two mathematical models for the CSAHLRP including a formulation with a 4-index variable and another with a 3-index variable. In order to perform a computational study with a commercial solver, a generation process of HLRP instances is described based on related problems from the literature and the real data base from Comité National Routier (CNR) [2]. Particular attention is paid to the comparison of the complexity and the performance of the two models based on generated small and medium size instances. We discuss the strengths and weaknesses of each model according to computational results. In addition, a solution analysis is given based on different parameter values to provide some insights into the network design of the CSAHLRP. This chapter not only provides a computational foundation for the following chapters, but also shows the difficulty of solving the hub location-routing problem with a commercial solver and the necessity of developing a metaheuristic or other specific algorithm to solve large instances.

2. http://www.cnr.fr/en

Chapter 4 is devoted to the development of such a metaheuristic for solving the capacitated single allocation hub location-routing problem efficiently. A memetic algorithm is proposed, based on a genetic algorithm (GA) combining an iterative local search (ILS) procedure. For the coding of the chromosomes, we used a two-dimensional array to represent the location and routing informations. For the iterative local search, different operators for the hub location and vehicle routing sections are implemented sequentially to improve the solutions obtained at each iteration of the GA. Many computational experiments are conducted on the generated instances to evaluate the performance of the proposed MA, including tuning of the main parameters. The results obtained on small to medium instances have been compared with the ones from the CPLEX commercial solver to show the strengths of our MA in terms of finding best solutions in acceptable computational time. In addition, the results obtained on large instances demonstrate that this MA is able to provide promising and reliable solutions for the CSAHLRP with up to 100 non-hub nodes.

Chapter 5 presents an exact method based on the branch- and-cut algorithm (B&C) for the CSAHLRP. We propose firstly a new mathematical model for the CSAHLRP based on a three-index vehicle flow model of the LRP [173]. Then some valid inequalities derived from the VRP or the LRP are introduced to strengthen the LP relaxation of this formulation. In the proposed B&C algorithm, a separation algorithm for each family of valid inequalities is presented as well as a branching strategy. In addition, at the root node of the B&C algorithm, the best solution from the memetic algorithm is used as an initial solution and its objective value is considered as the upper bound of the optimal solution. Computational experiments have been conducted on available instances to evaluate the effects of each family of valid inequalities on strengthening the LP relaxation of the new formulation. In addition, we compare the results on small to medium instances with ones obtained by CPLEX and one particular B&C without initial solutions, and also investigate the whole performance of the B&C algorithm compared to the results obtained by the MA based on the large instances. The results demonstrate a good performance of our B&C algorithm for the small and medium instances and reveal the importance of a good initial solution in the B&C algorithm.

Chapter 6 deals with the application of hub location-routing problem to postal service systems where the collection and delivery at each node location are operated simultaneously in the same truck. A mathematical formulation is presented here based on the main features of this application and adapted from our previous formulation. Our memetic algorithm is adapted to solve this problem. A large number of computational experiments are completed, based on instances inspired from the Australian Post data set [78] to analyze the solutions with different parameter values and evaluate the performance of the MA for solving the postal system cases. This also prove the efficiency and adaptability of the MA to variant of the HLRP.

Finally, *Conclusion* summarizes the main contributions of the thesis and defines some perspectives for future research.

2

Literature review

In the past decades, the design and optimization of transportation networks have received an increasing interest from the scientific community and this interest corresponds to challenging problems from industry. Based on the concerned research of this thesis, this chapter provides a state of the art for the hub location-routing problem (HLRP), as well as an extensive literature review on closely related problems, such as the hub location problem, the location routing problem and the vehicle routing problem. Firstly, section 2.1 describes in detail each related problem, involving their definition, variants, modeling approaches and different solution methods found in the literature. Then section 2.2 presents a review for the HLRP based on the current published researches, highlighting and discussing the models, solution methods and applications. Finally the section 2.3 provides the conclusion of this chapter with a short summary.

2.1 Related problems

As mentioned in the first chapter of this thesis, the hub location-routing problem considers the decisions needed for the design of logistic service network for less-than-truckload (LTL) shipments, concerning the choice of consolidation terminals, the allocation of each origin/destination point and the routing plan of flows over the network. These decisions are related to those of three classical problems: the hub location problem (HLP), the vehicle routing problem (VRP) and the integrated location-routing problem (LRP). In this section, a literature review for each of these problems is presented. We first give a detailed description of the HLP with its definition, classical models and exact and heuristic solution methods. Then we summarize the main variants of the VRP, the mathematical models and the algorithms found in the literature. Finally a review of the LRP is presented including its features, models and main solution methods. A final section present a review of the existing works on the HLRP and we conclude about our research agenda for addressing this problem.

2.1.1 The hub location problem

Introduction

As an important research area in location theory, the hub location problems (HLPs) involve the location of hub facilities and the allocation of non-hub nodes to hubs in order to route the flows between many origins and many destinations. Application areas concern many transportation systems involving commodities such as mail, people, digital signal or information flows. Hubs are facilities devoted to flow consolidation, sorting, transshipment and switching centers to replace the traditional direct connections between all origin-destination (O-D) nodes with fewer and indirect transportation links [31]. In order to satisfy origin to destination demands, instead of transferring the flows directly, concerned commodities are concentrated into hubs from many different origins, sorted and then re-routed to different destinations directly or via another hubs. Usually, the hubs themselves are fully interconnected and the links between non-hub nodes (spokes) and a hub are direct arc. This problem is fundamental for the design of a hub-and-spoke network and can reduce the total costs of the system using fewer resources. In the following, a detailed description of HLP is provided based on the recent literature. We will present the HLP features, network components, classifications, applications and solution algorithms.

The hub location problem is a subclass of network design problems in which hub facilities concentrate flows in order to take advantage of economies of scale. It is actually a location-allocation problem, and the essence is to decide the optimal number and location of hubs and achieve the efficient allocation of resources, to transport demands from many origins to many destinations. In the HLP system (as seen in Figure 2.1), the transportation of commodities from one origin to its destination is performed through one single hub, like path $i_2 \longrightarrow h \longrightarrow j_2$ or two different hubs like path $i_3 \longrightarrow k \longrightarrow m \longrightarrow j_3$. The origin or destination nodes are directly connected to the hubs by single arcs. In some cases, direct shipments are allowed between origins and destinations like path $i_1 \longrightarrow j_1$. The corresponding network example and alternative paths between the origins and destinations are shown in Figure 2.1. Based on geographical characteristic and the type of problem addressed, the system can rely on a discrete network or a continuous plane. However, a hub-and-spoke network mainly consists of two types of nodes and arcs linking them, [36]:

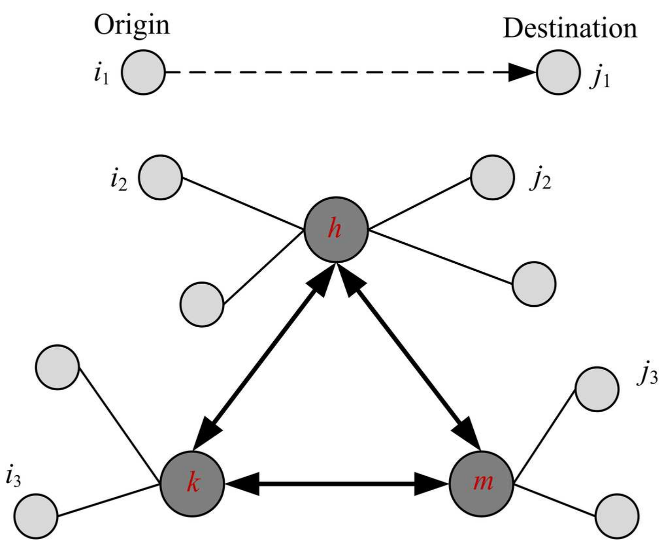


Figure 2.1: A HLP network example and the alternative paths

(1) Non-hub nodes (nodes $(i_1, i_2, i_3 ...)$ and $(j_1, j_2, j_3 ...)$ in Figure 2.1). They are origin and

destination nodes of commodities or informations and form the basic structure of the HLP system. For example, in air passenger travel, cities airport are the origin and destination nodes. In a LTL system for freight transport, different suppliers (shippers) correspond to the origin nodes while their clients are the destination nodes.

(2) Hub nodes (nodes h, k and m). They refer to a switch terminal where the commodities or information flows are consolidated to be transfered to another hub or to their final destination. They can be located among a set of candidate hubs, which can be a subset of non-hub nodes or not. Their locations greatly affect the total cost and performance of the hub network.

(3) Inter-hub arcs (arc (k, m), (h, k) and (h, m) in Figure 2.1). They connect any two hub nodes and form the backbone structure of the network. Normally, full trucks operate on the inter-hub arcs and this results in a discount factor from the cost of transport between non-hub and hub nodes because of the economies of scale [154]. Most of the researches assume that the hubs are fully connected [11].

(4) Access arcs (arcs (i_2, h), (h, j_2), (i_3, k) and (m, j_3)). They connect the non-hub nodes (origins or destinations) to the hub nodes and correspond to the assignment decisions. In a traditional hub network, these arcs are one-way links and concentrate the main parts of the flows. In some cases, direct connection between origin and destination are allowed, however, without transferring through the hubs.

(5) Demands. A demand is associated to a commodity or information flow between the origin and destination nodes. Generally, the demands for each O-D pair are relatively small quantities and it is beneficial to ship them via consolidation hubs if the volumes are not compatible with direct shipments. Two major data sets are available in HLP research area. One of them is based on the airline passenger interactions between 25 U.S. cities in 1970 as evaluated by the Civil Aeronautics Board (called CAB data sets) [156]. The other one is based on the operational data of the Australia Post system (called AP data sets) [76].

From the above description of the network structure characteristics, it can be seen that the HLP is different from classical facility location problems. For example, the demand in facility location problems is only specified as a quantity of commodities for each customer point, not as a flow between many O-D pairs. Also the facility location problem doesn't consider the interaction between hub facilities. Campbell and O'Kelly [37] summarized the following key features of the hub location problem, establishing the theoretical basis for a research agenda on the HLP long the following lines:

- Hubs are not only consolidation/dissemination points for the flows, but also switching and sorting centers. Their locations need to be decided through an optimization process.
- Flows between many origins and destinations (demands) are allowed to pass via hubs to get a benefit on transportation costs because of the consolidation of flows. This benefit is expressed in terms of a discount factor α.
- The overall objective is to minimize the total cost, the transportation distance or the response time of the system. Any of these factors depends not only on the location of hubs but also on the routing of flows.
- In addition, Campbell proposed in 1994 two other features [33] which have been retained as fundamental characteristics of most of the literature in this area: firstly, the routing of flows can go through at most two hubs, resulting from the triangle inequality for distances and secondly, direct transportation between O-D pairs is not allowed.

During the past years, many variants of the hub location studies have been studied: there are capacitated or uncapacitated hub location problems depending on whether the capacity of hubs is limited or not, if the number of hubs to locate is known in advance, it is termed p-hub location problem. Different objectives can be considered as p-hub median location problem which is to minimize the total cost (distance, time, etc.) of transportation [34, 164], p-hub center location problem which is to minimize the maximum cost (distance or time) of O-D pairs [31, 75], and p-hub covering location problem which is to locate some hubs so that the non-hub nodes can be covered within a specified distance or time by hubs [89]. In addition, if hubs are located from a set of discrete points, it is called as discrete hub location problem. However, if hubs can be selected from any point in a continuous plane, it is a continuous hub location problem. More commonly, if it is assumed that each non-hub node can be assigned to only one hub, it is termed a single allocation HLP, such as less-than-truckload transportation networks or some telecommunication networks. Otherwise, it is a multiple allocation HLP in which non-hub nodes can be linked to more than one hub, for example the airline passenger networks where there are multiple flights from non-hub cities to several airline terminals. In order to clearly show these different types of HLPs and introduce the notations used in the following of the thesis, the classification of HLPs is given in Table 2.1. For example, if a location problem doesn't consider a limited capacity for the hub nodes but considers different periods of times, and assumes that each non-hub node is assigned to only one hub, and that the number of located hubs is imposed to a given value p, this problem is called a "multi-periods uncapacitated single allocation p-hub location problem"', which can be noted MPUSApHLP.

Table 2.1: The varieties and notations of HLPs [80]

Hub capacity	Allocation strategy	Number of located hubs	Objectives	Periods
Capacitated (C)	Single allocation (SA)	Certain-p	Median (M)	Single period (SP)
Uncapacitated (U)	Multiple allocation (MA)	Uncertain-fixed cost	Center (T)	Multi-periods (MP)
			Covering (V)	

It seems that Hakimi [96] presented the first related paper addressing the optimal location of the switching centers in a communication network and the police stations in a highway system, which is similar to the concept of the HLP. Then Goldman [91] extended Hakimi's works to the optimal location for n centers in a network. Later, Toh et al. [197] gave a definition for a hub and spoke network and discussed the impact of the hub network centralization on the domestic airline industry. However, the first popular paper about transportation hub location problem has been proposed by O'Kelly [155, 156] in 1986. He gave several operational examples of air freight and passenger networks involving ten large U.S. cities. Since that time, HLPs have attracted many researchers and have been successfully applied to the design of many different transportation systems (such as air passenger/freight networks, postal systems, trucking system and maritime cargo industry) and telecommunications networks (for example computer systems, telephone networks and video conferences).

In airlines and airport industries, O'Kelly [157] studied the airline passenger interactions between 25 U.S. cities and firstly proposed the standard CAB data set for the HLPs. Later, O'Kelly and Lao [159] solved the transportation mode choice problem of an air express industry in the USA. Adler and Hashai [4] researched a p-hub median location problem for air transport of the Middle East region. Menou et al. [139] described a method to select the location to centralize cargo at a Moroccan airport hub, in order to minimize the total transport costs. Besides, there are some other applications of the HLP in this area [58, 129]. For the postal and express systems, Kuby and Gray [117] developed a mixed-integer program for a hub-and-spoke network with stopovers and feeders, and then applied it to the western U.S. portion of the Federal Express packages collection system. Ernst and Krishnamoorthy [76] solved a p-HLP for the Australian Post (AP) network. Gendron and Semet [88] discussed some formulations of a multi-echelon location problem for an application in the fast delivery service. In the trucking and railway networks, Cunha *et al.* [60] proposed a genetic algorithm heuristic to solve an uncapacitated single allocation HLP for a LTL truck company in Brazil. Jeong et al. [103] addressed a hub-and-spoke design problem for the European freight railway system. Campbell [35] presented a multiple allocation p-hub median model and a multiple allocation hub arc location model for the freight transportation of time-definite trucking firms. Gelareh et Nickel [87] presented a new realistic model tailored for an urban transport network and liner shipping companies and proposed Benders decomposition approach to solve them. Sender and Clausen [182] developed heuristic approaches to solve a capacitated multiple allocation HLP of German wagonload traffic in railway logistics. There are also some studies of HLPs for the maritime cargo industry. Hsu et al. [100] studied a multi-objective decision-making for a maritime hub-and-spoke container network. Takano *et al.* [195] solved the p-hub median problem with shuttle services and applied it to the containerized cargo transport of Asian hub ports.

Classical mathematical models for the HLP

The classical hub location problem is defined on an undirected graph $G = (N, A)$, where $N = \{v_1, v_2 ... v_i ... v_n\}$ is the set of all nodes including origin/destination nodes and candidate hub nodes and $A = \{(i, j)|i \in N, j \in N\}$ is the set of all arcs connecting any two vertices including inter-hub arcs and access arcs. The flows between any O-D pair are denoted W_{ij} and C_{ij} is the unit transportation cost between any two nodes. In addition, $\alpha \leq 1$ is the inter-hub discount factor to reflect the economies of scale. Based on these key features and hypothesis, many mathematical formulations have been proposed for the HLPs. Here, some classical models for the single allocation HLP are introduced. The evolution of related studies is summarized in Figure 2.2.

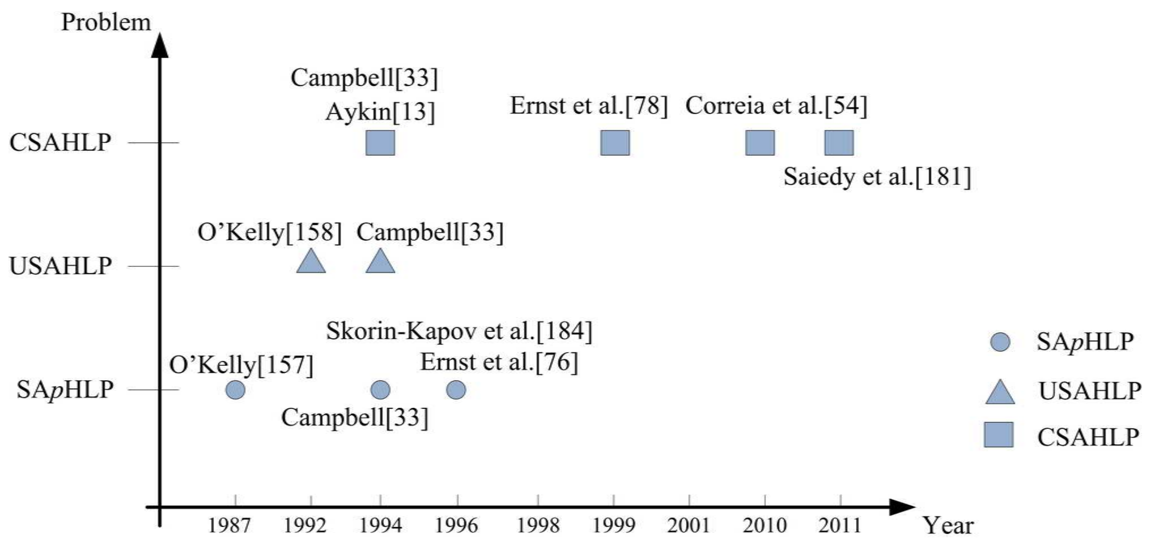


Figure 2.2: The evolution of the mathematical models for the single allocation HLP

The first recognized model for the HLP has been proposed by O'Kelly in 1987 [157]. It is a quadratic mathematical formulation of the uncapacitated single allocation p-hub location problem (USAp-HLP). This formulation assumes that each node is assigned to only one hub and the required number of located hubs is set to p. Each candidate hub has no capacity limitation. Then an integer variable X_{ij} is set to 1 to denote that node i is assigned to hub j, and 0 otherwise. The variable X_{ii} is set to 1 if node i is a hub, 0 otherwise. In order to simplify the model, $O_i = \sum_{j \in N} W_{ij}$ is the total amount of flows originating at node i; and similarly $D_j = \sum_{i \in N} W_{ij}$ is the total amount of flows whose destination is node j. Therefore, this first integer programming formulation of USApHLP [157] is:

$$\text{(USA}p\text{HLP)} \quad \text{Min} \quad \sum_{i\in N}\sum_{k\in N} X_{ik}C_{ik}(O_i+D_i)+\sum_{j\in N}\sum_{m\in N} X_{jm}\sum_{i\in N}\sum_{k\in N} X_{ik}(\alpha W_{ij}C_{km}) \tag{2.1}$$

subject to

$$(n-p+1)X_{jj}-\sum_{i\in N} X_{ij}\geq 0 \qquad \forall j\in N \tag{2.2}$$

$$\sum_{j\in N} X_{ij}=1 \qquad \forall i\in N \tag{2.3}$$

$$\sum_{j\in N} X_{jj}=p \tag{2.4}$$

$$X_{ij}\in\{0,1\} \qquad \forall i,j\in N \tag{2.5}$$

The objective function (2.1) is to minimize the transportation costs including the assignment cost of every node to its hub for outgoing and incoming flows and the quadratic interaction costs between hubs. Constraints (2.2) denote that no node is assigned to a site unless it is an open a hub. Constraints (2.3) are the single allocation constraints. Constraints (2.4) define the number of located hubs as p. The last equations (2.5) are the binary integer constraints for variables.

Later, O'Kelly [158] adapted this formulation to the uncapacitated single allocation hub location problem with fixed costs (USAHLP), where the number of located hubs is a decision variable. However, it was also a quadratic mathematical model. Then Campbell [33] introduced the first linear integer programming formulations for different HLPs including USApHLP and USAHLP, and then converted the uncapacitated versions to the capacitated single allocation hub location problem (CSAHLP) by adding capacity constraints. It can be considered as the first linear model of CSAHLP. In addition to the parameters of the USApHLP, it defined a variable X_{ijkm} as the fraction of flow from origin i to destination j that is routed via hubs k and m in that order. In addition, variable Y_k takes the value of 1 if node k is a hub and 0 otherwise. Z_{ik} is the allocation variable . In addition, let $C_{ijkm}=C_{ik}+C_{mj}+\alpha C_{km}$ be the transportation cost per unit from origin i to destination j via hubs k and m. Further, F_k is defined as the fixed cost of establishing a hub at location k, and the capacity of a hub at location k is noted as Γ_k. Based on the above parameters and decision variables, a linear integer programming formulation for CSAHLP [33] was presented as follows:

$$\text{(CSAHLP-1)} \quad \text{Min} \quad \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} W_{ij}X_{ijkm}C_{ijkm}+\sum_{k\in N} F_kY_k \tag{2.6}$$

subject to

$$\sum_{k\in N}\sum_{m\in N} X_{ijkm}=1 \qquad \forall i,j\in N \tag{2.7}$$

$$Z_{ik}\leq Y_k \qquad \forall i,k\in N \tag{2.8}$$

$$\sum_{j\in N}\sum_{m\in N}(W_{ij}X_{ijkm}+W_{ij}X_{jimk})=(O_i+D_i)Z_{ik} \qquad \forall i,k\in N \tag{2.9}$$

$$\sum_{i\in N}\sum_{j\in N}(W_{ij}+W_{ji})Z_{ik} \leq \Gamma_k Y_k \qquad \forall k \in N \tag{2.10}$$

$$0 \leq X_{ijkm} \leq 1 \qquad \forall i,j,k,m \in N \tag{2.11}$$

$$Y_k \in (0,1) \qquad \forall k \in N \tag{2.12}$$

$$Z_{ik} \in (0,1) \qquad \forall i,k \in N \tag{2.13}$$

Objective function (2.6) minimizes the total costs including not only the transportation cost over all O-D pairs but also the establishing cost of hubs. Constraints (2.7) ensure that the total flow for each O-D pair is routed via some hub pairs. Constraints (2.8) state that a node can only be assigned to an open hub. Constraints (2.9) are the single allocation requirement and flow balance equations. Equations (2.10) are the hub capacity constraints. And the last three equations (2.11)-(2.13) are constraints on the variable values .

In the same year, Aykin [13] formulated the capacitated HLP with fixed costs, allowing direct connections between O-D pairs. Later, Skorin-Kapov et al. [184] presented a new mixed integer model for the single allocation HLP. They noted the constraints (2.9) were very weak and provided much tighter ones by replacing them with the following pair of equations:

$$\sum_{m\in N} X_{ijkm} = Z_{ik} \qquad \forall i,j,k \in N \tag{2.14}$$

$$\sum_{k\in N} X_{ijkm} = Z_{jm} \qquad \forall i,j,m \in N \tag{2.15}$$

Constraints (2.14) ensure that if node i is allocated to hub k, the total flow from origin i to destination j routed via all paths using link $i-k$ will be nonzero; similarly, constraints (2.15) indicate that for each origin i and destination j, if the path of a flow is $i-k-m-j$, then the destination j should be allocated to the hub m.

Although the modified model of CSAHLP with constraints (2.14) and (2.15) is tight, it is still very large with regards to the number of variables and constraints. Therefore, in order to reduce the size of the above linear formulations, Ernst and Krishnamoorthy [76] introduced a new variable Y^i_{kl} to remove variable X_{ijkm} and proposed a different model for USA*p*HLP. They defined this new variable as the total amount of commodity flow from node i that is routed between hubs k and l. In addition, they defined the cost coefficients for collection paths from origins i to hubs k, and distribution paths from hubs l to destinations j. They were denoted as χ and δ, respectively. With these new variables and parameters, Ernst and Krishnamoorthy [78] developed a new formulation for the CSAHLP with fixed costs:

$$\text{(CSAHLP-2)} \quad \text{Min} \quad \sum_{i\in N}\sum_{k\in N} C_{ik}Z_{ik}(\chi O_i+\delta D_i)+\sum_{i\in N}\sum_{k\in N}\sum_{l\in N}\alpha C_{kl}Y^i_{kl}+\sum_{k\in N}F_kZ_{kk} \tag{2.16}$$

subject to (2.13),

$$\sum_{k\in N} Z_{ik} = 1 \qquad \forall i \in N \tag{2.17}$$

$$Z_{ik} \leq Z_{kk} \qquad \forall i,k \in N \tag{2.18}$$

$$\sum_{i\in N} O_i Z_{ik} \leq \Gamma_k Z_{kk} \qquad \forall k \in N \tag{2.19}$$

$$\sum_{l\in N} Y^i_{kl} - \sum_{l\in N} Y^i_{lk} = O_i Z_{ik} - \sum_{j\in N} W_{ij} Z_{jk} \qquad \forall i,k \in N \tag{2.20}$$

$$Y^i_{kl} \geq 0 \qquad \forall i,k,l \in N \tag{2.21}$$

Z_{kk} is the location variable which implies that site k is an open hub if it takes the value of 1. The objective function (2.16) has the same meaning as equation (2.6). Constraints (2.17) ensure the single allocation for each node i. Constraints (2.18) prevent allocation to non-hub nodes. Equations (2.19) are the capacity constraints and (2.20) are the divergence equations for the flow at node k from i. It indicates that the demand and supply at the nodes are determined by the allocation variable Z_{ik}. Correia et al. [54] revisited this classical formulation and showed that it may be incomplete for some instances. They introduced the following new set of constraints (2.22) to complete the formulation. Through some computational experiments, they proved that the new constraints were computationally effective as cuts but also that the original formulation can always obtain the optimal values for the tested instances. Recently, Saiedy et al. [181] presented a new two-index quadratic model for the CSAHLP with n hub centers and proved it was faster than other models for small and medium instances.

$$\sum_{l\in N, l\neq k} Y^i_{kl} \leq O_i Z_{ik} \qquad \forall i,k \in N \tag{2.22}$$

Exact methods and heuristic algorithms

Many exact and heuristic solution methods have been proposed for different hub location problems including. O'Kelly [157] showed that the HLP is NP-hard, even if the locations of the hubs are fixed, and he introduced two enumeration-based heuristics to solve a single allocation p-HLP. In this section, in accordance with the research focus of this thesis, we highlight and discuss some successful exact methods and heuristics proposed for the capacitated single allocation hub location problem (shown in Table 2.2). In this table, the first column represents the algorithm types proposed by the articles listed in the second column. The column "detailed algorithm" gives more information about the method proposed. The fourth column illustrates the problem variant studied by each article and the last column describes the largest instance size solved by the corresponding method. For the newest surveys on algorithms used to solve other HLPs, one can refer to Alumur and Kara [11] or Farahani et al. [80].

Table 2.2: Exact methods and heuristics proposed for CSAHLP

Method	Article	Detailed Algorithm	Problem notation	Application size
Exact methods	Aykin (1994) [13]	B&B+LR	CSAHLP+ direct connections	40 US cities
	Ernst et al. (1999) [78]	B&B+preprocessing	Classical CSAHLP	200 AP nodes
	Labbé et al. (2005) [121]	B&C	Quadratic CSAHLP	50 AP nodes
	Correia et al. (2010) [56]	LPR+preprocessing	CSAHLP + multiple capacity levels	50 AP nodes
	Contreras (2011) [50]	B&P+LR	Classical CSAHLP	200 AP nodes
	Correia et al. (2011) [55]	LP + commercial solver	CSAHLP + capacity decisions	50 AP nodes
	Camargo et al. (2012) [64]	GBD	CSAHLP under congestion	200 AP nodes
	Ge et al. (2014) [86]	LP	Fixed-hub single allocation problem	50 AP nodes
	Karimi et al. (2014) [111]	CPLEX	CSAHLP+hierarchical structure	IAD-37 cities
	Correia et al. (2014) [57]	LPR + valid inequalities	CSAHLP+ multi-product	-
Meta-Heuristics	Aykin (1994) [13]	Heuristic+LR	CSAHLP+ direct connections	40 US cities
	Ernst et al. (1999) [78]	SA and RDH	Classical CSAHLP	200 AP nodes
	Stanimirovic (2007) [186]	GA+caching technique	Classical CSAHLP	200 AP nodes
	Randall (2008) [177]	ACO + local search	Classical CSAHLP	50 AP nodes
	Chen (2008) [41]	An effective heuristic	Classical CSAHLP	200 AP nodes
	Costa et al. (2008) [61]	A bi-critera approach	CSAHLP+ bi-criteria objectives	40 AP nodes
	Contreras (2009) [51]	LR+SO	Classical CSAHLP	200 AP nodes
	Camargo et al. (2011) [65]	OA+GBD	CSAHLP under congestion	200 AP nodes
	Sun et al. (2012) [192]	ACO+GA	Asymmetric allocation HLP	-

Ernst and Krishnamoorthy [78] developed a linear programming (LP)-based branch and bound method (B&B) to solve the capacitated single allocation problem. They also proposed two heuristics to obtain a good solution as the initial upper bounds of the branch and bound. One of them is based on simulated annealing (SA), and the other is based on random descent (RDH). In addition, a few preprocessing steps were performed to fix some variables and tighten the LP relaxation. They conducted computational experiments not only based on the CAB data set, but also applied their method to the Australian Post (AP) network for the first time. They compared the performance of the two proposed heuristics on obtaining upper bounds, and showed that RDH outperformed SA for problems with tight hub capacities. Then the effectiveness of two formulations was compared with the same upper bounds in the branch and bound tree search. The preprocessing step was proved to be effective, especially for problems with tight hub capacities. Finally, some larger problems with 200 nodes were solved using SA and RDH heuristics. Labbé et al. [121] studied the polyhedral properties for the linear capacitated hub location (LHL) problem with single assignment which has a fixed capacity for each hub. Then they proposed a branch-and-cut algorithm (B&C) to solve this problem. In this exact method, some valid inequalities and a preprocessing algorithm were added to obtain optimal solutions. Computational tests were carried out on two sets of data from France Telecoms and the classical AP data set, showing that their algorithm can solve the problem effectively. Stanimirović [186] proposed a genetic algorithm (GA) to solve the CSAHLP where a caching technique was applied to improve the computational performance. Then the author demonstrated the robustness and effectiveness of the heuristic based on the computational results from the AP data set with up to 200 nodes. Later, Randall [177] developed four variants of the ant colony (ACO) algorithm to solve this problem including hub oriented ACO, node oriented ACO, multi-colony ACO and hub-subset oriented ACO. Some local search operators were used to improve the solutions produced by any of the ACO algorithms. For the assignment order, two methods were used: one is a random assignment order and the other is based on levels of flow. Finally, computational experiments were designed to evaluate the effectiveness of each ACO approach through the comparison of the results obtained by a random descent heuristic. In the tests, different assignment order strategies and local search heuristics are investigated. The results showed that each approach can obtain optimal solutions in a reasonable computational time for the AP data set up to 50 nodes. Chen [41] presented an effective heuristic to resolve the CSAHLP. It was evaluated through extensive computational experiments based on AP data set and proved that it outperformed a simulated annealing from the literature.

Recently, Contreras et al. [51] proposed a Lagrangean relaxation (LR) based on a subgradient optimization (SO) to obtain tight upper and lower bounds for the CSAHLP. Also, a fast primal heuristic was applied to obtain good quality feasible solutions. Some tests were presented to reduce the computational effort for larger instances from the AP data set. Later, these authors [50] developed a branch-and-price algorithm (B&P) based on the Lagrangean relaxation to solve exactly large-scale capacitated hub location problems with single assignment of up to 200 nodes.

For some extensions of the CSAHLP, Aykin [13] presented a B&B algorithm and a greedy-interchange heuristic for a capacitated hub-and-spoke network design problem where hubs had a limited capacity. In addition, he allowed both non-stop service and hub connection service between origins and destinations. Both algorithms firstly identified a set of hub locations and obtained a reduced subproblem. Then, for the subproblem, a subgradient optimization was used to solve its Lagrangian relaxation to get the optimal solution. Finally, he tested the two heuristics with 135 problems using data on airline passenger flows between the top forty US cities ranked by the number of passengers in 1989. The results showed that the two methods could successfully find good solutions, and the overall performance of the greedy-interchange heuristics was better than the branch-and-bound algorithm for most cases. Costa et al. [61] presented two bi-criteria models for the CSAHLP. Instead of imposing hub capacity constraints, they introduced a second objective function to minimize the total service time (first model) or the maximum service time (second model) for processing the flows into the hubs. Then in order to provide non-dominated solutions, they proposed an interactive decision-aid approach based on progressive and selective learning. Computational tests were carried out on the AP data set. The acceptability of the hubs flow charges of these non-dominated solutions was studied and compared to those of the unique solution given by the standard CSAHLP. Correia et al. [56] studied a CSAHLP with multiple capacity levels where the capacity of the hubs becomes another decision variable. They proposed several formulations with different sets of inequalities and compared them to the bounds provided by the linear programming relaxation (LPR). In addition, some preprocessing tests were introduced to reduce the model size. The results of computational experiments based on the AP data set allowed the identification of a good model for solving this problem optimally. After this work, the authors [55] imposed balancing requirements as an extension of the CSAHLP, which considered the maximum difference between the maximum and minimum number of non-hub nodes allocated to the hubs. They proposed two mixed-integer linear programming (MIP) formulations and evaluated their possibility to obtain optimal solutions with a commercial solver.

Recently, Camargo et al. [65] proposed a hybrid outer-approximation/Benders decomposition (GBD) algorithm to tackle the single allocation hub location problem under congestion. The computational experiments based on AP standard data set showed that the proposed method could solve the problem optimally with 200 nodes in a reasonable time. Later, Camargo and Miranda [64] extended this problem to two different network design perspectives: the network owner and user. They proposed related models and deployed a Benders decomposition algorithm to obtain the optimal solutions for large scale instances based on AP data set. Sun et al. [192] presented an integer programming model for a capacitated asymmetric allocation hub location problem and solved it using CPLEX. For large-size problems, they developed a hybrid method combining an ant colony optimization algorithm (ACO) and a genetic algorithm. Ge et al. [86] presented a linear programming-based algorithm to solve a fixed-hub single allocation problem where hubs are fixed and fully connected. In his algorithm, a geometric rounding method was used to round fractional solutions of LP relaxations to integer ones. The results based on randomly generated instances and AP data set indicated its effectiveness for large-sized problems. Karimi et al. [111] developed a new and applied model for the capacitated single allocation hub location problem with a hierarchical structure, where different levels of services for transportation network were considered. Computational results based on Iranian Airport Data (IAD) including 37 cities were presented to confirm the performance of the model solved by CPLEX. Correia et al. [57] proposed several models for an extension of classical CSAHLP assuming that multiple products were shipped through the hub-and-spoke network. They considered two cases according to the hubs could handle all products or a single-product. Additionally, in order to strengthen the models, they added some inequalities based on the lower bound provided by the linear relaxation.

Conclusion

As presented in this section, many researches have been conducted on many variants of the Hub Location Problem and many models have been proposed for the past twenty years or so, such as on the basic HLP as well as more complex problems such as the multi-product HLP, the HLP hybridized by the routing problem and the multi-period HLP. The earliest classification on the HLP has been proposed by O'Kelly and Miller in 1994 [160]. For a detailed introduction on the HLP variants, one can refer to the review of Alumur and Kara [11]. For an extensive review on the applications, one can refer to Campbell et al. [36] or to Farahani et al. [80], who also presented the main methods for solving the HLP to date. Two main data sets have been developed and are available for testing the HLP models and solution techniques, i.e. the CAB data set in the airline and airport industries and the Australian Post data set (AP) in the postal service and most of the developed models and methods have been tested on these data sets. As far as exact methods are concerned, mixed integer programming formulation have been solved using standard commercial solvers, or Branch and Bound methods using preprocessing, Branch and Cut or lagrangian relaxation. Different heuristic or meta heuristic methods have been proposed and tested, including local search methods and population based methods such as genetic algorithms. The best exact and heuristic methods have been able to solve large problems such that the 200 nodes problems of the AP data set.

2.1.2 The vehicle routing problem

Introduction

The second related problem to our research is the vehicle routing problem (VRP). The objective of this literature review is not to present an exhaustive review of VRP variants and solution methods, but to focus on the most important aspects related to our research. The VRP can be described as a problem of designing a set of optimal collection or delivery routes from a depot. This problem plays a vital role in the field of distribution management and logistics system optimization. It was introduced by Dantzig and Ramser in 1959 [62], and can be found in many industrial applications, such as mail delivery, product distribution, waste collection, bus schedule and the routing of snowplough. There have been huge research efforts for more than 50 years to study different types of VRPs and propose solution approaches. Even for the literature review, there are abundant publications: among the main reviews and books for the VRP, one can refer to [52, 90, 122, 123, 200]. Some recent surveys about the latest works and developments can be found in [19, 119] or [67]. In this section, a general introduction to the VRP is given, consisting in its basic features, different types, as well as the main exact solution methods and heuristic algorithms.

As a natural generalization of the Traveling Salesman Problem (TSP) [62], a classical vehicle routing problem (seen in Figure 2.3) can be defined on a complete graph $G' = (V', A')$ where $V' = \{0, 1, 2, ...i...j...n\}$ is the vertex set with node 0 the depot and $A' = \{(i, j)|i, j \in V', i \neq j\}$ is the set of arcs. $V'_c = V' \setminus \{0\}$ corresponds to the set of customers (stops) with a known non-negative demand d_i. A weight c'_{ij} is associated to each arc $(i, j) \in A'$. It represents the traveling cost from vertex i to vertex j. This problem known as Capacitated Vehicle Routing Problem (CVRP) is to visit each customer once, with a fleet of homogeneous vehicles, starting and ending at the same depot and satisfying vehicle capacity constraints. The classical goals are to minimize the total traveling distance or time, to minimize the number of used vehicles or to achieve multi-objectives simultaneously.

In addition to the CVRP described above, many variants of the VRP have been studied in the past years. Yeun et al [209] introduced the definitions and mathematical models for some variants, as well as their solving methods. In particular, we can mention:

- the multi-depot vehicle routing problem (MDVRP). It is a direct extension of the VRP, in which a set of available depots is given instead of only one. The problem is to design several vehicle routes from multiple depots to service a set of customers and then return to the same depot [113].
- the vehicle routing problem with pick-up and delivery (VRPPD). In this problem, each customer is associated to a non-negative commodity quantity d_i to be delivered and also p_i to be picked up. Then at each customer location i, both pickup and delivery services

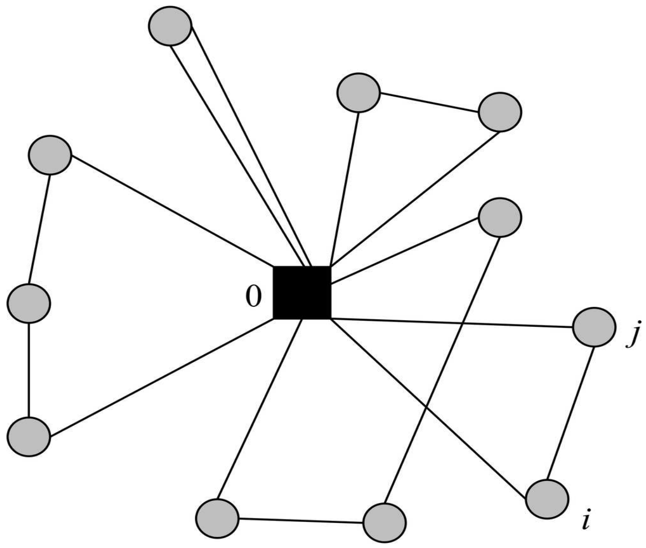


Figure 2.3: A solution example of a classical vehicle routing problem

should be accomplished by the same vehicle route, assuming that the delivery is performed before the pickup. Its goal is to minimize the overall length of the routes and requires that the vehicle capacity can not be exceeded in any point of a route [200].

Other variants of the vehicle routing problems have been studied, such as the vehicle routing problem with time window (VRPTW), the heterogeneous fleet VRP (HVRP) where vehicles have different capacities; the VRP with backhauls (VRPPB) which deals with two types of customers: linehaul customers (LC) that request a quantity of goods from the depot and backhaul customers (BC) that send a quantity of goods to the depot; and the periodic VRP (PVRP) where vehicle routing for serving customers should be designed based on multiple time periods. For more details, one can refer to the review papers [52, 67].

Mathematical models for the CVRP

Based on the above definition of the CVRP, many modeling approaches are proposed in the literature. The first one is known as a *vehicle flow model* where an integer variable is defined to count the number of times the arc (i, j) is visited by a vehicle. We can distinguish two forms: *two-index vehicle flow formulation* and *three-index vehicle flow formulation*. A two-index formulation has been proposed by Laporte et al. [126]. An integer variable x_{ij} is used to represent the number of times the arc (i, j) appears in the solution. It can take value $\{0, 1\}$ if ß$, j \in V'_c$ and value $\{0, 1, 2\}$ when $i = 0, j \in V'_c$. Note that $x_{0j} = 2$ corresponds to a return trip including a single customer j. The two-index vehicle flow model (VF2) proposed by [126] for the symmetric capacitated VRP can be described as follows :

$$\text{(VF2)} \quad \text{Min} \sum_{(i,j)\in A', i<j} c_{ij} x_{ij} \tag{2.23}$$

subject to

$$\sum_{j \in V'_c} x_{0j} = 2m \tag{2.24}$$

$$\sum_{i<k} x_{ik} + \sum_{j>k} x_{kj} = 2 \qquad \forall k \in V'_c \tag{2.25}$$

$$\sum_{i \in S} \sum_{h \notin S, h<i} x_{hi} + \sum_{i \in S} \sum_{j \notin S, j>i} x_{ij} \geq 2b(S) \qquad S \subset V'_c \tag{2.26}$$

$$x_{ij} \in \{0,1\} \qquad \forall i,j \in V'_c \tag{2.27}$$

$$x_{0j} \in \{0,1,2\} \qquad \forall j \in V'_c \tag{2.28}$$

In this model, the objective function (2.23) is to minimize the total route costs. Constraints (2.24) define the degree of depot vertex. Constraints (2.25) impose that exactly two edges are incident into each customer k. In constraints (2.26), $b(S)$ is the minimum number of vehicles needed to serve a given set S. This capacity-cut constraints prevent the formation of subtours by forcing a sufficient number of edges to be linked with each subset and also ensure the vehicle capacity requirements. In practice, it is common to handle $b(S)$ as $\lceil \sum_{i \in S} d_i / Q' \rceil$. The equations (2.27) and (2.28) are the value constraints of variables. In some cases, the variable x_{ij} can be replaced by $x_e, \forall e \in A'$ [199]. In addition, constraints (2.26) may be rewritten as the generalized subtour elimination constraints:

$$\sum_{i \in S} \sum_{j \in S, j>i} x_{ij} \leq |S| - b(S) \qquad S \subseteq V'_c, S \neq \emptyset \tag{2.29}$$

A three-index formulation (VF3) uses a binary variable x_{ij}^k to indicate the number of times the arc (i,j) is visited by vehicle k. A variable y_{ik} is used to represent if vertex i is visited by vehicle k. The three-index formulation can be given as follows [200]:

$$\text{(VF3)} \quad \text{Min} \quad \sum_{i \in V'} \sum_{j \in V'} c_{ij} \sum_{k=1}^{m} x_{ijk} \tag{2.30}$$

subject to

$$\sum_{k=1}^{m} y_{ik} = 1 \qquad \forall i \in V'_c \tag{2.31}$$

$$\sum_{k=1}^{m} y_{0k} = m \tag{2.32}$$

$$\sum_{j \in V'} x_{ijk} = \sum_{j \in V'} x_{jik} = y_{ik} \qquad \forall i \in V', k = 1, ...m \tag{2.33}$$

$$\sum_{i \in V'} d_i y_{ik} \leq Q' \qquad \forall k = 1, ..., m. \tag{2.34}$$

$$\sum_{i \in S} \sum_{j \notin S} x_{ijk} \geq y_{ik} \qquad \forall S \subseteq V'_c, i \in S, k = 1, ..., m. \tag{2.35}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i,j \in V', k = 1, ...m \tag{2.36}$$

$$y_{ik} \in \{0,1\} \qquad \forall i \in V', k = 1, ...m \tag{2.37}$$

In this formulation, constraints (2.31)-(2.33) indicate the single service of each customer by exactly one vehicle, that m vehicles start from the depot, and the route continuity at every customer location i, respectively. Constraints (2.34) are the vehicle capacity restriction, and equations (2.35) are the subtour elimination constraints. Note that constraints (2.35) are exponentially increasing, then in order to overcome this drawback, the following family of constraints with a polynomial number may be considered to replace them [118]:

$$u_i - u_j + nx_{ijk} \leq n - 1 \qquad \forall i, j \in V'_c, i \neq j, k = 1, ..., m \tag{2.38}$$

where $u_i, \forall i \in V'_c$ are additional variables with an arbitrary real number.

The second modeling approach is called as a *commodity flow* (CF) model, developed by Garvin et al. [85] in an oil delivery problem. In this approach, a flow variable is added into the model to specify the amount of the demand transported on arc $(i, j)|i, j \in V', i \neq j$. There are mainly three different formulations: one commodity, two commodity and multi commodity flow models [16, 123]. The third approach is originally proposed by Balinski and Quandt in 1964 [21], which is called a *set partitioning formulation*. In this approach, one new set $R = \{R_1, ..., R_s\}$ is defined to represent all feasible routes with a cost r_j for each route R_j. For variables, let a_{ij} be a binary coefficient equal to 1 if and only if vertex i is served by route R_j, 0 otherwise. Another binary variable x_j takes the value 1 when the route R_j is selected by the solution. Then the formulation is [52]:

$$\text{(SP)} \quad \text{Min} \quad \sum_{j=1}^{s} r_j x_j \tag{2.39}$$

subject to

$$\sum_{j=1}^{s} a_{ij} x_j = 1 \qquad \forall i \in V'_c \tag{2.40}$$

$$\sum_{j=1}^{s} x_j = m \tag{2.41}$$

$$x_j \in \{0, 1\} \qquad j = 1, ..., s \tag{2.42}$$

Constraints (2.40) impose that each customer i can be visited by only one feasible route, and (2.41) require that in total m routes are selected. One of the main drawbacks associated with this formulation lies in the large number of binary variables x_j due to the huge size of feasible route set R. Thus, in order to solve this problem exactly based on the SP formulation, one of the most common approaches is a column generation algorithm [200]. In addition to the above models, a two-commodity flow formulation was proposed by Baldacci et al. for the symmetric CVRP [16].

To solve these NP-hard problems, a large variety of solution methods have been proposed: exact algorithms, classical heuristics and metaheuristics [123]. In the following, some of the most popular methods proposed for the CVRP, MDVRP and VRPPD are introduced and also summarized in Table 2.3. The last column "Node size" shows the number of cities of the largest size instances solved by the corresponding work.

Exact methods

In the abundant literature about exact methods to solve VRPs, branch and bound methods as well as branch and cut methods have been widely used. The branch-and-bound method (B&B) adopts a best-bound-first search strategy to find current best solution for a given problem in a dynamically generated search tree. During the searching iterations, the aim is to narrow the gap between the lower bounds and the current best solutions through node selection, bound calculation and branching. In order to improve the lower bounds, most of the B&B methods proposed for the CVRP were implemented based on some combinatorial relaxation strategies, such as *Assignment Problem* relaxation obtained from formulation VF2 [124], the K-tree relaxation combined with Lagrangian relaxation (LR) [138], or the *additive bounding approach* proposed by Fischetti et al. [81]. Later, Baldacci et al. [17] presented a procedure to compute valid lower bounds for the CVRP based on a Set Partitioning formulation (SP) and solved it by a branch and bound algorithm. In addition, Dastghaibifard et al. [63] developed a parallel B&B for solving the CVRP using multi computers and dynamic load balancing to save the execution time. Laporte et al. [127] expended a branch and bound method to solve a class of asymmetrical MDVRP with up to 80 nodes. More details about the structure and implementation of the B&B algorithms for VRPs can be found in [123, 199].

Recently, Baldacci and Mingozzi [18] presented a new B&B algorithm for solving different classes of VRPs based on the set partitioning formulation (SP) including the CVRP, the MDVRP and the HVRP. This algorithm used three types of bounding procedures based on LP-relaxation (LPR) and on the LR to reduce the formulation size so that the resulting problem could be solved by a commercial solver. Computational results reveal that the method improved lower bounds of the main instances from the literature and solved all considered problem types involving up to 199 customers. Almoustafa et al. [9] reformulated an asymmetric VRP (AVRP) with distance constraints and introduced a multi-start B&B method (MSBB) based on the one proposed by Laporte et al. [124]. It used the best-first strategy and a new tolerance based criterion for branching. In addition, a random tie-breaking rule was used to choose the next solving node. Computational results showed this exact method solved the largest instances in literature with up to 1000 customers within a reasonable time. Muter et al. [145] embedded a column generation (CG) within a B&B algorithm for solving the multi-depot vehicle routing problem with inter-depot routes and obtained optimal integer solutions.

The second effective exact method for solving the VRP is the branch-and-cut algorithm (B&C). Araque et al. [12] presented a branch-and-cut algorithm for the identical customer VRP based on a path-partitioning formulation which used the *generalized subtour elimination* (GSE) constraints , *large multistars* inequalities (LM), *intermediate multistars* (IM) and *small multistars* (SM) in the cutting plane phase. Computational results showed that it could solve optimally randomly generated problems with up to 60 customers.

Augerat et al. [163] were the first to present the B&C algorithm for the CVRP based on formulation VF2 and strengthened it by valid inequalities such as *generalized capacity* inequalities (GEC), *comb* (COM) inequalities and *capacity strengthened comb* (CSCOM) inequalities. In addition, they contributed mainly to the design of separation procedures for the valid inequalities, as well as the introduction of different branching strategies. Experimental results showed this algorithm solved instances involving 135 customers. Later, an improved version was developed by Naddef and Rinaldi [200]. They introduced several new families of valid inequalities to strengthen the linear relaxation problem, such as *framed capacity* constraints (FC), *path-bin* inequalities and some valid inequalities from the TSP. They also exactly solved the instances with up to 135 customers. Letchford et al. [128] showed the validity of LM, IM and SM inequalities for both kinds of capacitated VRP either with unit demands or general demands. In addition, they developed other families of multistar inequality to strengthen this method, such as *generalized large multistar* (GLM) inequalities, *knapsack large multistar* (KLM) inequalities, *partial multistar* (PM) inequalities. Then, Lysgaard et al. [133] developed the most complete B&C algorithm for the CVRP based on formulation VF2 by using a variety of cutting planes, including *rounded capacity* constraints, FC constraints, GEC, *strengthened comb* (SCOM) inequalities, *multistar* and PM inequalities, *extended hypotour* inequalities and classical *Gomory mixed-integer cuts*. They also described the detailed separation algorithms for each class of inequalities, as well as the branching rules and node selection strategy. Computational results based on a large number of instances showed it was competitive with other B&C and optimally solved some instances for the first time.

In addition, Baldacci et al. [16] proposed a new two-index integer formulation for the symmetric CVRP based on a two-commodity flow approach and an adapted B&C. Computational experimentation with instances from the literature and randomly generated demonstrated the good performance of this new B&C. Subramanian et al. [190] proposed undirected and directed two-commodity flow formulations for the VRP with simultaneous pickup and delivery (VRPSPD). They presented a B&C scheme to test proposed formulations and showed the undirected one obtained better results based on open problems with up to 200 customers.

Table 2.3: Exact methods and meta-heuristics for the CVRP, MDVRP and VRPPD

Method		Article	Problem type	Detailed Algorithm	Node size
Exact method	B&B	Laporte et al. (1986)[124]	CVRP	AP relaxation	100
		Laporte et al. (1988)[127]	MDVRP	B&B	80
		Fischetti et al. (1994)[81]	CVRP	additive bounding	135
		Martinhon et al. (2000)[138]	CVRP	LR+K-tree relaxation	135
		Baldacci et al. (2006)[17]	CVRP	SP	200
		Dastghaibifard et al. (2008)[63]	CVRP	Parallel B&B	101
		Baldacci et al. (2008)[18]	CVRP+MDVRP	LPR+LR	199
		Almoustafa et al. (2013)[9]	AVRP	MSB&B	1000
		Muter et al. (2014)[145]	MDVRP	CG+B&B	200
	B&C	Araque et al. (1994)[12]	VRP	GSE+LM+IM+SM	60
		Augerat et al. (1998)[163]	CVRP	GEC+COM+CSCOM	135
		Naddef et al. (2002)[200]	CVRP	FC+path-bin	135
		Letchford et al. (2002)[128]	CVRP	GLM+KLM+PM	135
		Lysgaard et al. (2004)[133]	CVRP	SCOM+hypotour ...	135
		Baldacci et al. (2004)[16]	CVRP	CF model	135
		Fukasawa et al. (2006)[83]	CVRP	robust BCP	135
		Baldacci et al. (2008)[15]	CVRP	SP model+LR	135
		Subramanian et al. (2010)[190]	VRPSPD	CF model	200
	others	Amico et al. (2006)[68]	VRPPD	B&P	40
		Christiansen et al. (2008)[43]	MDVRP	BCP	60
		Gutiérrez-Jarpa et al. (2010)[93]	VRPPD	B&P+CG	50
		Bettnelli et al. (2011)[28]	MDVRP+TW	BCP	144
		Contardo (2012)[46]	MDVRP	VF+SP models	200
Metea-heuristic	Local search	Pisinger and Ropke (2007) [166]	CVRP+MDVRP	ALNS	1000
		Wassan et al. (2008)[205]	VRPSPD	RTS	200
		Chen et al. (2010)[42]	CVRP	IVND	480
		Groer et al. (2011) [92]	CVRP	PLS	1200
		Cordeau et al. (2011)[53]	CVRP+MDVRP	PITS	480
		Jin et al. (2012)[104]	CVRP	PMNTS	1200
		Marinakis (2012)[136]	CVRP	MPNS-GRASP	480
		Kuo and Wang (2012)[120]	MDVRP	VNS	360
		Subramanian et al. (2013)[191]	CVRP+MDVRP	ILS+MIP	480
		Nagy et al. (2013)[151]	VRPDPD	RTS	200
		Xiao et al. (2014) [207]	CVRP	VNS+SA	480
		Wassan and Nagy (2014)[204]	VRPPD	review	..
	Population search	Prins (2004)[168]	CVRP	GA+LS	483
		Cattaruzza et al. (2014)[39]	MTVRP	MA	199
		Tasan and Gen (2012)[202]	VRPSPD	GA	34
		Potvin (2007)[167]	VRPs	EAs review	..
	Learning mechanism	Hoff et al. (2009)[98]	VRPPD	Lasso solution+TS	484
		Gajpal et al. (2009)[84]	VRPSPD	ACS	135
		Szeto et al. (2011)[193]	CVRP	ABCA	480
		Kanthavel et al. (2011)[108]	CVRP	NPS	135
		Yang et al. (2011)[208]	MDVRP	APAC	16
		Kanthavel et al. (2012)[107]	VRPPD	NPS	135

Some other B&Cs are developed based on the Set Partitioning (SP) formulation. Fukasawa et al. [83] presented a new SP formulation and proposed a robust branch-and-cut-and-price (BCP) method for the CVRP, which combined column and cut generation to improve lower bounds and could solve to optimality all instances from the literature with up to 135 vertices. Later, Baldacci et al. [15] presented a new exact algorithm based on the SP model and strengthened it by additional cuts, such as *strengthened capacity* inequalities and *clique* inequalities. This algorithm used three different bounding procedures to generate a reduced problem based on q-route relaxation and Lagrangean relaxation. The resulting problem was solved by CPLEX. Computational results showed the new lower bounds were better than the ones presented by Fukasawa et al. [83]. For a detailed introduction of this method, one can refer to a recent review of Baladacci et al. [19].

Except the above two classical exact methods, there are also many works which focus on developing other effective exact algorithms for solving various VRPs. With respect to the multi-depot VRP, Christiansent et al. [43] introduced a branch-and-cut-and-price algorithm for the MDVRP with stochastic demands. In this algorithm, they decomposed the pricing into a separate pricing problem for each depot and then solved them by a dynamic programming procedure. In addition, they suggested two heuristic algorithms to speed up the pricing and solved the instances with up to 60 nodes. Bettnelli et al. [28] developed also a BCP algorithm to solve the MDVRP with heterogeneous vehicles and considering time windows, which combined different cutting and pricing strategies. Contardo [46] proposed two models for the MDVRP under capacity and route length constraints using vehicle-flow and set-partitioning formulations, respectively. A new exact method was proposed to solve the two models. The first one is based on the cutting planes method and the second one on column-and-cut generation. Some valid inequalities were added to strengthen their lower bounds.

For the VRP with pickup and delivery, Amico et al. [68] developed a branch-and-price approach (B&P) to solve this problem exactly with up to 40 customers. Later, Gutiérrez-Jarpa et al. [93] proposed a new branch-and-price algorithm to solve five variants of the VRP with deliveries, selective pickups and time windows, which combined a column generation algorithm and a label-setting algorithm. Computational results showed this method solved the instances involving up to 50 customers optimally.

Metaheuristics

As introduced in the last section, most of the exact methods can only solve the VRP instances with up to 200 customers. With the increase of the real problem size, many efforts have focused on the development of heuristics or metaheuristics in recent years for solving different VRPs. Generally, classical heuristics can be divided into constructive heuristics and improvement heuristics. The former mainly consists of the Clarke and Wright *saving* algorithm, the sequential insertion algorithm, *sweep* algorithm and other *two-phase* decomposition heuristics. For the improvement algorithms, two types are applied to the VRP solutions: *intra-route* heuristics and *inter-route* heuristics. For more details of these classical heuristics, one can refer to the introduction in [52, 200]. Here, we mainly presented some successful metaheuristics for the related VRPs in recent years. We distinguish local search methods, population search based methods and methods based on a learning mechanism.

The first category of metaheuristics widely applied is local search based algorithms which start from an initial solution and move to another solution at each iteration in a neighborhood solutions space to find best solutions until a stopping condition is satisfied. They mainly include simulated annealing (SA) algorithm, tabu search (DS) and variable neighborhood search (VNS) algorithms. Pisinger and Ropke [166] proposed recently a general heuristic to solve five different variants of the VRP based on a large neighborhood search framework, called the adaptive large neighborhood search (ALNS). They presented an approach to transform all variants into a rich pickup and delivery model and then solved them using ALNS. Computational results based on many instances from the literature showed it was able to improve 183 best known solutions out of 486 benchmark tests. Chen et al. [42] developed an iterated variable neighborhood descent algorithm (IVND) based on a multi-operator optimization to solve the CVRP, where a cross-exchange operator was applied as a perturbation strategy to generate new starting solutions and avoid local minim. The results based on 34 benchmark problems with up to 480 customers showed the competitiveness of the IVND. Later, in order to quickly find high-quality solutions for the VRP, a parallel local search algorithm (PLS) was developed by Groër et al. [92] which was combined with integer programming based on SP formulation. Computational experiments with 129 processors proved this method could solve large instances with up to 1200 nodes within a short time and discovered 13 new best solutions from 55 benchmark problems. Jin et al. [104] presented a parallel multi-neighborhood cooperative tabu search algorithm (PMNTS) for the CVRPs, where several different neighborhood structures were used in a DS algorithm. Their computational experiments based on 32 large scale instances proved the high effectiveness of this method and provided new best solutions for four instances involving up to 1000 nodes. Marinakis [136] proposed a new modified version of the greedy randomized adaptive search procedure (GRASP), called multiple phase neighborhood search-GRASP (MPNS-GRASP) to solve the CVRP. In this method, a new stopping criterion, based on Lagrangean relaxation and subgradient optimization, was used. In addition, the *circle restricted local search moves* strategy was used to expand the neighborhood search. The test results on two sets of benchmark instances (up to 480 nodes) showed that the method can obtain solutions with high quality in a competitive computational time compared to other heuristics. Xiao et al. [207] presented a hybrid VNS combined with simulated annealing (SA) to solve the CVRP. Test results based on 39 well-known benchmark instances showed the proposed algorithm outperformed most algorithms in the literature in terms of solution quality and computational efficiency.

Cordeau and Maischberger [53] developed a parallel iterated tabu search (PITS) heuristic for solving the VRP and several of its variants including the CVRP, the PVRP and the MDVRP. In this heuristic, the iterated local search is combined with tabu search and the computational results showed that it outperformed the tabu search alone and recent heuristics. In addition, the use of the parallel computing found new best known solutions for many test problems. Kuo and Wang [120] proposed a variable neighborhood search (VNS) to solve an extended MDVRP which considered the loading cost. The proposed method comprised three phases: an initial solution generation with a stochastic method; a neighborhood search phase with four operators and finally SA is used for neighborhood solution acceptance. The results based on 23 benchmark problems showed that this method improved the best known results. Recently, Subramanian et al. [191] proposed a hybrid algorithm based on an iterated local search (ILS) and a mixed integer programming (MIP) solver for solving a class of VRPs with an homogeneous fleet. It was evaluated on hundreds of well-known instances of the considered variants (CVRP, MDVRP, VRPPD, etc.) and found 52 new best solutions. The largest instances involved up to 480 nodes. For the VRP with pickups and deliveries, a reactive tabu search (RTS) metaheuristic was designed by Wassan et al. [205] to solve the VRP with simultaneous pickups and deliveries (VRPSPD) involving up to 200 nodes. Recently, Nagy et al. [151] used this RTS algorithm to solve effectively the VRP with divisible pickups and deliveries (VRPDPD). A recent review about the VRPPD was written by Wassan and Nagy [204], which introduced the models and some meta-heuristics for solving VRPPDs.

The second category of metaheuristic for the VRP is population search based algorithms, i.e. evolutionary algorithms (EAs) [39, 168, 202]. A recent survey can be found in [167]. We will describe these methods in detail in Chapter 4 of this thesis since we chose to develop such a memetic algorithm for solving our hub location and routing problem.

The last class of metaheuristics is *learning mechanisms* based algorithms including neural networks and ant colony optimization (ACO). Szeto et al. [193] proposed an artificial bee colony algorithm (ABCA) for the CVRP. Gajpal and Abad [84] considered a VRP with simultaneous delivery and pickup (VRPSDP) problem and introduced an ant colony system (ACS) to solve it. In this algorithm, a construction rule combined with two multi-route local search schemes is used. Kanthavel et al. [107, 108] applied the nested particle swarm (NPS) optimization to solve the CVRP and the VRPPD with up to 135 customers. For other variants, Hoffe et al. [98] presented some *lasso* shaped solutions produced by a TS algorithm for solving the VRPPD. Yang et al. [208] presented a new mathematical model and proposed a self-adaptive and polymorphic ant colony (APAC) algorithm for the MDVRP with multi-model vehicle and multi-task.

Conclusion

The vehicle routing problem concerns many real-life applications and is a very challenging combinatorial optimization problem. It has attracted a considerable amount of research for more than fifty years. Several formulations such as 2-index, 3-index and set covering formulations have been proposed for many variants of the problem and many exact and approximate solutions techniques have been and are still developed. Among others, a recent survey from Cordeau [52] summarized different models and algorithms of VRPs. Recently, Baldacci et al. [19] provided a review on recent exact algorithms for the vehicle routing problem (VRP). And Jaegere et al. [67] classified existing numerous articles of VRPs and analyzed the trends in the literature.

In this section, we have discussed VRP variants that are directly or indirectly related to our subject. We have presented the main models and solution methods. Exact solution techniques focus on many variants of the branch and bound, branch and cut, and branch and cut and price procedures or learning mechanisms, while many types of meta-heuristic solutions are available, corresponding mainly to local searches and population search based methods. Together with other criteria, this will allow us to propose the mathematical formulation of the routing part of our HLRP, and guide us for the choice of promising solution methods. We will discuss this at the end of this chapter.

2.1.3 The location-routing problem

Introduction

Another problem related to the HLRP is the location-routing problem (LRP). It deals with the simultaneous determination of the location of facilities and vehicle routing to or from these locations in order to prevent suboptimal solutions when these to problems are closely linked [150]. The LRP takes into account a number of feasible depot sites and aims aims at determining the number and location of depots to retain, as well as build the distribution (or collection) routes from depots to service all customers, so as to minimize the total system cost. Generally, the total cost includes both fixed and operating costs of depots and vehicle routes. This combinatorial optimization problem arises in the distribution management and has been applied in many different fields, such as the design of city logistic systems [189], the management of hazardous wastes [10] and the optimization of a supply chain [82]. In this section, we mainly introduce the mathematical model and solving methods for the LRP, especially the capacitated location-routing problem (CLRP).

Mathematical models

The capacitated location-routing problem is defined on an undirected network $G = (I, J, E)$ (shown in Figure 2.4). $I = \{i_1, ..., i_m\}$ is a set of m possible depots, and $J = \{j_1, ..., j_n\}$

indicates a set of n customers. Let $V = I \cup J$ be the set of all nodes and $E = \{(i,j) : i \in V, j \in V\}$ represents the set of all edges. Each edge $(i,j) \in E$ is associated to a nonnegative weight c_{ij} representing the traveling cost between node i and j. Each customer $j \in J$ has a known demand d_j. In addition, a capacity limitation W_i and an opening cost O_i are associated with each potential depot i. Further, a set of vehicles with capacity and fixed cost is available at depots. It is normally assumed that the vehicles are shared by all depots and the cost matrix satisfies the triangle inequality [176]. Therefore, in order to satisfy the demand of all customers and minimize the total cost, the CLRP needs to select which depots should be open and which routes should be constructed. In addition, the following constraints must be satisfied:

1. each customer j can be served by only one vehicle and its demand d_j must be satisfied.
2. the available vehicles can be used by any depot, but each vehicle must start from one depot and return to the same one.
3. each vehicle can be used once at most by one depot. Therefore, the CLRP is a single assignment problem.
4. the total quantity of customer demand served by one depot i can not exceed its capacity W_i. And also the total load of each vehicle must fit its capacity.

Various versions of the LRP have been put forward during the last years, such as the periodic location-routing problem [174], the LRP with simultaneous pickup and delivery [110] and the LRP with time windows [44] which considers time requirements of customers.

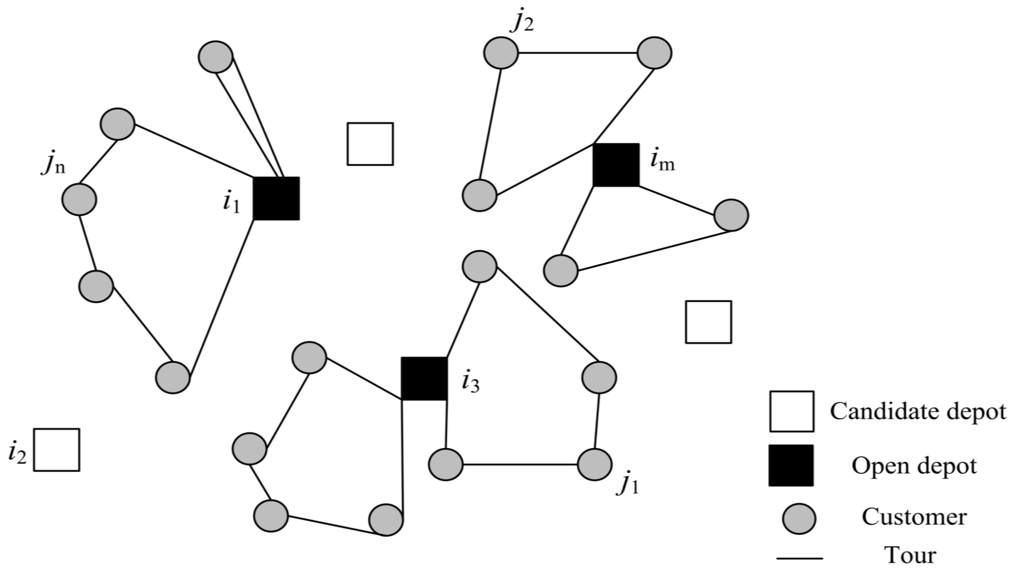


Figure 2.4: Network of a location-routing problem (CLRP)

In the first LRP studies, most authors addressed a version with either uncapacitated depots or uncapacitated vehicles. For example, Laporte et al. [125] provided an integer linear program (ILP) formulation and a branch-and-cut method for the LRP with capacitated vehicles and uncapacitated depots. Albareda-Sambola et al. [8] proposed a model for the LRP which considered capacitated depots while the vehicles were assumed uncapacitated. For the CLRP with both capacitated depots and vehicles, Wu et al. [206] gave a mathematical formulation for this problem considering a homogeneous fleet or limited heterogeneous fleet. In 2007, Prins et al. [172] proposed a zero-one linear programming model (CLRP1) for the CLRP with a single fleet K. Each vehicle $k \in K$ is associated with the capacity Q and the fixed cost F. Then based on the basic constraints as mentioned above, this model defines binary variables $y_i = 1$ if depot $i \in I$ is opened, $f_{ij} = 1$ if customer $j \in J$ is assigned to depot $i \in I$, and $x_{jlk} = 1$ if edge

$(j,l) \in E$ is in the route performed by vehicle $k \in K$. So in order to minimize the total cost, the CLRP1 model was formulated as follows:

$$\text{(CLRP1 [172])} \quad \text{Min} \quad \sum_{i \in I} O_i y_i + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk} + \sum_{k \in K} \sum_{i \in I} \sum_{j \in J} F x_{ijk} \tag{2.43}$$

subject to

$$\sum_{j \in J} d_j f_{ij} \le W_i y_i \qquad \forall i \in I \tag{2.44}$$

$$\sum_{j \in J} \sum_{i \in V} d_j x_{ijv} \le Q \qquad \forall k \in K \tag{2.45}$$

$$\sum_{k \in K} \sum_{i \in V} x_{ijk} = 1 \qquad \forall j \in J \tag{2.46}$$

$$\sum_{j \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0 \qquad \forall k \in K, \forall i \in V \tag{2.47}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ijk} \le 1 \qquad \forall k \in K \tag{2.48}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \le |S| - 1 \qquad \forall S \subseteq J, \forall k \in K \tag{2.49}$$

$$\sum_{u \in J} x_{iuk} + \sum_{u \in V \setminus \{j\}} x_{ujk} \le 1 + f_{ij} \qquad \forall i \in I, \forall j \in J, \forall k \in K \tag{2.50}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i \in V, \forall j \in V, \forall k \in K \tag{2.51}$$

$$y_i \in \{0,1\} \qquad \forall i \in I \tag{2.52}$$

$$f_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in J \tag{2.53}$$

Objective function (2.43) minimizes the sum of depot opening costs, vehicle travel costs and fixed costs. Constraints (2.44) and (2.45) are capacity constraints for depots and routes, respectively. Constraints (2.46) ensure that each customer can be visited by exactly one route. Constraints (2.47) and (2.48) guarantee the continuity of each route and a termination at the depot of origin. Constraints (2.49) are sub-tour elimination constraints. Constraints (2.50) specify that a customer can be assigned to a depot only if there is a route connecting them. Finally, constraints (2.51)-(2.53) state the binary nature of decision variables used in the formulation.

Recently, Belenguer et al. [26] added a new variable into the CLRP model and developed a *two-index vehicle-flow* formulation (CLRP2). In this new model, variables w_{ij} are used to model return trips with only one customer. They are equal to 1 if edge (i,j) is used twice and to 0 otherwise. Then for general routes, binary variables x_{ij} take value 1 only if edge $(i,j) \in E \setminus I \times I$ is traversed exactly once in the solution. Note that $x_{ij} = 1$ implies $w_{ij} = 0$ and $w_{ij} = 1$ implies $x_{ij} = 0$ if $i \in I$ and $j \in J$. In addition, they define $\delta(S) = \{(i,j) \in E : i \in S, j \notin S\}$ to denote the set of edges with only one end-node in S. $\gamma(S) = \{(i,j) \in E : i \in S, j \in S\}$ represents the set of edges with both end-nodes in S. And for each pair of disjoint subsets S and S', let $(S : S') = \{(i,j) \in E : i \in S, j \in S'\}$ denote the set of edges with one end-node in S and the other in S'. Finally, for a given edge set H, they define $x(H) = \sum_{(i,j) \in H} x_{ij}$ and similarly for $w(H)$. For a given subset $S \subseteq J$, $D(S) = \sum_{j \in S} d_j$ is the total demand in S, and an integer $k(S) = \lceil D(S)/Q \rceil$ is defined as a lower bound on the number of vehicles to serve customers in S. Thus, the CLRP2 model can be formulated as the following integer program.

$$\text{(CLRP2 [26])} \quad \text{Min} \quad \sum_{i \in I} O_i y_i + \sum_{(i,j) \in E} c_{ij} x_{ij} + \sum_{i \in I} \sum_{j \in J} 2 c_{ij} w_{ij} + \frac{F}{2} \sum_{i \in I} \sum_{j \in J} (x_{ij} + 2 w_{ij}) \tag{2.54}$$

subject to

$$2w(\{j\} : I) + x(\delta(j)) = 2 \qquad \forall j \in J \tag{2.55}$$

$$x(\gamma(S)) \leq |S| - k(S) \qquad \forall S \subseteq J \tag{2.56}$$

$$x_{ij} + w_{ij} \leq y_i \qquad \forall i \in I, \forall j \in J \tag{2.57}$$

$$x(\{j\} : I) + w(\{j\} : I) \leq 1 \qquad \forall j \in J \tag{2.58}$$

$$x(S : J \setminus S) + x(S : I \setminus \{i\}) + 2w(S : I \setminus \{i\}) \geq 2 \quad \forall S \subseteq J, \forall i \in I, D(S) > W_i \tag{2.59}$$

$$x(\gamma(S \cup \{l, j\})) + x(\{j\} : I') + x(\{l\} : I \setminus I') \leq |S| + 2 \quad \forall j, l \in J, \forall S \subseteq J \setminus \{l, j\}, \forall I' \subset I \tag{2.60}$$

$$x_{ij} \in \{0, 1\} \qquad \forall (i,j) \in E \tag{2.61}$$

$$y_i \in \{0, 1\} \qquad \forall i \in I \tag{2.62}$$

$$w_{ij} \in \{0, 1\} \qquad \forall i \in I, \forall j \in J \tag{2.63}$$

The objective function (2.54) represents the same total cost as (2.43). Constraints (2.55) are the degree constraints and flow conservation equations for customer nodes. Constraints (2.56) limit the capacity of the vehicles and also are called sub-tour elimination constraints. In constraints (2.57), the edges incident with a depot may be used only if this depot is open. Constraints (2.58) forbid return trips to be linked with two different depots. Constraints (2.59) restrict the capacity for depots. They prohibit the existence of routes starting from a same depot i and serving a demand larger than W_i. Constraints (2.60) are called path elimination constraints and prevent paths with two different depots. Finally, constraints (2.61)-(2.63) are the binary restrictions on the variables.

Based on the CLRP2 model, Contardo et al. [48] introduced a *three-index vehicle-flow* formulation, as well as a *two-index two-commodity flow* formulation and a *three-index two-commodity flow* formulation. And then they presented several families of valid inequalities that can be used to strengthen the LP relaxation of the proposed formulations. Finally, they compared the performance of all the formulations based on numerous computational experiments. In addition, there exist also some other mathematical models based on *set-partitioning* formulation [7, 20, 49].

Solution methods for the LRP

The combination of two NP-hard problems (facility location problem and vehicle routing problem) in the LRP induces that this problem is difficult to solve by exact methods. When the problem size increases, heuristic approaches become the best viable alternative. Therefore, there are fewer works devoted to the study of exact methods, in contrast to an extensive literature on heuristics or metaheuristics for solving the LRP [176]. Here, we introduce the exact methods and some heuristic algorithms proposed in the recent literature about the CLRP. A summary of these methods can be seen in Table 2.4. This table use the same notations with the ones in Table 2.2, while the last column illustrates the largest instance size solved by corresponding algorithms consisting of the number of customers (first value) and the number of potential depots (second value).

After some exact methods were proposed to solve the LRP with uncapacitated depot (UDLRP) [125] or uncapacitated vehicle (UVLRP) [8], Akca et al. [7] presented the first set-partitioning based formulation for the CLRP and then described a branch-and-price algorithm (B&P) to solve this problem. In their algorithm, different pricing procedures were developed including exact pricing algorithms and heuristic versions. Four variants of the B&P were implemented based on the proposed pricing schemes. The computational experiments based on instances from the literature and randomly generated evaluated the performance of each kind of branch-and-price algorithm on their capacity to provide upper bounds. The results indicated that the algorithm was effective to find good solutions even for instances with up to 85 customers and 7 depots, though it didn't produce quality lower bounds.

Table 2.4: Exact methods and heuristics proposed for the LRP

Method	Article	Detailed Algorithm	Problem note	Application size
Exact method	Laporte et al. (1986) [125]	B&C	UDLRP	20-8
	Albareda-Sambola et al. (2005) [8]	B&B	UVLRP	30-10
	Akca et al. (2009) [7]	B&P	CLRP	85-7
	Belenguer et al. (2011) [26]	B&C	CLRP	75-10
	Baldacci et al. (2011) [20]	DP+DA	CLRP+ULRP	199-14
	Contardo et al. (2011) [49]	BCP	CLRP	200-10
	Contardo et al. (2013) [48]	B&C	CLRP	100-10
Heuristics	Prins et al. (2006) [170]	MA+PM	CLRP	200-20
	Duhamel et al. (2008) [73]	GAHLS	CLRP	100-10
	Derbel et al. (2012) [71]	GA+ILS	CLRP	30-10
	Wu et al. (2002) [206]	SA+decomposition	MDLRP	150-30
	Prins et al. (2007) [172]	LRGTS	CLRP	200-20
	Derbel et al. (2010) [70]	ILS	UVLRP	30-10
	Duhamel et al. (2010) [74]	GRASP×ELS	CLRP	200-20
	Yu et al. (2010) [210]	SA	CLRP	318-4
	Contardo et al. (2011c) [47]	GRASP+ILS	CLRP	200-20
	Jokar et al. (2011) [105]	ITPS	CLRP	200-10
	Jokar et al. (2012) [106]	SA	CLRP	100-10
	Escobar et al. (2013) [79]	2-Phase HGTS	CLRP	200-20
	Prikwieser et al. (2010) [165]	VNS+VLNS	CLRP+PLRP	**
	Derbel et al. (2011) [69]	VNS	CLRP	150-10
	Jabal-Ameli et al. (2011) [101]	VND	CLRP	200-20
	Jarboui et al. (2013) [102]	VND+VNS	UVLRP	200-20
	Barreto et al. (2007) [24]	Cluster analysis	CLRP	150-10
	Nadizadeh et al. (2011) [146]	GCM	CLRP	150-10
	Sodsoon (2010) [185]	MMAS	CLRP	85-7
	Ting et al. (2013) [196]	MACO	CLRP	200-20

Belenguer et al. [26] elaborated the first branch-and-cut algorithm based on the CLRP2 model. They strengthened the model by some new families of valid inequalities, such as *improved depot capacity* constraints, *improved path elimination* constraints, *co-circuit* constraints, *depot degree* constraints and inequalities derived from the CVRP. Based on a basic framework of B&C, they embedded all additional constraints in a cutting plane based on a detailed separation strategy, looked for a set of violated inequalities, and then added them to the relaxation linear program to resolve it until no violated inequality was found. The computational experiments based on 34 standard instances showed that 26 instances were solved to optimality by this B&C, including some problems with up to 50 customers and 5 depots. And also it provided feasible solutions for instances with up 85 customers and 7 depots. In the same year, Baldacci et al. [20] described a new exact method based on a set-partitioning formulation for solving the CLRP. They provided new efficient lower-bounding procedures, based on dynamic programming (DP) and dual ascent methods (DA). Then the produced lower bounds were applied by an algorithm to decompose the CLRP into a limited set of multi-depot vehicle routing problem (MDVRP). Finally an additional exact method was used to solve each MDVRP. The experimental results on 60 benchmark instances from the literature showed this exact method outperformed the previous ones proposed by Akca et al. [7] and Belenguer et al. [26], on the quality of the lower bounds produced and the number of instances solved to optimality. Furthermore, it solved to optimality the instances of uncapacitated LRP involving 199 customers and 14 depots.

Later, a branch-and-cut-and-price algorithm (BCP) was developed by Contrado et al. [49] to solve the CLRP. They introduced some new families of valid inequalities and added them into a two-index formulation to determine all the possible subsets of depot locations. Then two bounding procedures were applied sequentially to reduce the CLRP to a series of MDVRP based on the set-partitioning formulation. Finally, they solved the corresponding MDVRP by the means of column-and-cut generation. Through computational experiments on a large number of instances from the literature, the proposed algorithm was demonstrated to be able to produce tighter bounds for most instances and improved the best known feasible solutions for 7 instances, compared with the exact method of Baldacci et al. [20]. In addition, it solved four previously open instances to optimality including one with up to 200 customers and 10 depots. Recently, these authors [48] presented three new formulations for the CLRP (mentioned in last section), and compared the performance of the proposed ones with an existing *two-index vehicle-flow* formulation. They derived new valid inequalities and improved some previous ones to strengthen each formulation. Then, new suitable branch-and-cut algorithms were developed for each of the formulations, as well as new separation algorithms. Computational results based on a wide number of instances from the literature showed compact formulations were able to produce tighter gaps for most instances. And *three-index* formulations can solve problems more quickly, especially for some hard instances with up to 100 customers. Furthermore, they indicated that the proposed B&C could produce tighter gaps than the one of Belenguer et al. [26] on the *two-index vehicle-flow* formulation.

In many studies on heuristics and metaheuristic techniques developed to solve the CLRP, the first family is the genetic algorithm-based approach, such as a memetic algorithm with population management (MA|PM) [170], a genetic algorithm with a split procedure and a local search scheme (GAHLS) [73], and a genetic algorithm with iterated local search (GA+ILS) [71]. The detailed description on the application of the CLRP will be given in Chapter 4.

Another group of popular heuristics for the CLRP is local search based algorithms. Wu et al. [206] presented a simulated annealing-based (SA) decomposition approach for solving the multi-depot location-routing problem (MDLRP) with multiple fleet types. They divided the problem into two sub-problems, i.e. the location-allocation problem (LAP) and the general vehicle routing problem (VRP). Then each sub-problem was solved by SA hybridized some neighborhood moves in a sequential and iterative manner. The computational results on problems from the literature and newly created indicated that the proposed algorithm outperformed some previous heuristics for small and medium instances. In addition, it was effective to solve large-size instances with up to 150 customers and 30 depots. Prins et al. [172] developed a cooperative metaheuristic combining Lagrangean relaxation (LR) and a granular tabu search (GTS) heuristic to solve the CLRP. Firstly, they used a Lagrangean relaxation on a facility-location problem to select the depot locations. Then, the routes from the resulting MDVRP were improved by the proposed GTS. The new routes obtained were applied into a new location phase. In order to strengthen the cooperation between the two phases, information about edges most often used was recorded to build new routes. Computational results based on three sets of instances from the literature showed that the proposed LRGTS improved the best known solutions for most instances involving up to 200 customers with 10 capacitated depots or 20 uncapacitated depots. Furthermore, it was able to find optimal solutions on small instances with up to 29 customers and 5 depots within a competitive time.

Later, Derbel et al. [70] applied an iterated local search (ILS) to solve the LRP with an uncapacitated vehicle at each depot. This local search used four different neighborhood structures to modify routing sequence for obtaining a local optimum solution. Then a perturbation mechanism was added to improve the solutions until a stopping criterion was satisfied. The computational results showed that the proposed ILS was competitive in terms of solution quality and computing time compared with the tabu search heuristics of Albreda-Sambola et al. [8]. In addition, this method was able to solve some instances with up to 30 customers and 10 depots. Duhamel et al. [74] developed a new metaheuristic, consisting in a greedy randomized adaptive search procedure (GRASP), hybridized with an evolutionary local search (ELS) to solve the CLRP. It was called as GRASP×ELS. In this method, the GRASP was used to provide some initial solutions, which then were improved by the ELS. Instead of creating one child-solution at each iteration, the ELS generated more children-solutions and selected the best one at each iteration. And two levels of mutation were applied in the ELS including the *mutation on tour* and *mutation on hubs*. In the *mutation on tour*, the giant TSP tours without trip delimiters were produced and then converted into the LRP solutions via a splitting procedure subject to vehicle capacity, fleet size and depot capacities. Computational results on three sets of benchmark instances showed the GRASP×ELS outperformed the three other metaheuristics (MA|PM, GRASP and LRGTS) proposed by Prins et al. [170, 171, 172] for the number of best solutions found. It also improved a majority of best-known solutions including instances involving up to 200 customers and 20 depots. Meanwhile, Yu et al. [210] proposed a simulated annealing (SA) based heuristic for solving the CLRP and compared it with other heuristics in the literature including MA|PM [170], GRASP [171], LRGTS [172], clustering based heuristics (CH) [24] and GAHLS [73]. Their computational results indicated the proposed SA was competitive on the aspect of finding best solutions. For 85 LRP benchmark instances solved, this method obtained 78 best solutions including 52 new best solutions during the parameter analysis. Furthermore, it solved two large instances with up to 318 customers and 4 depots. A metaheuristic was developed by Contardo et al. [47] for the CLRP based on a greedy randomized adaptive search procedure (GRASP) and a new integer linear programming (ILP), i.e., a new location-reallocation model. In the first step of this approach, the GRASP was applied to construct a bundle of initial solutions followed by local search procedures. Then the ILP was solved to obtain a new solution as a combination of all routes in the solution bundle. In the final step, the ILP model was iteratively solved by a column generation algorithm to improve the solutions found in last two steps. Through comparing to seven previous heuristics published for the CLRP, the computational results on 89 benchmark instances showed the high competitiveness of the proposed algorithm in terms of solution quality and computing time. In total, it improved 17 best known feasible solutions including two instances with up to 200 customers and 20 depots.

Recently, Jokar et al. proposed an iterative two phase search (ITPS) based heuristic [105] and a new simulated annealing (SA) based algorithm [106] to solve the CLRP. In the first work, an initial solution population was generated by a greedy approach in phase 1 and then was improved by various neighborhood searches in phase 2. The computational results based on some benchmark instances showed this method was able to provide effective solutions, where the cost was less than 3% in average compared with the one obtained by the three published methods of Prins *et al.*. In the second work, the SA was compared to the method of Barret et al. [24]. Results on 11 instances showed the new SA improved some best known solutions with up to 100 customers and 10 depots. A new two-phase hybrid heuristic algorithm (2-Phase HGTS) was developed by Escobar et al. [79] in 2013 to solve the CLRP. In the first phase, an initial feasible solution was constructed by an initial hybrid procedure followed by a splitting procedure. And then a modified granular tabu search (GTS) was applied to improve the solution quality in the second phase. In addition, a randomized perturbation approach was applied in the GTS to escape from a local optimum. Computational results based on 79 benchmark instances from the literature showed the effectiveness of the proposed method. It improved 10 best known solutions within reasonable computing times and outperformed previous methods for larger instances with 200 customers, compared with five most effective published heuristics for the CLRP.

In addition, the neighborhood search based algorithms have also been widely studied to solve the CLRP. Pirkwieser et al. [165] presented a variable neighborhood search (VNS) for the CLRP and the periodic LRP, where the VNS was combined with three very large neighborhood searches (VLNS) based on integer linear programming. Derbel et al. [69] developed a general VNS for solving the CLRP, where many neighborhood structures and a shaking procedure were applied to diversify the search space and improve solutions. Computational study on 13 instances from the literature showed the proposed VNS obtained 11 best known solutions involving one with 150 customers and 10 depots. Meanwhile, a variable neighborhood descent (VND) based algorithm was proposed by Jabal-Ameli et al. [101] to solve the CLRP. They used a two-stage algorithm to generate the initial solutions in the first stage, and then improved them using the VND algorithm in the second stage. The computational experiments were carried on 55 benchmark instances from the literature. Results indicated that the proposed VND was able to provide high quality solutions with objective values not more than 5% in average of the ones obtained by other three published heuristics: GRASP of Prins et al. [171], LRGTS of Prins et al. [172] and SA of Yu et al. [210]. Recently, Jarboui et al. [102] integrated a VND algorithm as the local search in the general VNS framework to solve the LRP with multiple capacitated depots and uncapacitated vehicles (UVLRP). In this algorithm, they used simultaneously the routing and location neighborhood structures in the local search and shaking phases. Computational results based on large number of test instances from the literature showed the proposed method could provide high quality solutions in less average computing times, in comparison with the ILS of Derbel et al. [70] and the TS of Albreda-Sambola et al. [8].

In addition to the heuristics mentioned above, Barreto et al. [24] presented several hierarchical and non-hierarchical clustering techniques to solve the CLRP, which are included in a sequential heuristic algorithm. They proposed also a number of guide-lines concerning the choice of a suitable clustering technique. This approach has demonstrated its effectiveness in finding feasible solutions for instances with up to 150 customers and 10 depots in a reasonable computing time. Nadizadeh et al. [146] introduced a greedy clustering method (GCM) to solve the CLRP. This method consists in four phases to find the best solutions, i.e., clustering the customers using a greedy search algorithm subject to the vehicle capacity, selecting the location of depots to cover total demand of customers and minimize the total establishing cost, allocating the clusters to the open depots considering the capacity of depot and finally setting routes between depots and customers using ant colony system (ACS). They were repeated for a predefined number of iterations to obtain better solutions to replace last one until a stopping criterion was satisfied. Computational results showed the performance of the proposed algorithm was satisfactory. It solved all tested problems (19 standard instances) with the lowest average gap (1.33%)compared to the best known solutions from three other algorithms of the literature. In addition, it obtained 9 best solutions including 2 new ones. There are also some works focusing on the development of ant system based heuristics to solve the CLRP, such as a max-min ant system (MMAS) algorithm proposed by Sodsoon [185] and a multiple ant colony optimization algorithm (MACO) proposed by Ting et al. [196] in 2013.

Conclusions

For more details of the location-routing problem, one can refer to the review papers [140, 150, 176]. Min et al. [140] summarized the earliest researches on the combined LRP and proposed some research directions. It gave the classifications about problems, perspectives and solution methods. Later, a new review was given by Nagy and Salhi [150] who made a detailed description for the LRP and summarized the main achievements until 2007. It not only introduced the different formulations and application areas of the LRP, but also analyzed the main exact methods and heuristics. Recently, Prodhon and Prins [176] gave the newest survey about research works on the LRP and new extensions after 2007. It compared the results obtained by different metaheuristics based on the standard instances for the CLRP, the multi-echelon LRP (LRP-2E) and the multi-objective LRP. In addition, some research directions are deduced from the literature analysis.

As appears from our analysis, many variants of the location routing problem have been studied and many models and solution techniques have been proposed in terms of exact methods and metaheuristics. These methods can solve problems involving up to 20 depots and 200 customers. Since the research problem we are concerned with concerns both the location of hubs (with inter-hub flows) and collection or delivery routing, we will consider approaches related to the HLP, LRP and VRP in order to model and solve this problem.

2.2 The hub location-routing problem

2.2.1 Problem introduction and features

Unlike the previous related problems, there has been only few studies focusing on the global hub location-routing problem (HLRP) which is the focus of our research. As mentioned above, this problem can be considered as a combination of the hub location, location-routing and vehicle routing problems, which are often interdependent in real logistics applications. The location of hubs is often influenced by market areas, vehicle transport costs and organizational considerations. As the LRP is an approach to locate facilities simultaneously to corresponding vehicle routes, the HLRP can be regarded as an approach to model and solve hub location problems taking into account vehicle routing decisions. So in order to propose more accurate models and more realistic solutions in a hub-and-spoke network, in addition to determining the location of hub terminals and the assignment of spokes, the HLRP also considers local tour planning between spokes and hubs. In some researches, this problem has been called the many-to-many location-routing problem (MMLRP) [149]. In this section, a state of the art about the HLRP is given including its characteristics, the relationship with other related problems, application areas and solution methods.

It seems that Nagy and Salhi presented the first work related to the hub location-routing problem in 1998 [149], which was called the MMLRP. In this problem, it is assumed that each customer at location i sends a different commodity q_{ij} to some other customers j. The customer locations may be independent or feasible sites for a consolidation terminal. Finally, a fleet of vehicles is available. The objective of the MMLRP is to design a system able to exchange commodities between customer pairs, determining the number and location of terminals, and the number and routes of vehicles in order to satisfy the transportation demands while minimizing the total fixed and variable costs of the terminals and transport activities. This is probably the first definition for the HLRP. These authors viewed this system as a *two-level network*, the hub level, determining the hub locations and the access level designing the routing decisions, respectively. In addition, they pointed out that a customer may be served by two vehicles (one for pickups and one for deliveries) if these operations are not made at the same time. This problem may occur in the freight transport [130] or in the postal industries [40, 66, 180, 203]. In the freight transport, the pickups and deliveries are made separately while they are performed simultaneously in the postal system. We can also distinguish the uncapacitated or the capacitated variants of this problem [66] or the p-hub variant [40, 180]. For the vehicle routing constraints, some studies focus on the tour length limitation [40, 66] while others limit the number of customers in each tour [180].

Çetiner et al. [40] combined the hubbing and routing problem and applied it to the Turkish postal delivery system. They described the HLRP as a problem of *locating hubs and generating multiple-stop routes for the non-hub points allocated to the hubs*. They formulated an integrated bi-objective problem combining the multiple allocation p-hub median problem with the multiple vehicle routing problem with uncapacitated hubs and vehicles. Camargo et al. [66] extended the definition including also the routing of the flow of many origins to many destinations at a minimal cost and they called this problem the many-to-many hub location-routing problem (MMHLRP). Recently, Rodríguez-Martín [180] combined the single allocation p-hub median problem and the vehicle routing problem in a postal system and assumed that each hub operated only one route to serve its customers.

Based on the above descriptions and definitions of different researchers, the HLRP can be described by the following characteristics:

1. There are three different sets in the network, including origins sites, destination sites and feasible hubs sites. Their geographical locations can either be disjoint or not.
2. The HLRP considers the corresponding relationship of demand flows between each origin and destination (O-D) pairs and not only the demand of individual origin or destination. Therefore, the routing of flows from origins to destinations should be designed, as well as both pick-up and delivery routes.
3. At the hub level, the terminal location has to be determined and all terminals are assumed to be connected to all the others. The links between terminals are direct links.
4. At the access level, the routing decisions serve the customers with vehicles starting from and returning to the same terminal.
5. The pick-up and delivery processes can be operated simultaneously or not. But they should be implemented in a cycle tour starting and ending at same hub.
6. For the location of the hubs, fixed costs of establishing the hubs and also transportation cost between hubs are considered.

As mentioned in the hub location problem, the hub-and-spoke network for freight transport and postal systems consists in three phases: the first one is the collection phase from origins to hubs, the second one is the exchange phase between different hubs and the third one is the distribution phase from hubs to destinations. So in order to simultaneously meet the needs of many origins and destinations, the Hub Location Routing Problem (HLRP) can be defined to determine the number and the location of hubs, the assignment of the non-hub nodes to the hubs, the local vehicle routing for collection and delivery simultaneously or not, and the routing of flows from origins to destinations, with the objective of minimizing the total cost including hub costs, inter-hub transportation costs and vehicle routing costs. From this definition, we can see that the HLRP considers all the decisions associated with the HLP and the VRP. And differently from the LRP, which only considers one type of route (collection or delivery), the HLRP deals with both collection and delivery routes. In addition, the HLRP takes into account the exchanges of flows between origins and destinations and the transfers between hubs, which are not involved in the LRP. The differences between the related problems and the HLRP on the aspect of decisions making are summarized in Table 2.5. From this table, it can be seen that many logistics problems are special cases of the HLRP, such as the hub location problem if the local routing costs are not considered, and the LRP if there is no inter-hub flow and if only collection or delivery processes are considered.

Table 2.5: The differences between the HLP, LRP and HLRP

Decision / Problem	Locate hub	Allocate non-hub nodes	Design vehicle routes	Consider flow between origin and destination
HLP	×	×		×
VRP			×	
MDVRP		×	×	
LRP	×	×	×	
HLRP	×	×	×	×

2.2.2 Modeling and solution methods

In contrast to the HLP, VRP and the LRP, which have been the subject of many researches, only very few works have directly addressed the HLRP. Nagy and Salhi [149] proposed a mathematical formulation for the many-to-many location-routing problem (MMLRP) with capacity. They introduced one variable to represent whether the pickup and delivery at one customer location were served simultaneously or not in the model. However, this formulation leads to a large number of variables and constraints even for a small number of customers. So in order to solve this complex problem, they developed a hierarchical heuristic solution framework based on the concept of "nested methods", and they solved the problem in three stages including the location, the routing and their inter-relation. Finally, one instance with 249 customers was solved to illustrate the usefulness of the proposed method. Liu et al. [130] studied a mixed truck delivery system that allows both hub-and-spoke and direct shipment delivery modes. They introduced a heuristic algorithm to determine the delivery mode for each supplier-customer pair and to perform vehicle routing with two modes. They compared the mixed system with the pure hub-and-spoke system and the pure direct shipment system. The experimental results showed that the mixed system can save around 10% of the total traveling distance on average, compared with the two pure systems. Although the system considered pickup and delivery routing in a hub-and-spoke network, it included only one hub. It can therefore be treated as a 1-HLRP with direct shipments.

In addition, some researchers have focused on the partitioning-hub location-routing problem (PHLRP) which is indeed a hub location problem involving graph partitioning and routing features. Gourdin et al. [162] first introduced three integer programming formulations of the PHLRP and compared them. Catanzaro et al. [38] explored possible valid inequalities to strengthen the IP model and introduced a branch-and-cut algorithm to solve the PHLRP which contained 20 vertices. Ta et al. [194] presented a binary integer linear programming model and a new method based on the difference of convex function algorithm to solve larger problems with up to 25 vertices.

Until now, most of the work on the HLRP has been applied to postal service networks where the pickup and delivery usually occur simultaneously. Wasner et al. [203] developed a two-layer hub location and vehicle routing model for an Austrian parcel delivery service. In their non-linear model, they considered the number of trips between hubs, the quantity transferred between depots directly or across the hubs, the number of pickup and delivery routes, and the capacity of the vehicles. To solve this non-linear model, a heuristic solution was proposed, based on a sequence of local search procedures. According to the authors, the key idea of the proposed method was to divide the transportation process into two interrelated problems: the pickup and delivery design and the line haul design. Then, the heuristics determines the decision variables and parameters. The solution concept is based upon a parallel method and includes many feedbacks and iterations to improve the solution obtained from each step. Finally, one case was studied to illustrate the method. Çetiner et al. [40] developed a two-stage method that includes locating and routing for an uncapacitated-length-limited HLRP for the Turkish postal delivery system. The heuristic contains hubbing and routing stages which iterate during the whole procedure using an updating scheme of the distance used in the first stage. A case study for the Turkish postal service with 81 nodes was developed and tested to prove the effectiveness of this model and method.

More recently, Camargo et al. [66] proposed a new formulation and a tailored Benders decomposition algorithm for the many-to-many hub location-routing problem (MMHLRP) with uncapacitated hubs in a parcel delivery network. The model consisted of locating the hubs, generating local tours to service the non-hub nodes and to connect the non-hub nodes to the installed hubs, and routing the flows. In this problem, it is assumed that the pickups and the deliveries may occur simultaneously and that a constraint for each local tour limits the maximum distance allowed. In the proposed algorithm, the formulation was divided into the master problem to provide a lower bound (LB) and the subproblem to provide an upper bound (UB) to reach an optimal solution. Computational results based on the instances inspired from AP data set confirmed the efficiency and robustness of the algorithm which can solve instances up to 100 nodes. Rodríguez-Martín et al. [180] presented a mixed integer programming formulation and proposed a branch-and-cut algorithm for the single allocation p-hub location and routing problem in the postal industry. The model determines the location of p-hubs, the allocation of the nodes to the hubs and the routing among the nodes allocated to the same hubs. They assumed that the uncapacitated hub nodes were directly connected to each other and each hub could operate one route at most to serve all the nodes assigned to it on a cycle. The objective function minimizes the sum of the cost of assigning nodes to hubs, the cost of routing between hubs and the cost of routing within a cycle. In the proposed B&C, the authors introduced some valid inequalities to strengthen the formulation and checked the violated ones at each node of the search tree with a cutting plane method. Through some computational experiments based on instances from the CAB and the AP data set, they demonstrated that the proposed method succeeds in solving instances with up to 50 nodes. In addition, they analyzed the impacts of different model parameters on the solutions and the cost structures of different solutions.

Today, more and more researchers consider incorporating the routing decisions into the location problem. Recently, Kartal et al. [112] presented a mathematical mini-max model which considered the integration of uncapacitated single allocation p-hub center and multi depot multiple traveling salesman problems. The goal of the model was to minimize maximum route lengths without capacity. Two heuristics based on simulated annealing and random descent were introduced and tested on CAB, AP and Turkish data sets. Armin et al. [132] presented a formulation for the p-hub location and vehicle routing problem for which they proposed two exact solution approaches (branch-and-cut and Benders decomposition). Their model was also based on the postal delivery system. Computational experiments were performed on the AP data set ranging from 10 nodes to 25 nodes with CPLEX. Rieck et al. [179] focused on an extension of the many-to-many location-routing problem (MMLRP) which considered the inter-hub transport and multi-commodity pickup-and-delivery. They divided the network into three layers and integrated direct shipment to determine the location of hubs and vehicle routing of pickups and deliveries. They presented a mixed-integer linear program (MIP) for this problem with capacitated vehicle and uncapacitated hubs. Computational study shown some small-scale problem instances (5 pick points, 2 potential hubs and 8 delivery points) generated can be solved to optimality using solver CPLEX based on this model. In addition, a heuristic multi-start procedure (MSP) based on a fix-and-optimize scheme and a genetic algorithm (GA) were developed for solving larger-scale instances. In each iteration of the fix-and-optimize scheme, a subset of binary decision variables is set to one based on an initial solutions and then re-optimize the problem with a branch-and-bound framework. In the proposed genetic algorithm, a multi-dimensions matrix was used to represent an individual. The results obtained by CPLEX, the MSP heuristic and the genetic algorithm were compared to evaluate the performance of each method in solving different size of instances. They shown that the proposed heuristics can give some feasible solutions for the instances with up to 40 pickup point, 6 potential hubs and 100 delivery points. The detailed comparison between the above articles about the HLRP and our study can be seen in Table 2.6.

Table 2.6: Detailed comparison between related works on the HLRP

Article	Characteristic							
	Location	Allocation	Number of hubs	Hub constraint	Routing constraint	Solution	Application area	Size
Nagy et al. (1998)[149]	yes	single	not fixed	capacitated	length	Hierarchical heuristic	One instance	249
Liu et al. (2003)[130]	no	single+direct shipment	one hub	uncapacitated	length	Heuristic	Random instances	25
Wasner et al. (2004)[203]	yes	multiple+direct shipment	not fixed	capacitated	capacitated	Heuristic	Austria postal	10
Çetiner et al. (2010) [40]	yes	multiple	p hubs	uncapacitated	length	two-stage heuristic	Turkish postal	81
Camargo et al. (2013) [66]	yes	single	not fixed	uncapacitated	length	Benders decomposition	AP	100
Rodriguez-Martin et al. (2014) [180]	yes	single	p hubs	uncapacitated	number of nodes	B&C	CAB+AP	50
Rieck et al. (2014) [179]	yes	single+direct shipment	p hubs	uncapacitated	capacitated	Multi-start procedure +GA	timber-trade industry	140
Our research	yes	single	not fixed	capacitated	capacitated	MA and B&C	freight and postal	100

2.2.3 Conclusions

The state of the art about the HLRP indicates that, among the few works on this problem, most of them concerns parcel delivery applications where collections and deliveries may be performed simultaneously and within the same routes. Moreover, very few works take into account the capacity constraint for vehicles and there is a lack of models and methods addressing the HLRP for general LTL freight shipments. Regarding solution methods, some proposed heuristics and solved real cases. Others developed exact methods to solve small and medium size instances. However, it is not possible to compare the different approaches as most researches tackled practical applications or studied different variants. We therefore feel that there is a need for developing generic models and methods for tackling hub location and routing problems for LTL shipment for both cases involving simultaneous and separate collections and deliveries, with and without vehicles or hub capacities.

2.3 General conclusions and research directions

This section provided a literature review on the problems related to the hub location-routing problem and on the HLRP itself. This review was focused on problems related to our research problematic. Firstly, we introduced the definition, features, variants and applications for each related problem including the hub location problem (HLP), the vehicle routing problem (VRP) and the location-routing problem (LRP). Then the mathematical formulations and solution methods were summarized for each problem, especially the single allocation HLP, the

CVRP and the CLRP. Finally, a comparison between related problems and the HLRP was given, followed by a review of all works published on this topic.

From this literature review, it can be seen that although many works focus on related problems, the hub location routing problem has received little attention until now especially for applications to general LTL freight transport. Most of the papers on the HLRP deal with particular applications in which the vehicle capacity is not taken into account, facilitating the mixed collection and delivery of products in the same tour, such as it is the case in the postal services area. Even if heuristics or exact solution methods have been proposed, only small and medium problems can be solved. From this literature review, we can notice that no research has been conducted onto the HLRP with separated collections and deliveries, corresponding nevertheless to real applications as for trucking companies specialized in the transportation of loads less than 3 tons, necessitating this kind of organization to reduce costs and achieve the time restrictions for deliveries (less than 24h or less than 48h).

Thus the development of a general model for the the HLRP in with distinct collections and deliveries and corresponding efficient exact and approximate solution methods appears to be an interesting avenue of research. The literature on the HLP, LRP and VRP inspires us for the modeling of this problem and provides us hints for the most promising solution methods. The literature review also tells us the necessity to develop specific solution methods and in particular metaheuristics if we want to solve realistic problems.

In the following of the thesis, we present mathematical formulations for the hub location-routing problem for general freight transport. We develop a metaheuristic method in order to solve realistic large size instances in a reasonable time. This metaheuristic of the memetic algorithm framework combines a genetic algorithm and an iterative local search. These techniques have been successfully used for VRPs and LRPs. Inspired from the efficient exact methods for the VRP, we also propose a new branch and cut method. This technique has already proven its efficiency for LRP problems to outperform commercial solvers on computing time and quality of solutions. Finally we adapt the proposed model and metaheuristics in order to solve the classical postal service hub location routing problem.

3

Models for the capacitated single allocation hub location-routing problem

In this chapter, we introduce a general description of the capacitated single allocation hub location-routing problem (CSAHLRP) considered in this research. Two mathematical models for this problem are presented. The first model includes a formulation with a 4-index variable and the second one contains a formulation with a 3-index variable to reduce the size of the model. Finally, we compare the complexity and the performance of the two models solving small to medium instances with a commercial solver.

3.1 Problem definition

To illustrate the operation of the hub location-routing problem in LTL shipments, an example of the organization and network is shown in Figure 3.1 where the squares, circles and triangles represent the candidate hubs, the origins of goods (suppliers) and the destinations (clients) respectively. The bold edges represent the transfer arcs connecting hubs. The dashed edges represent the local collection arcs connecting suppliers to one hub while the simple solid edges stand for the local delivery arcs. Since we consider the general case where collections and deliveries are made separately, we distinguish two kinds of tours. For both collection or delivery tours, there are two possible routing modes. One is a single-node route such as routes $R1$ or $R2$. The other is a multiple-nodes route like routes $R3$ or $R4$.

As can be seen from Figure 3.1, there are therefore three types of node sets: the set of suppliers which are the origins of flows; the set of clients which are the destinations of flows; and the set of candidate hubs which serve as service and transit centers for both suppliers and clients. Then the transportation process can be divided into three components in order to ship the flows from any supplier to its clients: the collection process from suppliers to hubs, the transfer process between hubs and the delivery process from hubs to clients. Because the activities of collection from suppliers and distribution to clients are considered separately, a supplier and a client cannot be serviced to the same tour. In this research, for each O-D pair, we allow shipment through a one hub or two hubs rather than direct transport. No more than two hub

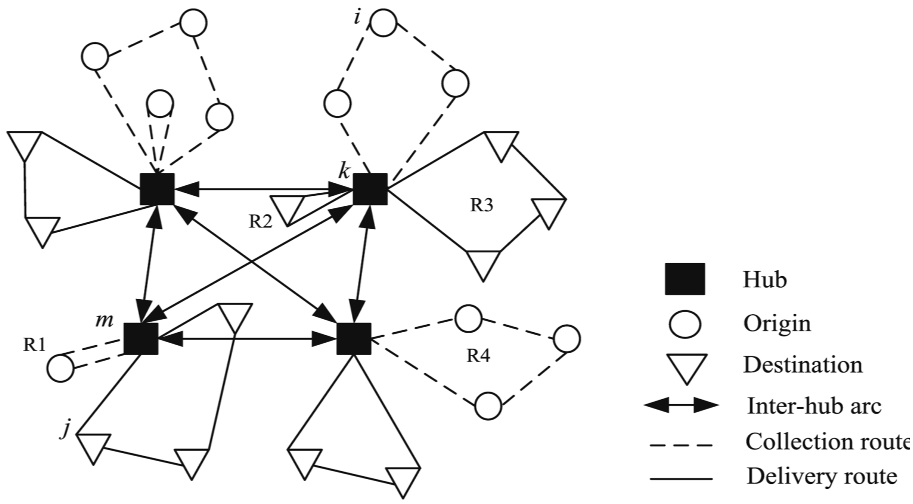


Figure 3.1: The network of the HLRP in LTL shipments

stops are considered because we assume that the distance matrix satisfies the triangle inequalities. So more than two hub stops would be meaningless. In addition, likewise for the HLP, there are many variants of the HLRP based on different hypothesis, such as single/multiple allocation, capacitated/uncapacitated and fixed cost/p-hub problems. Here, we consider a capacitated single allocation hub location-routing problem (CSAHLRP). It includes the following characteristics and hypothesis:

(1) Every potential hub is capacitated and subject to a fixed opening and operating cost. Then the total quantity of flow assigned to a hub can't exceed its capacity.

(2) The number of hubs to be selected is not fixed, which is decided by the decision process.

(3) Each tour associated to a hub corresponds to one vehicle.

(4) Each node (supplier or client) is assigned to one selected hub (single allocation).

(5) Each supplier or client is visited exactly once by one vehicle (no splitting of collections or deliveries).

(6) The total quantity of flow on each tour is limited to at most the vehicle capacity.

(7) Each tour starts and ends at the same selected hub.

(8) The total quantity of flow from suppliers (supply) to clients (demand) has to be transported.

(9) The collections and deliveries are done independently (no mixed tours).

(10) There are no direct connection between origins and destinations, and all the flows must go through at most two hubs.

(11) For the vehicles, we consider a homogeneous fleet with a capacity and a fixed cost.

(12) The transportation cost between hubs is assumed to depend on the distance and the flow quantity transferred.

(13) The routing costs of the vehicles for the local collection and delivery tours are assumed to only depend on the distance of the traversed arcs.

Also, the geographic position of suppliers, clients and potential hubs are known in advance, as well as the quantities of flows to be shipped between each O-D pair. Then in order to optimize the transportation network and minimize the total cost, the CSAHLRP consists in deciding which potential hub will be selected, and in assigning every supplier and client to only one hub among the open ones. At the same time, for each hub-suppliers group or hub-clients group, it is needed to design the optimal tours to service them. In the next section, mathematical models for the CSAHLRP are proposed, based on the above characteristics.

3.2 Mathematical formulations

In this section, two mathematical models for the CSAHLRP and some formulation improvements are presented. The hub location parts are derived from some classical HLP formulations [78, 184]. For the routing parts, they are based on a three-index VRP formulation [200], and improved by ideas from the multi-depot vehicle-routing problem [206].

3.2.1 Notation and overview

We consider a complete graph $G = (N, E)$ with a vertex set N and a complete connected edge set E. Associated with the network G, a distance matrix is defined, where the elements are the distance between two nodes $(i, j) \in E, i \in N, j \in N$. We consider also a demand matrix being the flow quantity for each supplier-client pair. As suggested in the problem definition above, the process of routing the flow from suppliers to clients, can be viewed as a set of three components: the collection process from suppliers to hubs, the transfer process between hubs and the delivery process from hubs to clients. Different parameters reflect the unit costs for the transfer, collection and delivery processes, respectively. Based on the above description and hypothesis of Section 3.1, the notations used to formulate the model are presented below:

- **Sets**

  $H-$ set of potential hubs $k \in H$;

  $I-$ set of suppliers $i \in I$;

  $J-$ set of clients $j \in J$;

  $N-$ set of all nodes, $N = H \cup I \cup J$;

  $V-$ set of vehicles $v \in V$.

- **Parameters**

  F_k- fixed cost of operating hub k;

  Γ_k- capacity of hub k;

  f_v- fixed cost of a vehicle v;

  $Q-$ capacity of a vehicle;

  $q_{ij}-$ flow quantity from supplier $i \in I$ to client $j \in J$;

  $d_{ij}-$ distance between two nodes $(i, j) \in E, i \in N, j \in N$;

  $\alpha-$ unit cost parameter for the inter-hub transport;

  $\beta-$ unit cost parameter for the collection tour;

  $\gamma-$ unit cost parameter for the delivery tour;

  O_i- total quantity of flow originating at supplier i, $O_i = \sum_{j \in J} q_{ij}$;

  D_j- total quantity of flow for client j, $D_j = \sum_{i \in I} q_{ij}$;

- **Decision Variables**

  $Y_{ijkl}-$ the fraction of flow shipped from supplier i to client j via hubs k and l. It is a classical decision variable for the HLP which indicates that the flow from one supplier to one client is routed along the path $i \to k \to l \to j$. Node k and l can refer to a single hub node.

  $z_{ik}-$ the allocation variable of a node i to a hub k. It is equal to 1 if the node i is allocated to the hub k, 0 otherwise; especially, $z_{kk} = 1$ if the hub k is selected to be open. It is also a classical variable for the HLP.

x_{ij}^v- is equal to 1 if arc (i,j) is served by vehicle v, 0 otherwise. This variable indicates the visiting order of each local tour;

Z_i^v- the binary allocation variable of a non-hub node i to a vehicle v. It is equal to 1 if node i is allocated to vehicle v.

$U_{iv}-$ Auxiliary variables for sub-tours eliminations.

Based on the above description, collection from suppliers and delivery to clients are considered separately, a supplier and a client cannot be allocated to the same local tour, in accordance with our hypothesis. Moreover, the number of hubs required is not imposed and will result from the optimization, taking into account the capacity restrictions and the fixed costs for potential hubs, in particular. In the next section, we present the formulation of the problem with 4-index variables for flows.

3.2.2 4-index formulation of CSAHLRP

To clarify the presentation of the model, we divide the constraints into four parts: the hub location constraints; the collection routing constraints; the delivery routing constraints and the constraints on the values of variables. Then, based on the above notations, the initial mathematical formulation of the CSAHLRP with 4-index (*CSAHLRP-F4-0*) is presented as follows, with the goals of minimizing the total cost and meeting the service requirements:

CSAHLRP-F4-0

$$\text{Min} \quad \sum_{k\in H} F_k z_{kk} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in H}\sum_{l\in H} \alpha d_{kl} q_{ij} Y_{ijkl} + \sum_{v\in V}\sum_{i\in I\cup H}\sum_{j\in I\cup H, j\neq i} \beta d_{ij} x_{ij}^v + \sum_{v\in V}\sum_{i\in J\cup H}\sum_{j\in J\cup H, j\neq i} \gamma d_{ij} x_{ij}^v + \sum_{v\in V}\sum_{k\in H}\sum_{i\in I\cup J} f_v x_{ki}^v \tag{3.1}$$

subject to

—hub location constraints:

$$z_{ik} \le z_{kk} \qquad \forall i \in N, \forall k \in H \tag{3.2}$$

$$\sum_{k\in H} z_{ik} = 1 \qquad \forall i \in I \cup J \tag{3.3}$$

$$\sum_{l\in H} Y_{ijkl} = z_{ik} \qquad \forall i \in I, \forall j \in J, \forall k \in H \tag{3.4}$$

$$\sum_{k\in H} Y_{ijkl} = z_{jl} \qquad \forall i \in I, \forall j \in J, \forall l \in H \tag{3.5}$$

$$\sum_{i\in I}\sum_{j\in J}\sum_{l\in H} q_{ij} Y_{ijkl} \le \Gamma_k z_{kk} \qquad \forall k \in H \tag{3.6}$$

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in H} q_{ij} Y_{ijkl} \le \Gamma_l z_{ll} \qquad \forall l \in H \tag{3.7}$$

—collection routing constraints:

$$\sum_{i\in I}\sum_{j\in J} q_{ij} Z_i^v \le Q \qquad \forall v \in V \tag{3.8}$$

$$\sum_{i \in I \cup H} x_{ij}^v - \sum_{i \in I \cup H} x_{ji}^v = 0 \qquad \forall v \in V, \forall j \in I \cup H \tag{3.9}$$

$$\sum_{u \in I \cup H} (x_{ku}^v + x_{ui}^v) \leq 1 + z_{ik} \qquad \forall i \in I, \forall k \in H, \forall v \in V \tag{3.10}$$

$$\sum_{v \in V} Z_i^v = 1 \qquad \forall i \in I \tag{3.11}$$

$$\sum_{i \in I \cup H} x_{ij}^v = Z_j^v \qquad \forall j \in I, \forall v \in V \tag{3.12}$$

$$\sum_{j \in I \cup H} x_{ij}^v = Z_i^v \qquad \forall i \in I, \forall v \in V \tag{3.13}$$

$$\sum_{i \in H} \sum_{j \in I} x_{ij}^v \leq 1 \qquad \forall v \in V \tag{3.14}$$

$$\sum_{i \in I} \sum_{j \in H} x_{ij}^v \leq 1 \qquad \forall v \in V \tag{3.15}$$

—delivery routing constraints:

$$\sum_{j \in J} \sum_{i \in I} q_{ij} Z_j^v \leq Q \qquad \forall v \in V \tag{3.16}$$

$$\sum_{i \in J \cup H} x_{ij}^v - \sum_{i \in J \cup H} x_{ji}^v = 0 \qquad \forall v \in V, \forall j \in J \cup H \tag{3.17}$$

$$\sum_{u \in J \cup H} (x_{ku}^v + x_{uj}^v) \leq 1 + z_{jk} \qquad \forall j \in J, \forall k \in H, \forall v \in V \tag{3.18}$$

$$\sum_{v \in V} Z_j^v = 1 \qquad \forall j \in J \tag{3.19}$$

$$\sum_{i \in J \cup H} x_{ij}^v = Z_j^v \qquad \forall j \in J, \forall v \in V \tag{3.20}$$

$$\sum_{j \in J \cup H} x_{ij}^v = Z_i^v \qquad \forall i \in J, \forall v \in V \tag{3.21}$$

$$\sum_{i \in H} \sum_{j \in J} x_{ij}^v \leq 1 \qquad \forall v \in V \tag{3.22}$$

$$\sum_{i \in J} \sum_{j \in H} x_{ij}^v \leq 1 \qquad \forall v \in V \tag{3.23}$$

—constraints on decision variables:

$$U_{iv} - U_{jv} + |I| x_{ij}^v \leq |I| - 1 \qquad \forall v \in V, \forall i \in I, \forall j \in I \tag{3.24}$$

$$U_{iv} - U_{jv} + |J| x_{ij}^v \leq |J| - 1 \qquad \forall v \in V, \forall i \in J, \forall j \in J \tag{3.25}$$

$$\sum_{i \in H} \sum_{j \in H} x_{ij}^v = 0 \qquad \forall v \in V \tag{3.26}$$

$$0 \leq Y_{ijkl} \leq 1 \qquad \forall i \in I, \forall j \in J, \forall k \in H, \forall l \in H \tag{3.27}$$

$$z_{ik} \in \{0,1\} \qquad \forall i \in N, \forall k \in H \tag{3.28}$$

$$x_{ij}^v \in \{0,1\} \qquad \forall i \in N, \forall j \in N, \forall v \in V \tag{3.29}$$

$$Z_i^v \in \{0,1\} \qquad \forall i \in I \cup J, \forall v \in V \tag{3.30}$$

$$U_{iv} \geq 0 \qquad \forall i \in I \cup J, \forall v \in V \tag{3.31}$$

In this model, the objective function (3.1) minimizes the sum of the fixed hub operating costs, transportation cost between hubs, local collection routing cost, local delivery routing cost and fixed costs of vehicles. The meaning of constraints is explained below:

Constraints for Hub location

- Constraints (3.2) verify that a spoke (a supplier or client node) is allocated to an open hub.
- Constraints (3.3) impose that a spoke is assigned to one hub (single allocation).
- Constraints (3.4) and (3.5) represent the coherence between allocation variables and flow variables. They show that if a non-hub node is allocated to a hub, then all of the flows from or to this non-hub node should pass through this hub.
- Constraints (3.6) and (3.7) are hub capacity constraints for the collection and the delivery processes, respectively.

Constraints for collection routing

- Constraints (3.8) are the vehicle capacity constraints.
- Constraints (3.9) are the flow conservation constraints which ensure the continuity of every node visited by one vehicle.
- Constraints (3.10) ensure the connection of location variables and routing variables. They specify that a supplier can be assigned to a hub only if there is a vehicle from that hub going through that supplier.
- Constraints (3.11) impose that a supplier can only be served by a single vehicle.
- Constraints (3.12) and (3.13) ensure the connection of two variables. They show the continuity of visit.
- Constraints (3.14) and (3.15) represent that each vehicle can be used once at most starting and ending at one hub.

Constraints for delivery routing

- Constraints (3.16)-(3.23) impose the same conditions defined for the local collection routing, but for the set of clients. They have the same meanings as the collection routing constraints.

Constraints on decision variables

- Constraints (3.24) and (3.25) are sub-tour elimination constraints for collection and delivery routing, respectively.
- Constraints (3.26) represent that there are no collection or delivery routes between hubs.
- Constraints (3.27)-(3.31) are value constraints for variables Y_{ijkl}, z_{ik}, x_{ij}^v, Z_i^v and U_{iv}, respectively.

We can see that the *CSAHLRP-F4-0* formulation is linear but in order to simplify the model, we note that variable Z_i^v can be replaced by x_{ij}^v, because a tight relationship exists between the two variables in constraints (3.12), (3.13), (3.20) and (3.21). So *CSAHLRP-F4-0* can be improved as follows:

- **Improvement 1:** Vehicle capacity constraints (3.8) and (3.16) can be replaced by the following two inequalities, respectively:

$$\sum_{i\in I}\sum_{j\in I\cup H} O_i x_{ij}^v \leq Q \qquad \forall v \in V \tag{3.32}$$

$$\sum_{j\in J}\sum_{i\in J\cup H} D_j x_{ij}^v \leq Q \qquad \forall v \in V \tag{3.33}$$

Demonstration:
To prove this property, let us consider expression (3.8) as an example:

$$\textstyle\sum_{i\in I}\sum_{j\in J} q_{ij} Z_i^v \leq Q \qquad \forall v \in V$$

As

$$\textstyle\sum_{j\in J} q_{ij} = O_i \qquad \forall i \in I$$

Then we get:

$$\sum_{i\in I} O_i Z_i^v \leq Q \qquad \forall v \in V \tag{3.34}$$

since

$$\textstyle\sum_{j\in I\cup H} x_{ij}^v = Z_i^v \qquad \forall i \in I, v \in V$$

Therefore, the left side of constraints (3.8) can be expressed as $\sum_{i\in I}\sum_{j\in I\cup H} O_i x_{ij}^v$. And then we get the new vehicle capacity constraints (3.32). Similarly for delivery process, equations (3.33) are valid for delivery vehicle capacity constraints.

- **Improvement 2:** Single visiting constraints (3.11) and (3.19) can be changed to the following expression, respectively:

$$\sum_{v\in V}\sum_{j\in I\cup H} x_{ij}^v = 1 \qquad \forall i \in I \tag{3.35}$$

$$\sum_{v\in V}\sum_{i\in J\cup H} x_{ij}^v = 1 \qquad \forall j \in J \tag{3.36}$$

Demonstration:
Here, we consider constraints (3.11) as an example to prove this proposition:

$$\textstyle\sum_{v\in V} Z_i^v = 1 \qquad \forall i \in I$$

As

$$\textstyle\sum_{j\in I\cup H} x_{ij}^v = Z_i^v \qquad \forall i \in I, v \in V$$

Then, we can get:

$$\textstyle\sum_{v\in V}\sum_{j\in I\cup H} x_{ij}^v = \sum_{v\in V} Z_i^v = 1 \qquad \forall i \in I$$

Therefore, equations (3.35) are valid for the single visiting constraint and similarly for constraints (3.36). They show that there must be only one vehicle visiting a given supplier or one client.

Considering the above improvements, variable Z_i^v is no more necessary for the *CSAHLRP* model, as well as equations (3.12), (3.13), (3.20) and (3.21). We then obtain a new mathematical formulation with the 4-index variable, denoted as *CSAHLRP-F4*, which includes objective function (3.1) and the following constraints:

- hub location constraints (3.2)-(3.7);

- collection routing constraints including equations (3.9), (3.10), (3.14), (3.32) and (3.35);
- delivery routing constraints including equations (3.17), (3.18), (3.22), (3.33) and (3.36);
- sub-tour elimination constraints and value constraints for variables consisting of expressions (3.26)-(3.31) and (3.33).

The objective of model *CSAHLRP-F4* is still to minimize the total cost including the fixed hub costs, transportation cost between hubs, local tours cost and fixed costs of vehicles. The constraints have the same meanings as the ones in *CSAHLRP-F4-0*. Even if it has been improved, *CSAHLRP-F4* still contains a large number of variables and constraints. So in order to reduce the size of the model, a 3-index mathematical formulation is introduced in next section.

3.2.3 3-index formulation of CSAHLRP

Based on the analysis for the 4-index mathematical model presented in the previous section, we can see that variable Y_{ijkl} induces a large number of variables and constraints for the CSAHLRP. So in order to reduce the size of the model, inspired by a model of the hub location problem (HLP) [78], the second component of the HLRP, i.e. the inter-hub transfers, can be treated as a multi-commodity flow problem. The flow from each supplier i can be represented with a commodity i. Then a new variable Y_{ikl} is defined as the fraction of total flow of commodity i (i.e. the flow from supplier i) that is routed from hub k to l. Except for this new decision variable, the notations used in 3-index model remain the same as in *CSAHLRP-F4*.

For this new model, we also divide the constraints into four parts: the hub location constraints; the collection routing constraints; the delivery routing constraints and the constraints on the values of variables. Then, based on the above definitions, the 3-index mathematical formulation for the CSAHLRP (*CSAHLRP-F3*) is proposed as follows:

CSAHLRP-F3

$$\begin{aligned}\text{Min}\quad & \sum_{k\in H} F_k z_{kk} + \sum_{i\in I}\sum_{k\in H}\sum_{l\in H} \alpha d_{kl} O_i Y^i_{kl} + \sum_{v\in V}\sum_{i\in I\cup H}\sum_{j\in I\cup H, j\neq i} \beta d_{ij} x^v_{ij} \\ & + \sum_{v\in V}\sum_{i\in J\cup H}\sum_{j\in J\cup H, j\neq i} \gamma d_{ij} x^v_{ij} + \sum_{v\in V}\sum_{k\in H}\sum_{i\in I\cup J} f_v x^v_{ki} \end{aligned} \tag{3.37}$$

subject to

- hub location constraints (3.2), (3.3) and

$$\sum_{i\in I} O_i z_{ik} \le \Gamma_k z_{kk} \qquad \forall k \in H \tag{3.38}$$

$$\sum_{j\in J} D_j z_{jl} \le \Gamma_l z_{ll} \qquad \forall l \in H \tag{3.39}$$

$$\sum_{l\in H} Y^i_{kl} = z_{ik} \qquad \forall i \in I, \forall k \in H \tag{3.40}$$

$$\sum_{l\in H} Y^i_{lk} O_i = \sum_{j\in J} q_{ij} z_{jk} \qquad \forall i \in I, \forall k \in H \tag{3.41}$$

- collection routing constraints including (3.9), (3.10), (3.14), (3.32) and (3.35);
- delivery routing constraints including (3.17), (3.18), (3.22), (3.33) and (3.36);

- finally the sub-tour elimination constraints and the value constraints for variables consisting of expressions (3.24)-(3.26), (3.28), (3.29), (3.31) and

$$0 \leq Y^i_{kl} \leq 1 \qquad \forall i \in I, \forall k \in H, \forall l \in H \tag{3.42}$$

The objective function also minimizes the sum of the fixed hub costs, transportation cost between hubs, collection routing cost, delivery routing cost and fixed costs of vehicles. Constraints (3.38) and (3.39) are hub capacity constraints for collection and delivery, respectively. Constraints (3.40) and (3.41) are the flow conservation equations at the hubs. They show that if an O-D node is allocated to a hub, then all the flow from or to the node should pass through this hub. Constraints (3.42) are constraints on the values of variables Y^i_{kl}. It can be any real number between 0 and 1.

In comparison to *CSAHLRP-F4*, the above formulation reduces the problem size due to the introduction of variable Y^i_{kl}. For example, if we let $|H| = k$, $|I| = |J| = n$, *CSAHLRP-F4* will have n^2k^2 variables Y_{ijkl}. And it requires n^2k constraints (3.4) and (3.5). However, *CSAHLRP-F3* will have nk^2 variables Y^i_{kl} and nk constraints (3.40) and (3.41). Then the problem size of *CSAHLRP-F3*, both in number of variables and constraints, has been reduced by a factor of n. The comparison of problem size between the two models with 6 potential hubs can be seen in Figure 3.2. It shows that *CSAHLRP-F3* has less variables and constraints for the same number of non-hub nodes.

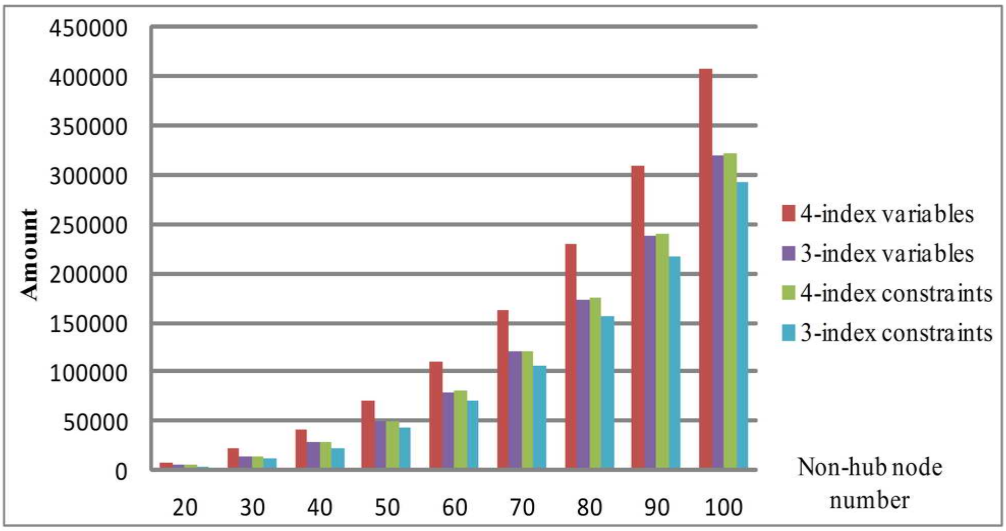


Figure 3.2: The size comparison of *CSAHLRP-F4* and *CSHLRP-F3* with 6 potential hubs

3.2.4 Valid inequalities

In this section, in order to strengthen the previous two formulations, we introduce two families of valid inequalities. The first valid inequality is to restrict the number of open hubs:

$$\sum_{k \in H} z_{kk} \geq z_{min} \tag{3.43}$$

where z_{min} is the minimum number of open hubs to be able to handle the total quantity of flows from the suppliers or to the clients. That is,

$$\sum_{k=1}^{z_{min}} \Gamma_k \geq max(\sum_{i \in I} O_i; \sum_{j \in J} D_j) \tag{3.44}$$

where the hub capacities are indexed in decreasing order. Especially, when the hub capacities are assumed identical, this inequality becomes the following:

$$z_{min} * \Gamma_k \geq max(\sum_{i \in I} O_i; \sum_{j \in J} D_j) \tag{3.45}$$

As $\sum_{i \in I} O_i = \sum_{j \in J} D_j = \sum_{i \in I, j \in J} q_{ij}$ and the z_{min} is an integer, then there is an obvious lower bound defined as $\lceil \frac{\sum_{i \in I, j \in J} q_{ij}}{\Gamma_k} \rceil$, where $\lceil x \rceil$ represents the smallest integer not less than x. Thus, the valid inequality (3.43) can be replaced by the following polynomial-size inequality when the hub capacities are identical:

$$\sum_{k \in H} z_{kk} \geq \lceil \frac{\sum_{i \in I, j \in J} q_{ij}}{\Gamma_k} \rceil \tag{3.46}$$

The second valid inequality provides a lower bound on the total number of vehicles used for collection and delivery in any integer feasible solution, respectively. This family of valid inequalities is inspired from the capacitated vehicle routing problem [200]. They are represented as follows:

$$\sum_{i \in H} \sum_{j \in I} \sum_{v \in V} x_{ij}^v \geq \lceil \frac{\sum_{i \in I} \sum_{j \in J} q_{ij}}{Q} \rceil \tag{3.47}$$

$$\sum_{i \in H} \sum_{j \in J} \sum_{v \in V} x_{ij}^v \geq \lceil \frac{\sum_{i \in I} \sum_{j \in J} q_{ij}}{Q} \rceil \tag{3.48}$$

Here, $\lceil x \rceil$ denotes the smallest integer not less than x. Inequalities (3.47) impose a minimum number of vehicles in the collection process to handle the total flows from suppliers and (3.48) limits the vehicle number necessary for the delivery process. The above valid inequalities are polynomial-size and can be added directly to our models to solve the CSAHLRP.

3.3 Instances generation

In order to solve the CSAHLRP, we needs to generate sets of instances, because none are directly available in the literature for this problem. To that purpose, we have decided to use a set of randomly generated instances inspired from the AP data set introduced by Ernst and Krishnamoorthy [78] to generate our instances networks nodes and arcs. In addition, we have generated cost parameters on the basis of the cost data base of the French Comité National Routier (CNR) [1].

The standard AP data set has been used to generate instances ranging from 10 to 200 nodes. From these generated instances, we only retained the coordinates of nodes (obtained at *OR-*

1. http://www.cnr.fr/en

Library[2]). Because our HLRP considers collections and deliveries separately, it is necessary to distinguish the set of potential hubs, suppliers and clients. Hence, we used the initial node coordinates of AP data sets to generate the node coordinates of the above three sets randomly. Then we generated the demand flows between each supplier-client pair and the other parameters concerning hubs and vehicles (fixed costs and capacities) based on the informations from the CNR data base, to obtain realistic values for our problem.

Thus, based on the CNR informations and the characteristics of our HLRP, we set the capacity of all vehicles Q to 15 t. The fixed cost of vehicles f_v and variable unit costs α, β and γ have been calculated using the formula presented in Table 3.1. The fixed daily cost for a vehicle has been retained at 200 €. As the collection or delivery tours take place on a half-day, we set f_v to 100 € as the fixed cost of each local tour. In addition, because the unit cost α is assumed to depend on the distance and the flow quantity between hubs and β, γ only depend on the distance of tours, we obtained their values from the formulas shown in Table 3.1. So, α is set to 0.057 €/km.t, and β, γ are set to 0.8 €/km.

Table 3.1: The parameter values for vehicle

Basic dates from CNR		Dates for HLRP parameters		
Name	Value	Name	Formula	Value
Load capacity Q	15 t	Fixed cost for tour f_v	$f_0/2$	100 €
Cost per kilometer C_k	0.4 €/km	Unit transfer cost α	$(C_k + C_h/v_1)/Q + f_v/(Q * d_{ave})$	0.057 €/km.t
Fixed daily cost f_0	200 €	Unit collection cost β	$C_k + C_h/v_2$	0.8 €/km
Cost by hour C_h	20 €/h	Unit delivery cost γ	$C_k + C_h/v_2$	0.8 €/km

Note: d_{ave} is the average inter-hub distance. Here, it is set to 500 km. And v_1, v_2 is the average speed of one vehicle for inter-hub transfer and local tour, respectively. Here, we set $v_1 = 80km/h$ and $v_2 = 50km/h$.

The instance names are denoted $|H| - |I| - |J|$, $|H| \in \{3, 6, 10\}$ representing the number of potential hubs, $|I| = |J| \in \{5, 10, 15, 20, 20, 25, 30, 35, 40, 45, 50\}$ the number of suppliers and clients respectively. As mentioned above, for each instance, the coordinates of suppliers and clients have been selected randomly from the AP data sets and the potential hubs have been selected, also randomly, from the supplier and client sets. For example, the initial 10-nodes AP instance can be used to generate the HLRP instances 3-5-5, the initial 20-nodes instance can generate the HLRP instances 3-10-10, 6-10-10 and 10-10-10, and so on. The generation process of nodes coordinates can be seen in Algorithm 1. Normally, potential hubs can be preselected by decision makers based on geography, economy or environment factors. Thus a part of known node locations were generated here as the locations of potential hubs. The size of instances differ by the number of suppliers. The small instances consider a number of suppliers from 5 to 15, the medium ones from 20 to 25 suppliers, and the large instances are the ones with 30 to 50 suppliers. Obviously, this method can generate many different instances for the CSAHLRP. Here, we generated one group of test sets shown in Table 3.2 to conduct our first experiments and compare the different models. The results will be shown in the next section based on the small and medium instances. This group contains 23 HLRP instances, where the suppliers and clients can be located at the same geographic locations. Then, the distances d_{ij} are computed

2. http://people.brunel.ac.uk/ mastjjb/jeb/orlib/phubinfo.html

as follows: first the coordinates are divided by 100, the Euclidean distances are calculated and then rounded to the nearest integer which should require the triangular inequality.

Algorithme 1 Generation of node coordinates

1: **Input**: Initial node coordinates of AP data set N_0;
2: **Set** $|I| = |N_0|/2$ and $I \Leftarrow RandomGenerator(N_0)$;
3: **Set** $|J| = |N_0|/2$ and $J \Leftarrow RandomGenerator(N_0)$;
4: **Get** the union set $H^{'} = I \cup J$;
5: **Set** $|H| \in \{3, 6, 10\}$ and $H \Leftarrow RandomGenerator(H^{'})$;
6: **Return** the coordinates of each set and compute the distance d_{ij}.

The quantity of flow between suppliers and clients is determined uniformly in the interval [0.5, 3.0] such that the total flow O_i, $\forall i \in I$ and D_j, $\forall j \in J$ can not exceed 15t. In addition, when the coordinates of a supplier/client couple are the same, the flow is set to 0, because, this kind of flow doesn't require a hub transfer and so is not considered in this model. The hub capacities Γ_k are the same for all the hubs and three values are considered to ensure the opening at least 3, 2, 1 hubs, respectively. The hub capacities can also accommodate an integer number of vehicles. For each instance, the value of Γ_k that we retained can be seen in Table 3.2.

Table 3.2: Description of experiment instances

Instance size	Instance name	Hub capacity (t)
Small	3-5-5	15/30/45
	3 (6, 10)-10-10	45/60/120
	3 (6, 10)-15-15	45/75/135
Medium	3 (6, 10)-20-20	60/90/165
	3 (6, 10)-25-25	75/105/195
Large	6 (10)-30-30	90/120/240
	6 (10)-35-35	90/135/270
	6 (10)-40-40	90/135/255
	6 (10)-45-45	105/150/285
	6 (10)-50-50	105/150/300

Finally, to set the hub fixed costs for the purpose of experimenting our models, we have conducted a sensitivity analysis on a possible range of costs to analyze its influence on solutions. We have tested four values for the hub fixed costs $F_k \in \{2000, 1000, 500, 100\}$ and three values for the hub capacities. All the results presented in Table 3.3 have been obtained with CPLEX 12.5 on model *CSAHLRP-F3* within 3 hours of computation. In this table, the first column indicates the instance name and characteristics. For example, instance "3-10-10" refers to an instance with 3 candidate hubs, 10 suppliers and 10 clients. The "Hub_Cap" column indicates the value of the hub capacities. The third column gives the different values for F_k. And the next two columns represent the total cost and the cost without hub fixed cost of the best solution found in limited time. Next four columns show the percentage of each cost component on total cost, involving the fixed hub cost C_1, the inter-hub transportation cost C_2, the local collection and delivery cost C_3 and the vehicle fixed cost C_4, respectively. Then the "Open hub" column shows the name of open hubs in the best solution. From this table, we can see that for most instances, the hub fixed cost has no obvious influence on the best solutions until it is set to a small value (100 €). But for instances 3-10-10 and 6-10-10 with a hub capacity of 45t, it can be seen that opening a larger number of hubs can yield lower values for $Cost^{'}$ when F_k decreases. For example when $F_k = 500$, 3 hubs are open for the 3-10-10-45 instance, at a cost of 4885.32 €. However when $F_k = 1000$, 2 hubs are open at a cost of 5613.94 €. Therefore, through this preliminary analysis of the solutions, we have decided to set F_k to 1000 € for each hub for the following experiments.

Table 3.3: Sensitivity analysis on hub fixed cost for small instances

H-I-J	Cap_hub	F_k	$Cost_{total}$	$Cost^{'}$	$C_1\%$	$C_2\%$	$C_3\%$	$C_4\%$	Open hub
3-5-5	15	2000	5867.85*	1867.85	68.17	1.44	23.57	6.82	1, 2
		1000	3867.85*	1867.85	51.71	2.19	35.76	10.34	1, 2
		500	2867.85*	1867.84	34.87	2.95	48.23	13.95	1, 2
		100	2067.85*	1867.85	9.67	4.09	66.89	19.34	1, 2
	30	2000	4068.00*	2068.00	49.16	0.00	41.00	9.83	1
		1000	3068.00*	2068.00	32.59	0.00	54.37	13.04	1
		500	2568.00*	2068.00	19.47	0.00	64.95	15.58	1
		100	2067.85*	1867.85	9.67	4.09	66.89	19.34	1, 2
	45	2000	4068.00*	2068.00	49.16	0.00	41.00	9.83	1
		1000	3068.00*	2068.00	65.19	0.00	54.37	13.04	1
		500	2568.00*	2068.00	19.47	0.00	64.95	15.58	1
		100	2067.85	1867.85	9.67	4.09	66.89	19.34	1, 2
3-10-10	45	2000	9899.26	5926.26	40.41	3.18	40.53	16.16	2, 3
		1000	7613.94	5613.94	26.27	5.16	47.55	21.01	2, 3
		500	6385.32	4885.32	23.49	8.34	44.68	23.49	1, 2, 3
		100	5054.35	4754.35	5.94	11.24	53.15	29.68	1, 2, 3
	60	2000	8828.25	4828.24	45.31	4.39	33.31	16.99	2, 3
		1000	6828.25*	4828.24	29.29	5.67	43.07	21.97	2, 3
		500	5828.25*	4828.24	17.16	6.65	50.46	25.74	2, 3
		100	4884.31*	4584.31	6.14	10.57	52.58	30.71	1, 2, 3
	120	2000	7249.60*	5249.60	27.59	0.00	51.72	20.69	2
		1000	6249.60*	5249.60	16.00	0.00	60.00	24.00	2
		500	5749.60*	5249.60	8.70	0.00	65.21	26.09	2
		100	4884.31*	4584.31	6.14	10.57	52.58	30.71	1, 2, 3
6-10-10	45	2000	9896.50	5896.50	40.42	2.63	40.78	16.17	2, 3
		1000	7613.94	5613.94	26.27	5.16	47.55	21.01	2, 3
		500	6251.81	4751.51	23.99	8.54	43.46	23.99	2, 3, 5
		100	4690.78	4090.78	12.79	16.67	36.43	34.11	1, 2, 3, 4, 5, 6
	60	2000	8943.80	4943.80	44.72	4.85	33.65	16.77	2, 3
		1000	6828.25	4828.24	29.29	5.67	43.07	21.97	2, 3
		500	5943.80	4943.80	16.82	7.31	50.63	25.24	2, 3
		100	4671.45*	4071.45	12.84	16.46	36.44	34.25	1, 2, 3, 4, 5, 6
	120	2000	7249.60	5249.60	27.59	0.00	51.72	20.69	2
		1000	6249.60	5249.60	16.00	0.00	60.00	24.00	2
		500	5749.60	5249.60	8.70	0.00	65.21	26.09	2
		100	4671.45	4017.45	12.84	16.46	35.29	34.25	1, 2, 3, 4, 5, 6

3.4 Models comparison

To compare our two models (*CSAHLRP-F4* and *CSAHLRP-F3*), we coded them and the valid inequalities in C++ language and used CPLEX 12.5 to solve both of them with the same default parameters setting. All computational experiments were conducted on an Intel Core i3 CPU of 3 GHz and 8 GB of memory. The running time for CPLEX has been limited to three hours. Because CPLEX can't obtain solutions for larger instances in the time limit, all experiments in this section have been conducted only for small and medium size instances. The results are presented in Tables 3.4-3.7.

3.4.1 Comparison of *CSAHLRP-F4* and *CSAHLRP-F3*

The first experiments compare the performance of the two models: *CSAHLRP-F4* and *CSAHLRP-F3*. For both of them, we use the same upper bound value for each instance. It is obtained from the best objective value among the two formulations. In addition to the general final lower gap at the end of search tree, we also provide the initial lower gap at the root node to compare the effectiveness of the two models. The comparison results are presented in Tables 3.4 and 3.5. The following notations are used in the two tables:

- $z(UB)$: best objective value found by CPLEX in the time limit. For each instance, it is computed as $z(UB) = min\{z_{F4}, z_{F3}\}$, where z_{F4} is the best objective value based on *CSAHLRP-F4*, and z_{F3} is the one based on *CSAHLRP-F3*. The mark "*" denotes that the solution is optimal.
- $\%LB_0$: deviation in % of the best objective value from the lower bound obtained at the root node by CPLEX; it is computed as $\%LB_0 = \frac{z(UB)-LB_0}{z(UB)} \times 100\%$, where LB_0 is the initial lower bound obtained at the root node on each model.
- T_{LB_0}: computing time in seconds to get the initial lower bound LB_0.
- $\%LB$: the deviation in % between the best objective value and the final lower bound found by CPLEX within the time limit. Here, $\%LB = \frac{z(UB)-LB}{UB} \times 100\%$, where LB is the best lower bound.
- $\%UB$: the deviation in % between the best objective value and the corresponding upper bound based on each model. Here, $\%UB = \frac{z_{F4}||z_{F3}-z(UB)}{z(UB)} \times 100\%$.
- T_{total}: total computing time in seconds.

Table 3.4: Comparison of *CSAHLRP-F4* and *CSAHLRP-F3* with 3 potential hubs

Instance name		$z(UB)$	CSAHLRP-F4					CSAHLRP-F3				
H-I-J	Hub_Cap		$\%LB_0$	T_{LB_0}	$\%LB$	$\%UB$	T_{total}	$\%LB_0$	T_{LB_0}	$\%LB$	$\%UB$	T_{total}
3-5-5	15	3867.85*	12.99	0.37	**0.00**	**0.00**	8.03	11.96	0.90	**0.00**	**0.00**	8.55
	30	3068.00*	19.00	0.34	**0.00**	**0.00**	4.87	19.00	0.28	**0.00**	**0.00**	4.40
	45	3068.00*	19.00	0.28	**0.00**	**0.00**	5.01	19.00	0.30	**0.00**	**0.00**	4.48
3-10-10	45	7613.94	19.59	4.21	4.93	**0.00**	10800.60	20.93	2.04	**4.38**	**0.00**	10800.10
	60	6828.25*	13.80	1.68	0.98	**0.00**	10800.90	13.55	2.03	**0.00**	**0.00**	3632.55
	120	6249.60*	19.36	1.78	1.60	**0.00**	10800.40	18.97	1.58	**0.00**	**0.00**	5878.04
3-15-15	45	10614.21	24.39	3.93	18.03	**0.00**	10800.30	23.29	7.04	**17.49**	0.37	10800.20
	75	8940.28	21.54	6.63	16.20	1.00	10800.60	21.06	9.94	**13.71**	**0.00**	10800.10
	135	8232.80	26.29	7.47	11.21	**0.00**	10800.70	27.75	4.10	**9.06**	**0.00**	10800.70
3-20-20	60	12186.05	32.48	20.19	28.23	2.02	10800.80	32.53	19.95	**26.33**	**0.00**	10800.50
	90	10469.24	32.10	11.58	28.29	7.06	10800.90	31.75	21.72	**20.61**	**0.00**	10800.60
	165	9336.80	32.98	33.68	27.37	**0.00**	10800.00	34.71	10.56	**25.68**	**0.00**	10800.70
3-25-25	75	13165.30	26.39	57.11	24.71	3.24	10800.40	26.32	50.53	**23.28**	**0.00**	10800.10
	105	13121.13	33.77	62.20	31.96	0.88	10800.80	33.77	64.60	**31.76**	**0.00**	10800.50
	195	11325.60	32.10	79.95	**27.20**	7.51	10800.10	32.10	50.19	27.99	**0.00**	10800.00
Average			**24.39**	19.43	14.71	1.45		24.45	**16.38**	**13.35**	**0.02**	

Table 3.4 presents the performance comparison between the two models, for the small and medium size instances with 3 potential hubs. The numbers in bold indicate the best results obtained. In this table, based on the values of $\%UB$, we can see that *CSAHLRP-F3* can obtain the best objective values for 14 instances among 15. Conversely, *CSAHLRP-F4* has found only 9 best solutions. *CSAHLRP-F3* has found finds 5 optimal solutions against 3 for *CSAHLRP-F4*. Especially for the instance 3-10-10, *CSAHLRP-F3* finds the optimal solutions for larger hub capacity in a shorter time. And also, *CSAHLRP-F3* obtains tighter gaps than *CSAHLRP-F4* regarding to the values of $\%LB$ (13.35% vs 14.71% in average). In addition, to get the initial

lower bound LB_0, *CSAHLRP-F3* is faster than *CSAHLRP-F4* (16.38 seconds vs 19.43 seconds, in average).

For the instances with more potential hubs (6 and 10), Table 3.5 shows that *CSAHLRP-F3* outperforms *CSAHLRP-F4* for all of the instances to find better objective values within the time limit in terms of $\%UB$ (0.00% vs 3.34% in average). *CSAHLRP-F3* can find better solutions for all of 18 instances, while *CSAHLRP-F4* can reach them only for 6 instances. However, in terms of lower bound, especially at the root node, *CSAHLRP-F4* obtains a tighter gap than *CSAHLRP-F3* (24.53% vs 24.97% in average for the value of $\%LB$). But *CSAHLRP-F3* is still faster than *CSAHLRP-F4* (36.23 seconds vs 92.33 seconds, in average) to obtain the lower bound at the root node. Therefore, for the small and medium instances, *CSAHLRP-F3* can find better solutions than *CSAHLRP-F4* within the time limit. Detailed results for each formulation are presented in the next section.

Table 3.5: Comparison of *CSAHLRP-F4* and *CSAHLRP-F3* on instances with 6 and 10 potential hubs

Instance name		$z(UB)$	CSAHLRP-F4					CSAHLRP-F3				
H-I-J	Hub_Cap		$\%LB_0$	T_{LB_0}	$\%LB$	$\%UB$	T_{total}	$\%LB_0$	T_{LB_0}	$\%LB$	$\%UB$	T_{total}
6-10-10	45	7613.94	25.96	7.44	14.25	2.05	10800.90	24.63	17.32	**11.42**	**0.00**	10800.20
	60	6828.25	14.58	21.12	6.93	**0.00**	10800.10	16.38	16.83	**3.55**	**0.00**	10800.90
	120	6249.60	19.18	18.97	6.36	**0.00**	10800.50	22.47	12.11	**5.95**	**0.00**	10800.80
6-15-15	45	9608.55	21.23	32.70	**15.65**	2.20	10800.40	22.28	16.52	16.88	**0.00**	10800.30
	75	8940.28	25.73	29.38	**17.91**	1.00	10800.40	27.43	32.46	19.58	**0.00**	10800.4
	135	8232.80	29.68	28.88	15.34	**0.00**	10800.10	31.99	15.18	**14.38**	**0.00**	10800.00
6-20-20	60	10903.48	30.26	68.31	**27.07**	4.02	10800.50	30.92	39.53	28.24	**0.00**	10800.40
	90	10469.24	36.47	82.06	32.19	11.97	10800.9	37.73	43.20	**31.23**	**0.00**	10800.10
	165	9479.20	38.71	115.25	33.37	**0.00**	10800.60	40.13	47.64	**29.25**	**0.00**	10800.30
10-10-10	45	7549.08	25.46	68.91	**18.64**	0.72	10800.20	31.62	38.84	19.46	**0.00**	10800.00
	60	6896.10	20.94	36.54	**12.66**	**0.00**	10800.10	24.54	41.59	13.81	**0.00**	10800.60
	120	6249.60	24.57	31.36	7.72	**0.00**	10800.90	29.91	12.36	**5.62**	**0.00**	10800.30
10-15-15	45	9600.12	28.05	68.16	**23.03**	6.96	10800.70	30.06	30.58	25.95	**0.00**	10800.70
	75	8755.91	33.04	106.89	**24.89**	3.04	10800.70	34.13	51.40	29.02	**0.00**	10800.00
	135	8333.60	36.19	161.26	**23.38**	6.55	10800.20	41.52	28.00	25.61	**0.00**	10800.70
10-20-20	60	11466.44	40.12	207.92	**35.67**	7.02	10800.40	40.58	55.40	37.46	**0.00**	10800.00
	90	10611.96	45.22	167.59	**37.92**	7.48	10800.20	45.22	40.83	39.07	**0.00**	10800.40
	165	10469.24	54.03	179.76	42.46	7.06	10800.30	54.03	49.97	**38.95**	**0.00**	10800.00
Average			**32.65**	92.33	**24.53**	3.34		34.81	**36.23**	24.97	**0.00**	

3.4.2 Solutions analysis

In this section, we present the detailed results obtained with CPLEX on the two formulations for the small and medium size instances. Table 3.6 is based on the *CSAHLRP-F4* model, while Table 3.7 shows the results from the *CSAHLRP-F3* model. The optimal solution is reported with the mark "*". The best lower bound found is also reported if the time limit is reached. The following notations are used in Table 3.6 and 3.7.

- UB: upper bound found by CPLEX in a limited time on each formulation, marked "*" if the solution is optimal;
- LB: final lower bound or best lower bound found by CPLEX;
- $\%LB$: deviation in % of the corresponding upper bound from the lower bound found by CPLEX. Here, $\%LB = \frac{UB-LB}{UB} \times 100\%$;
- CPU time(s): computing time in seconds ;
- Open hubs: index of the located hubs;

- Max of routes: maximum number of routes assigned to a hub for collection and delivery together in the optimal or best solution.

Table 3.6: Experiments results obtained on *CSAHLRP-F4* for small and medium instances

H-I-J	Hub_Cap	UB	LB	$\%LB$	CPUtime (s)	Open hub	Max of routes
3-5-5	15	3867.85*	3867.85	0.00	8.03	1, 2	2
	30	3068.00*	3068.00	0.00	4.87	1	4
	45	3068.00*	3068.00	0.00	5.01	1	4
3-10-10	45	7613.94	7238.84	4.93	10800.60	2, 3	8
	60	6828.25	6761.66	0.98	10800.90	2, 3	9
	120	6249.60	6149.43	1.60	10800.40	2	15
3-15-15	45	10614.21	8700.37	18.03	10800.30	1, 2, 3	7
	75	9030.06	7491.54	17.04	10800.60	1, 3	11
	135	8232.80	7309.67	11.21	10800.70	1	18
3-20-20	60	12431.80	8746.01	29.65	10800.80	1, 2, 3	10
	90	11208.20	7507.93	33.01	10800.90	2, 3	15
	165	9336.80	6781.47	27.37	10800.00	2	20
3-25-25	75	13591.96	9912.11	27.07	10800.40	1, 2, 3	11
	105	13235.94	8928.26	32.55	10800.80	1, 3	16
	195	12175.86	8245.60	32.28	10800.10	1, 3	19
6-10-10	45	7769.82	6529.27	15.97	10800.90	2, 3	8
	60	6828.25	6354.95	6.93	10800.10	2, 3	9
	120	6249.60	5852.00	6.36	10800.50	2	15
6-15-15	45	9820.39	8104.40	17.47	10800.40	1, 3, 4	7
	75	9030.06	7338.70	18.73	10800.40	1, 3	11
	135	8232.80	6969.98	15.34	10800.10	1	18
6-20-20	60	11341.95	7951.78	29.89	10800.50	1, 2, 4, 5	9
	90	11722.40	7099.69	39.43	10800.90	2, 4	15
	165	9479.20	6315.92	33.37	10800.60	2	20
10-10-10	45	7603.18	6141.83	19.22	10800.20	5, 7, 8	6
	60	6896.10	6023.26	12.66	10800.10	2, 10	9
	120	6249.60	5767.31	7.72	10800.90	2	15
10-15-15	45	10268.51	7389.08	28.04	10800.70	1, 5, 7	8
	75	9022.46	6576.63	27.11	10800.70	5, 7	12
	135	8879.20	6385.09	28.09	10800.20	7	19
10-20-20	60	12271.94	7376.51	39.89	10800.40	3, 6, 8	9
	90	11405.52	6587.89	42.24	10800.20	2, 9	13
	165	11208.20	6024.20	46.25	10800.30	2, 3	15

The results in Table 3.6 show that CPLEX can find a feasible solution within the time limit of three hours for *CSAHLRP-F4* for all of the 33 instances, but only three optimal solutions. Table 3.7 also shows a good performance of *CSAHLRP-F3*, which can solve all of the instances within the time limit and find 5 optimal solutions. For the other instances, it can find a better solution than *CSAHLRP-F4*. However, the gap is still large. We conclude that it will be necessary to consider the development of a meta-heuristic and a specific exact method in order to better solve these instances.

Table 3.7: Experimental results obtained on *CSAHLRP-F3* for small and medium size instances

H-I-J	Hub_Cap	UB	LB	$\%LB$	CPU time(s)	Open hub	Max of routes
3-5-5	15	3867.85*	3867.85	0.00	8.55	1, 2	2
	30	3068.00*	3068.00	0.00	4.40	1	4
	45	3068.00*	3068.00	0.00	4.48	1	4
3-10-10	45	7613.94	7280.72	4.38	10800.10	2, 3	8
	60	6828.25*	6828.25	0.00	3632.55	2, 3	9
	120	6249.60*	6249.60	0.00	5878.04	2	15
3-15-15	45	10653.01	8758.11	17.79	10800.20	1, 2, 3	7
	75	8940.28	7714.49	13.71	10800.10	1, 3	11
	135	8232.80	7487.25	9.06	10800.70	1	18
3-20-20	60	12186.05	8977.83	26.33	10800.50	1, 2, 3	11
	90	10469.24	8312.03	20.61	10800.60	2, 3	13
	165	9336.80	6939.54	25.68	10800.70	2	20
3-25-25	75	13165.30	10099.80	23.28	10800.10	1, 2, 3	11
	105	13121.13	8954.44	31.76	10800.50	1, 2, 3	12
	195	11325.60	8155.36	27.99	10800.00	3	25
6-10-10	45	7613.94	6744.25	11.42	10800.20	2, 3	8
	60	6828.25	6585.81	3.55	10800.90	2, 3	9
	120	6249.60	5877.90	5.95	10800.80	2	15
6-15-15	45	9608.55	7986.42	16.88	10800.30	1, 3, 5	6
	75	8940.28	7189.37	19.58	10800.40	1, 3	11
	135	8232.80	7049.12	14.38	10800.00	1	18
6-20-20	60	10903.48	7823.82	28.24	10800.40	2, 4, 5	10
	90	10469.24	7199.82	31.23	10800.10	2, 3	13
	165	9479.20	6706.20	29.25	10800.30	2	20
10-10-10	45	7549.08	6080.37	19.46	10800.00	2, 3, 10	7
	60	6896.10	5943.59	13.81	10800.60	2, 10	9
	120	6249.60	5898.25	5.62	10800.30	2	15
10-15-15	45	9600.12	7108.60	25.95	10800.70	3, 5, 7	6
	75	8755.91	6214.67	29.02	10800.00	7, 8	11
	135	8333.60	6199.54	25.61	10800.70	7	18
10-20-20	60	11466.44	7171.53	37.46	10800.00	2, 4, 8	9
	90	10611.96	6465.97	39.07	10800.40	4, 5	12
	165	10469.24	6391.69	38.95	10800.00	2, 3	13

From the best solutions of all instances (see Tables 3.6 and 3.7), it can be seen that the number of open hubs in the solutions decreases and their index changes when the hub capacity increases. Most of the objective values also decrease because less hubs are operated to satisfy the total quantity of flow. But for some instances, the number of open hubs doesn't change even if the hub capacity increases. In this case, the hub fixed cost is no longer the main factor in the objective value. Moreover, open more hubs can possibly achieve a lower cost. For example for instance 3-25-25 with 105 t. of capacity, *CSAHLRP-F4* finds a solution with a cost of 13235.94 Euros and 2 open hubs. However, *CSAHLRP-F3* finds a better one with a cost of 13121.13 Euros and 3 open hubs. While the vehicle capacity remains unchanged, the route composition can change and sometimes the number of routes increases with a lower total cost. Figure 3.3 and 3.4 illustrate this effect through the optimal or best solutions obtained for instances 3-5-5 and 6-10-10. They provide some insights into the results obtained with CPLEX for solving this network design problem for general goods shipments. In these figures, the circles, the triangles and the squares represent the suppliers, the clients and the selected hubs, respectively. The dotted lines, solid lines and the solid lines with double arrows represent the collection arcs, delivery arcs and inter-hub arcs, respectively.

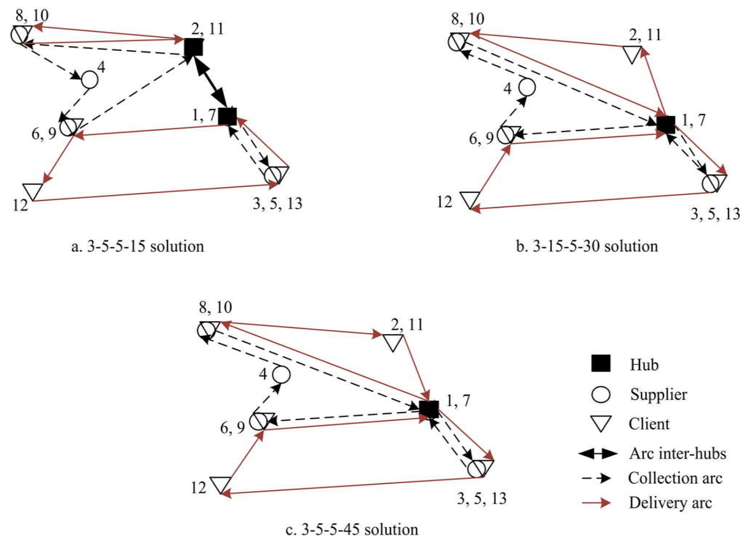


Figure 3.3: Solution illustration of instances 3-5-5

Figure 3.3 illustrates the results obtained with the 3-5-5 instance for three different values of hub capacities. According to the data generation procedure, the suppliers, clients and potential hubs may have been located at the same geographical position. So in this instance, potential hub sites 1, 2 and 3 have been randomly selected at the same geographical locations as supplier nodes 7, client node 11 and supplier node 5, respectively. From Figure 3 a, b and c, it can be seen that the optimal solutions found have obvious differences as the hub capacity is changing. For example, the optimal solution for the instances 3-5-5-30 and 3-5-5-45, will open one hub (number 1). All suppliers/clients are assigned to this hub to exchange the commodity flow through two collection routes and then two delivery routes (see Fig.3.3 b and c). However, for the 3-5-5-15 instance, two hubs (number 1 and 2), are selected and there are four routes designed to complete the commodity exchanges: one collection tour and one delivery tour for each hub (see Fig.3.3 a).

For the 6-10-10 instance (Figure 3.4), some suppliers, clients and potential hubs may also have been located at the same geographical position. Supplier 7 and client 18 are such an example, as well as hub 2, supplier 12 and client 17. The flow from supplier 12 is collected to hub 2 through a collection tour including other suppliers. After sorting and consolidation of goods in the hub, the flow to client 17 is delivered through another delivery tour (see Fig.3.4

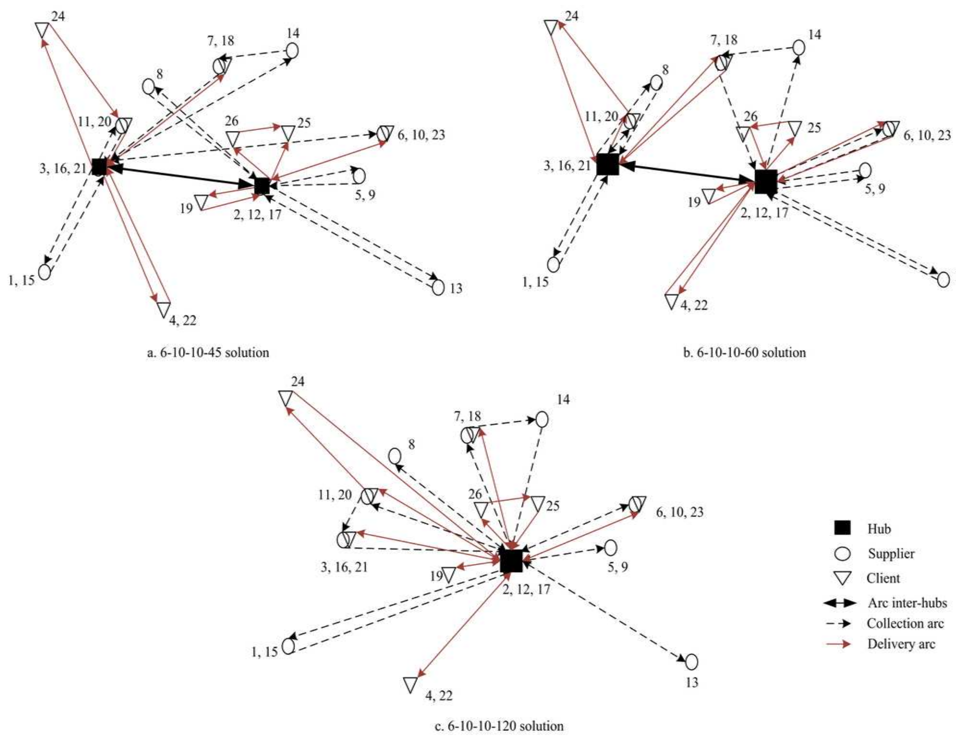


Figure 3.4: Solution illustration of instances 6-10-10

c). From Figure 3.4, we can see the variety of location and routing decisions depending on the different hub capacities. For example, the best solution for instance 6-10-10-120 in Fig.3.4 c will open hub number 2. Then all suppliers/clients are assigned to this hub to exchange the commodity flows through 7 collection routes and 8 delivery routes. However, for instance 6-10-10-45 (see Fig.3.4 a), with two open hubs, there are 8 collection local tours and 8 delivery local tours from the hubs, including the single node tours $2 \leftrightarrow 13$ and $3 \leftrightarrow 22$. The details of the best solutions for the two sets of instances can be seen in Table 3.8.

Table 3.8: Details of the best solutions for instance 3-5-5 and 6-10-10

H-I-J	Hub-Cap	Open hub	Collection tours	Delivery tours	Cost
3-5-5	15	1, 2	1-5-7-1	1-9-12-13-1	3867.85
			2-8-4-6-2	2-10-11-2	
	30	1	1-5-7-1	1-10-11-1	3068.00
			1-6-4-8-1	1-13-12-9-1	
	45	1	1-5-7-1	1-10-11-1	3068.00
			1-6-4-8-1	1-13-12-9-1	
6-10-10	45	2, 3	2-8-2, 2-9-2	2-17-2, 2-26-25-2	7613.94
			2-12-2, 2-13-2	2-23-2, 2-19-2	
			3-10-3, 3-14-7-3	3-21-3, 3-22-3,	
			3-11-16-3, 3-15-3	3-24-20-3, 3-18-3	
	60	2, 3	2-12-10-2, 2-9-2	2-23-2, 2-17-2	6828.25
			2-13-2, 2-14-7-2	2-19-2, 2-22-2	
			3-16-11-3, 3-8-3	2-25-26-2, 3-21-3	
			3-15-3	3-24-20-3, 3-18-3	
	120	2	2-10-2, 2-13-2	2-17-2, 2-22-2	6249.60
			2-11-16-2, 2-8-2	2-20-24-2, 2-25-26-2	
			2-9-2, 2-12-15-2	2-19-2, 2-18-2	
			2-7-14-2	2-23-2, 2-21-2	

3.5 Conclusion

In this chapter, we first discuss the concept and propose a detailed definition for the hub location-routing problem for less-than-truckload shipment. We propose then two mathematical models for the CSAHLRP, based upon a four index and a three index formulation, and we introduce some valid inequalities. In order to compare the two models and solve the HLRP, instances sets are generated based on related problems from the the literature and the real data base from Comité National Routier (CNR). Finally, some computational experiments are conducted on the basis of the small and medium generated instances to compare the performance of the two models and provide some insights into the network design of the CSAHLRP. The results prove the effectiveness of our models, especially that of the three-index formulation model for solving small to medium size instances. Experiments also show the difficulty of solving this NP-hard problem with a commercial solver even for the medium size instances. Therefore, it seems necessary to develop a metaheuristic and an efficient exact algorithm to obtain better solutions and be able to solve larger instances.

4

Memetic algorithm for the *CSAHLRP*

As mentioned previously, the Hub Location Routing Problem (HLRP) is a combinatorial optimization problem involving two NP-hard problems: the Hub Location Problem (HLP) and the Vehicle Routing Problem (VRP). The aim is to determine the hub locations, the assignment of non-hub nodes and the local routing for both collections and deliveries, which we consider handled separately. As we have found in previous experiments, it is very difficult and time-consuming to address this problem with a commercial solver. Therefore, to solve it effectively, we propose to develop a meta-heuristic. In this chapter, we propose a memetic algorithm (MA), combining a genetic algorithm (GA) and a local search procedure (LS) to solve the capacitated single allocation hub location-routing problem (CSAHLRP). Firstly, an introduction and literature review of this heuristic are given, including the fundamentals of genetic algorithm and iterative local search. Then the different components of the GA and local search operators are described in detail. Finally, extensive computational results based on the generated instances of Chapter 3 are presented including a parameter tuning phase and an algorithm performance analysis. Finally conclusions are given for this chapter.

4.1 An overview of memetic algorithm

In this section, the origin, the definition and general framework of memetic algorithm are introduced. Then a specific literature review about the related problems and applications is provided.

4.1.1 Introduction

The term of " Memetic algorithm " was created in 1989, introduced by Moscato [143], inspired by both Darwinian' principles of natural selection and Dawkins' notion of memes [1]. The word 'meme' is defined as " the basic unit of cultural transmission, or imitation". It denotes an analogy to the gene in the context of cultural evolution and is similar to the genes of genetic algorithms (GAs). However, a gene in GAs is improved only using global evolutionary operators in the reproduction step, but a meme is allowed to carry out a local search to improve it own quality [201]. Based on this fundamental, Memetic algorithms (MAs) can be viewed as the synergistic combination of a population-based global search algorithm coupled with an individual-based local improvement procedure. After this method has been proposed, with the continuous development of researchers in many diverse domains (mathematics [72], medicine [6], production [114] and transportation [137]), MAs have become a hot topic nowadays as optimization search methods. They have been used to solve many NP-hard optimization problems successfully, such as traveling salesman problem [29], hub location problem [192] and vehicle routing problem [202]. In these researches, MAs are also called as Lamarckian evolutionary algorithms (EA), cultural algorithms, hybrid genetic algorithm or genetic local search. A recent review about memetic algorithms and memetic computing optimization can be seen in [153].

Different from classical genetic algorithms which simulate the process of biological evolution, the memetic algorithm imitates a culture evolution process. In this process, MAs exploit all available knowledge about the problem under study. This incorporation of problem domain knowledge is a fundamental feature of MAs. It suggests that in memetic algorithm, a meme is not only simply transmitted between individuals, but also is modified and enhanced in life-time of an individual by studying and communicating with other meme parts [144]. In general, the enhancement part is accomplished by incorporating heuristics, approximation algorithms or local search techniques. In our research, for the capacitated single allocation hub location-routing problem, the memetic algorithm is a combination of a genetic algorithm (GA) with an iterated local search (ILS).

As a population-based search approach, the GA is a stochastic, global heuristic based on the concepts of natural selection and evolution [188]. At each iteration of the GA, each possible solution is treated as an individual of a population and is represented as a string of characters (genetic code). After the decoding of each individual, a fitness value is governed by the corresponding objective function value or other fitness function value to measure the individual's quality in the population. Then, pairs of good individuals are selected as parents to produce new individuals (offspring) by applying crossover and mutation operators. Next, some replacements are applied on the current population to generate a new population. This "evaluation-selection-crossover-mutation-replacement" cycle is repeated until a stopping criterion is met and the best individual of the last generation is considered as the solution to the problem. Because the pure GA may fail to converge to a global optimum, the MA uses a local search to refine the GA search spaces to obtain high quality solutions. Generally, an iterated local search algorithm starts from a candidate solution and then iteratively moves to a neighbor solution generated at random or constructed by an algorithm. The movements leading to preferable solutions are accepted and the better solution is kept at each step until a solution deemed optimal is found or a time bound is reached.

Based on the above description, the basic scheme of the memetic algorithm for the CSAHLRP can be seen in Figure 4.1. First, after the setting of global parameters, the MA starts with a population of initial random solutions, and then it selects the best individuals from the population at each generation, based on a fitness function, including a penalty component, and applies crossover and mutation operators to produce the next generation. Most importantly, to overcome the weaknesses of the GA, some iterations of local search modules are applied to each new generated individual or to the current best solution to improve the assignment and routing solutions. Finally, the new individuals are compared with the worst individuals of the current population and replace them if better. The whole process is repeated until the maximum number of iterations is satisfied or the best solution is not improved during a maximum number of successive iterations. The pseudo-code of the proposed MA is presented in Algorithm 2.

In Algorithm 2, the parameters *PopSize, MaxGen* and *MaxNoImp* refer to the population size of each generation, the maximum number of iterations and the maximum number of successive iterations without improvement of the best solution, respectively. Two counters, *NbGen*

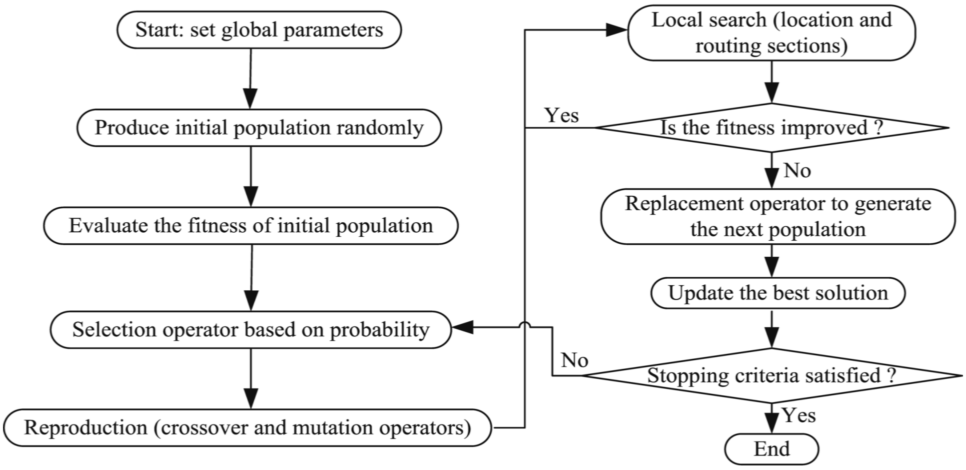


Figure 4.1: The framework of the memetic algorithm

and *NbNoImp*, must also be defined corresponding to the number of new generations already obtained and the number of successive iterations without improvement of the best solution. P_c and P_m are the probability of crossover and mutation, respectively. Based on the general framework of the MA, the key issues are: the code to represent a solution, the definition of the evaluation function, the design of the genetic operators and the iterated local search. These will be introduced in the following sections.

Algorithme 2 Memetic algorithm for the hub location-routing problem

1: Set the global parameters *PopSize, MaxGen* and *MaxNoImp*;
2: Generate randomly the first generation, define it as *Pop*;
3: Evaluate each chromosome of population Pop with fitness function $F_{eval}(x)$;
4: Find the current best chromosome x_{best}: $F_{eval}(x_{best}) = MinF_{eval}(x), \forall x \in$ *Pop*;
5: **While** *NbGen<MaxGen* and *NbNoImp<MaxNoImp* **Do**
6: **Select** parents x_1 and x_2 based on selection mechanism for all populations;
7: Apply **crossover** operator to x_1 and x_2 to obtain the offspring x_{new} based on P_c;
8: Apply **mutation** to x_{new} to get $x_{new}^{'}$ based on P_m;
9: **Repeat**
10: Apply **iterated local search** to $x_{new}^{'}$ to obtain the improved solution and return $x_{new}^{'}$;
11: **Until** the stopping criteria of **iterated local search** is satisfied;
12: Compare $x_{new}^{'}$ with the worst individual in the population Pop and **replace** it if better;
13: Update current best solution x_{best} and best fitness $F_{evalbest}$ for current generation;
14: **Return** x_{best}.

4.1.2 Literature review

As mentioned above, the MA has been one of the most popular algorithms for solving complex and large-scale combinatorial optimization problems in a wide range of application domains. In recent years, both GA and MA have been successfully applied to solve hub location problems (HLPs), vehicle routing problems (VRPs) and location-routing problems (LRPs). Some of the proposed memetic algorithms about these related problems are reviewed in the following and summarized in Tables 4.1-4.3.

Many solution methods have been developed successfully to solve the HLP based on its different characteristics, including the memetic algorithm or hybrid genetic algorithms. In 1998, Abdinnour-Helm [2] developed a new heuristic method based on a hybridization of genetic algorithms (GAs) and tabu search (TS)-GATS for the uncapacitated single allocation hub location problem (USAHLP). In this hybrid heuristic, GAs are used to determine the number and the location of the hubs and TS heuristic is used to find the optimal assignment of spokes to hubs. Based on airline passenger flow between 25 US cities in 1970 as evaluated by the Civil Aeronautics Board (the CAB data set), the experimental results clearly demonstrated that using TS in combination with GAs yields much better solutions compared to applying GAs alone. In 2005, Topcuoglu et al. [198] proposed a new and robust genetic algorithm for the USAHLP to determine the number of hubs, the location of hubs and the assignment of spokes to the hubs. The computational results based on some instances from CAB data set and Australia Post (AP) data set proved a better performance of their heuristics in comparison to those previously obtained from GATS by Abdinnour-Helm [2]. Even for large problems, their approach outperformed the related works at that time with respect to both solution quality and computational time.

Later on, Cunha *et al.* [60] addressed a formulation for a modified USAHLP, in which the discount factor on the inter-hub links is consider to vary according to the total amount of freight between hub terminals. Then they proposed a hybrid heuristic based on GA and simulated annealing (SA) to solve some benchmark problems from CAB data set and a real-world case for a LTL truck company in Brazil. In their approach, the SA was considered as an efficient local improvement procedure to apply to each generated individual of the population. Naeem et al. [147] also proposed a simple but effective genetic algorithm for the USAHLP in 2010. The main contribution in their research concerns two new chromosome representation methods and two crossover operators. The results based on benchmark problems found all best known solutions for 12 AP problems and improved some solutions for the CAB data. Recently, Miroslav et al. [135] proposed a newest efficient memetic algorithm for the USAHLP. They used two new and efficient local search heuristics to be incorporated in the evolutionary algorithm frame. One was to find the best location of hubs and the other one tried to improve the allocation of the non-hub nodes to hubs. Finally, they implemented a broad, comprehensive set of computational experiments, including all benchmark problems found in the literature and newly generated set of large-scale instances with up to 900 nodes. The results showed that the proposed MA obtained all best known solutions on small and medium size CAB and AP instances. Even for larger problems, it could produce high-quality solutions in a reasonable time.

Relaxing the single allocation constraint, Kratica et al. [116] proposed a genetic algorithm for solving uncapacitated multiple allocation HLP (UMAHLP) in 2005. They used binary encoding and caching technique to improve the performance of GA. The results based on standard AP instances with up to 200 nodes showed that the approach quickly obtained all optimal solution known in literature. Zorica Stanimirović [187] developed a new heuristic method based on a GA applying mutation with frozen bits to solve the uncapacitated multiple allocation p-hub median problem (UMApHMP). The good performance of this approach has been demonstrated by a comparison with the existing methods based on well known CAB and AP data sets with up to 200 nodes and 20 hubs.

For the capacitated versions, Stanimirović et al. [186] presented a genetic algorithm for the capacitated single allocation hub location problem (CSAHLP). In this proposed method, the objective function is correcting infeasible individuals to become feasible in the future generations of the GA. The computational experiments were conducted on standard AP instances with up to 200 nodes and revealed the satisfying performance of the GA implementation compared to previous researches, especially on large problems solved effectively in shorter time. Later, this heuristic method was applied for solving the capacitated single allocation p-hub median problem (CSApHMP) [188]. Similarly, the computational results on AP data set demonstrated the robustness fo the proposed algorithm. Kratica et al.[115] presented two evolutionary algorithms (EAs) to solve the capacitated hub location problem (CHLP). It is a variant of the classical HLP, where all the hubs are not necessarily connected but considers the capacities on both arcs and hubs. They improved the overall performance of both proposed EA implementations by a caching technique. And their empirical study indicated that the first EA method is very efficient for the small and medium size problems, while the second one based on heuristic can produce high-quality solutions for larger problems. Recently, Sun et al. [192] addressed a capacitated asymmetric allocation hub location problem (CAAHLP) and a solution method based on combined ant colony optimization algorithm and genetic algorithm was developed to solve this problem. The former was to determine the number and locations of hubs, and the latter tried to solve the node allocation problem. The solution method highlighted a good performance through a comparative study.

For other versions of the HLP (see Table 4.1), there are also many applications of memetic algorithms. Takano et al. [195] used a genetic algorithm to solve the hub-and-spoke problem (GAHP) for the liner shipping with shuttle services and applied it to the containerized cargo transport of Asian hub ports. Mohammadi et al. [141] proposed a hybrid algorithm, based on a GA and SA to solve a capacitated single allocation hub covering location problem (CSAHCLP). The related results comparison showed that the hybrid method gave better solutions than GA and SA alone. Bashiri et al. [25] presented a GA based heuristic to solve the capacitated single allocation p-hub center problem (CSApHCP) and achieved the optimum solutions for all tested instances in 25 runs with up to 25 nodes.

Table 4.1: The application of memetic algorithms on HLPs

Problem type	Paper	Algorithm details	Instance/size
USAHLP	Abdinnour-Helm (1998) [2]	GA+tabu search	CAB/ 25 nodes
	Topcuoglu et al. (2005) [198]	Pure GA	CAB and AP/ 200 nodes
	Cunha and Silva (2007) [60]	GA+SA	CAB and a case in Brazil/ 25 nodes
	Naeem et al. (2010) [147]	GA+two new encoding schemes	CAB and AP/ 200 nodes
	Marić et al. (2013) [135]	GA+two local searches	CAB and AP/ 900 nodes
UMAHLP	Kratica et al. (2005) [116]	GA+caching technique	AP/ 200 nodes
UMApHMP	Stanimirović (2008) [187]	GA+ caching technique	CAB and AP/ 200 nodes-20 hubs
CSAHLP	Stanimirović (2007) [186]	GA+caching technique	AP/ 200 nodes
CSApHMP	Stanimirović (2010) [188]	GA+caching technique	AP/ 200 nodes-20 hubs
CHLP	Kratica et al. (2011) [115]	two EAs+caching technique	New instances/ 100 nodes
CAAHLP	Sun et al. (2012) [192]	GA+ant colony optimization	-
GAHP	Takano et al. (2009) [195]	GA	CAB and case study/ 18 ports
CSAHCLP	Mohammadi et al. (2010) [141]	GA+SA	Randomly generate data/ 70 nodes
CSApHCP	Bashiri et al. (2013) [25]	GA	AP/ 25 nodes-6 hubs

Many variants of memetic algorithms have been proposed for solving different versions of the vehicle routing problem (VRP). In 2003, Baker et al. [14] considered the application of a pure GA and a hybrid one combined with neighborhood search methods (NSM) to the basic VRP. Computational results showed their good performances compared with tabu search and simulated annealing in terms of solution quantity and solving time. Then a hybrid genetic algorithm (HGA) was proposed by Berger et al. [27] to solve the capacitated vehicle routing problem (CVRP). In this HGA, two populations of solutions concurrently evolved and exchanged respective best individuals at each generation, and some mutation operators were used based on the large neighborhood search (LNS). The results over classical benchmark problems proved its competitiveness through a comparison with the best-known methods. Later, more researchers improved and developed new algorithms based on the GA to solve the classical VRPs [142, 167, 168, 169]. Among them, Prins [169] presented two memetic algorithms (GAs hybridized with a local search) to solve the heterogeneous fleet VRP (HFVRP) and the vehicle fleet mix problem (VFMP). Both of them encoded the chromosomes as giant tours without trip delimiters and evaluated solutions with a splitting procedure. The first MA used a dispersal rule in objective space and the second one considered the population management for MA to control solutions diversity using a distance measure in solution space. Computational tests showed both of them compete with other heuristics and found some new best solutions for VFMP. About the applications of evolutionary algorithms (EAs) to solve VRPs, one can refer to a survey reported by Potvin [167].

Regarding extensions of the VRP, Vidal et al. [202] proposed a hybrid genetic algorithm with adaptive diversity control to solve the multi-depot VRP (MDVRP), the periodic VRP (PVRP) and the multi-depot periodic VRP (MDPVRP) with capacitated vehicles and constrained route duration. This meta-heuristic combines a genetic algorithm, a neighborhood-based search (NS) and a population management mechanism to maintain a high level of diversity among individuals. Extensive computational results proved it can reach high quality solutions on the literature benchmarks and outperformed other heuristics for the three problems. Cattaruzza et al. [39] considered a memetic algorithm for the multi trip vehicle routing problem (MTVRP), where each vehicle can perform more than one service trip during the working day. In this MA, a combined local search (LS) operator was introduced to perform pejorative moves along with re-assignment of trips to vehicles. The reported results over 99 instances showed it outperformed previous algorithms with respect to average solution quality. In addition, Ho et al. [97] and Liu et al. [131] also addressed the hybrid GAs to solve the multi-depot VRP. For the VRP with time windows (VRPTW), there exists also the application of memetic algorithms. For example, Ombuki et al. [161] presented a multi-objective genetic algorithm using the Pareto ranking technique to solve the standard benchmarks with up 100 customers. Nagata et al. [148] proposed an effective memetic algorithm using an edge assembly crossover and a novel penalty function for solving the VRPTW. Its computational results demonstrated this method improves 184 best-known solutions out of 365 instances. Nalepa et al. [152] presented a parallel memetic algorithm, and Barkaoui et al. [22] developed an adaptive evolutionary approach to solve the VRPTW. The application details of memetic algorithm for VRPs can be seen in Table 4.2.

Table 4.2: The application of memetic algorithms on VRPs

Problem type	Paper	Algorithm details	Instances/ size
CVRP	Baker et al. (2003) [14]	GA +NSM	Benchmark instances/ 200 cities
	Berger et al. (2003) [27]	GA+LNS	Benchmark instances/ 200 cities
	Christian Prins (2004) [168]	GA+local search	34 instances from literature/ 483 customers
	Potvin et al. (2007) [167]	Survey on EAs for VRPs	Benchmark instances/ 1000 customers
	Mohammed et al. (2012) [142]	GA	Random instance/ 100 locations
HFVRP+VFMP	Christian Prins (2009) [169]	Two MAs	Instances from literatures/ 100 cities
MDVRP	Liu et al. (2014) [131]	A new hybrid GA	Instances from literature
	Ho et al. (2008) [97]	Two hybrid GAs	Two randomly examples/ 100 customers
MDPVRP	Vidal et al. (2011) [202]	GA+NS+adaptive diversity control	Benchmark instances/ 480 customers
MTVRP	Cattaruzza et al. (2014) [39]	GA+splitting procedure+LS	Benchmark instances/ 200 vertices
VRPTW	Ombuki et al. (2006) [161]	Multi-objective GA	Benchmark instaces/ 100 customers
	Nagata et al. (2010) [148]	Penalty-based edge assembly MA	356 benchmarks instances/ 1000 customers
	Nalepa et al. (2012) [152]	A parallel MA	Instances from literatures/ 1000 customers
	Barkaoui et al. (2013) [22]	An adaptive evolutionary approach	Instances from literatures / 100 customers

Finally, the location-routing problem (LRP), which combines location and routing decisions, has also been solved with memetic algorithms. Prins et al. [170] presented a memetic algorithm with population management (MA | PM) to solve the capacitated location-routing problem (CLRP). In this MA, a local search procedure was hybridized, and a PM technique was used to control the population diversity based on a distance measure in the solution space. The experimental results showed this algorithm can find good quality solutions for three kinds of instances and outperformed other meta-heuristics. Karaoglan et al. [109] proposed a hybrid approach based on genetic algorithm and simulated annealing to solve the location-routing problem with simultaneous pickup and delivery (LRPSPD). The experimental study indicated a good performance of this approach compared with a branch-and-cut algorithm on a set of instances in terms of quality solutions and solving time. Prodhon [174] developed a hybrid evolutionary algorithm to solve large size instances of the periodic location-routing problem (PLRP). In this hybrid heuristic, each individual was evaluated based on the randomized extended Clarke and Wright algorithm (RECWA). Then an evolutionary local search (ELS) was used to improve the assignment of visit days to customers. Computational results showed this meta-heuristic outperformed the previous methods for the PLRP. Derbel et al. [71] described a GA combined with an iterative local search (ILS) to solve a LRP with multiple capacitated depots and one uncapacitated vehicle for each depot. It used a ILS to intensify the search space and improve the solutions generated by GA. Recently, Martínez-Salazar et al. [137] proposed two memetic algorithms to solve a bi-objective transportation location routing problem (TLRP). In this problem, two objectives were considered : minimize total cost and balance the drivers' workload. One of the two MAs used a simple local search to replace the classic mutation operator and stopped when a fixed number of generations arrived, however, the other one considered a tabu search procedure as mutation operator and set the stopping criterion as a fixed number of iterations without changes. Computational results with some instances of the literature showed that the second one was better than the first one. In addition to the above works, there has also been other applications of the MA to LRPs [175, 82] (seen in Table 4.3).

Table 4.3: The application of memetic algorithms on LRPs

Problem type	Paper	Algorithm details	Instance size
CLRP	Prins et al. (2006) [170]	GA +local search+ PM	200 customers+20 depots
	Derbel et al. (2012) [71]	GA+ILS	30 customers+10 depots
PLRP	Prodhon et al. (2008) [175]	MA | PM	200 customer+10 depots
	Caroline Prodhon (2011) [174]	GA+RECWA+ELS	200 customer+10 depots
LRPSPD	Karaoglan et al. (2010) [109]	GA+SA	100 customers+10 depots
LRP+inventory	Forouzanfar et al. (2012) [82]	GA	60 customers+15 depots
TLRP	Martínez-salazar et al. (2014) [137]	Two MAs	50 customers+10 depots

In conclusion, the many successful implementations of memetic algorithms for these three families of related problems, suggests us to take advantage of its potential value for solving the HLRP. To our knowledge, this has never been done. In the next sections, we will describe the different components of the MA which we propose for solving the CSAHLRP, comprising a genetic algorithm, followed by an iterative local search.

4.2 Genetic algorithm for the CSAHLRP

In this section, a brief description of genetic algorithms and generic templates are presented. Then we detail each element of the GA that we propose.

4.2.1 Overview

As mentioned above, a genetic algorithm combines an adaptive probabilistic search with an optimization heuristic. It was first introduced by John Holland and his colleagues in 1975 [99], and simulates the biological selection and evolutionary processes in nature, based on Darwin's "survival of the fittest" theory. In recent years, GAs have been applied to different areas, and has proven to be one of the robust and effective algorithms for combinatorial optimization problems. Besides, it is computationally simple and easy to implement, even for complex problems.

Generally (see Algorithm 2), a GA starts with an initial feasible population (*GenerateInitial* ()), evaluates each individual in the population based on a fitness function, and then updates the population using a selection process (*Selection* (x_1, x_2)) and crossover (*Crossover* (x_1, x_2)), and mutation (*Mutation* ()) operators to generate better individuals and to better adapt the generated population to the problem environment. In this cycle process, individuals representing solutions are described at each generation in the form of chromosomes, which are made up of genes. Then all operators performed on individuals aim at generating better chromosome or genes for the population. So the best individual in the last generation can be seen as an approximate optimal solution for the problem. In practice, this process is not endless and often is repeated for a number of iterations or until the system does not improve anymore.

Based on this general process and according to the specific features of our CSAHLRP, we have designed GA, as a part of our MA, to solve this problem. As mentioned above, the general framework of the GA can be seen in Algorithm 3. In this algorithm, all notations remain the same as the ones of Algorithm 2 and P_{rnd} is a random probability to control occurrences of crossover or mutation processes. The main components of this GA consist of the following ones [178]:

Algorithme 3 Generic GA for the CSAHLRP

Solution encoding;
Set *PopSize, MaxGen* and *MaxNoImp*;
GenerateInitial ()
While *NbGen*<*MaxGen* and *NbNoImp*<*MaxNoImp* **Do**
    Evaluate chromosome;
    Find the current best chromosome x_{best}:
    for $i = 1$ **to** *PopSize* **Do**
        Selection (x_1, x_2)
        if $(P_{rnd} < P_c)$ **then**
            $x_{new} = Crossover(x_1, x_2)$;
        endif
        if $(P_{rnd} < P_m)$ **then**
            $x'_{new} = Mutation(x_{new})$;
        endif
        Update this population using the replacement strategy.
    endfor
End While
Return x_{best}.

(1) Representation of possible solutions. The chromosome encoding is the precondition to implement the GA. In practice, there are mainly four encoding approaches including binary encoding, permutation encoding, value encoding and tree encoding. In our GA, one chromosome of CSAHLRP is represented with value encoding for location decision but permutation encoding for routing decision, as presented in section 4.2.2.

(2) Evaluation of the chromosomes. This consists in the determination of the fitness function value and is the basis of the selection operator. It is the only criterion to evaluate the solution quality, and it can be an original fitness function or a standard one. The former is normally the objective function of the problem and the latter is to guarantee its non-negativity in some cases. Here, in order to prevent infeasible solutions, a penalty function is considered (see section 4.2.2).

(3) Generation of the initial feasible population. This the starting step of the GA. The determination of the population size can affect the diversity of the population and the compu-

tational efficiency. Generally, the initial population can be set randomly or by heuristics. Here, our initial population is generated randomly based on some rules and the population size is set after parameter tuning (seen in section 4.4.1).

(4) Design and application of genetic operators, including the selection technique, crossover, mutation and replacement strategies. These are the most important components for an effective GA. The basic approaches for each operator have been introduced in [178]. The detailed design of these operators for our CSAHLRP is presented in section 4.2.3 below.

(5) Termination criteria. This is needed especially because a GA is a stochastic search. Usual approaches put a limit on the number of generations or on the computer clock time, or stop when the objective value or fitness value can't continuously get better in a limited number of iterations, or terminate when the population's diversity falls below a preset value. For our GA, the first two criteria are used to terminate the algorithm.

In the following, we describe the components of our GA in detail.

4.2.2 Chromosome evaluation and initial population

In this algorithm, the solutions of the HLRP are coded with natural numbers for each chromosome $C(x)$ and include two sections: a location section $L(x)$ and a routing section $R(x)$. The location section $L(x)$ consists in hub location information and the routing section $R(x)$ consists in one permutation of non-hub nodes representing the order of a collection or delivery route. More clearly, consider a chromosome x and its coding $C(x) = L(x) + R(x)$. Let $L(x) = \{l_1, l_2, l_3, ..., l_n\}$ and $R(x) = \{r_1, r_2, r_3, ..., r_n\}$, where n is the number of non-hub nodes, l_i is the index of an open hub and r_i is the index of the suppliers and clients. Then, for each $i \in \{1, 2, ..., n\}$, the location/allocation information returned indicates that the non-hub node r_i is assigned to hub l_i and that hub l_i is open. The routing information can be obtained from $R(x)$ according to the order of non-hub nodes allocated to each hub of $L(x)$. An example of this coding method for a given chromosome can be seen in Figure 4.2. In order to express the proposed algorithm more clearly, the set of potential hubs, suppliers and clients are shown in disjoint locations in this figure. In this example, 3 possible hubs, 5 suppliers and 5 clients are considered. To distinguish the three kinds of sets, they are named with ascending integers. So, $H = \{1, 2, 3\}$ is the set of potential hubs; $I = \{4, 5, 6, 7, 8\}$ is the set of suppliers and $J = \{9, 10, 11, 12, 13\}$ is the set of clients. Because the collection and delivery processes are distinguished, the routing section enables the distinction of the two types of routes, knowing the nature of the nodes. For example, since suppliers 4, 7 and 6 are allocated to hub 1, the collection route serving these suppliers is obtained based on their permutation order in $R(x)$. More precisely, supplier 4 is followed by supplier 7 and then by supplier 6 in $R(x)$, indicating that the collection route serving them from hub 1 is {1-4-7-6-1}. Similarly, the delivery route from hub 1 is {1-10-12-1}. All the routes corresponding to this example can be seen in Figure 4.2. The verification of the vehicle capacity constraint is addressed by each decoding procedure. Once the vehicle capacity is violated by the insertion of a new node, this node is assigned to a new route. The hub capacity is calculated during the evaluation of each chromosome.

In order to perform a natural selection, every individual i is evaluated in terms of its *fitness value* f_i, determined by an evaluation function. The fitness value measures the quality of the solutions and enables them to be compared. In our MA, for each generation, the individuals are evaluated by the fitness function $F_{eval}(x)$ based on the formulation *CSAHLRP-F4* presented in Chapter 3, which includes the objective value of the solution, as well as a penalty cost for exceeding the capacity of hubs in order to favor the feasibility of the solutions. This evaluation is carried out at each iteration of the MA to determine the best solution. The evaluation function of a solution x is defined as follows:

$$F_{eval}(x) = Objectivevalue(x) + Penaltycost(x) \tag{4.1}$$

where

$$Objectivevalue(x) = \sum_{k \in H} F_k z_{kk} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in H} \sum_{l \in H} \alpha d_{kl} q_{ij} Y_{ijkl} + \sum_{v \in V} \sum_{i \in I \cup H} \sum_{j \in I \cup H, j \neq i} \beta d_{ij} x_{ij}^v$$
$$+ \sum_{v \in V} \sum_{i \in J \cup H} \sum_{j \in J \cup H, j \neq i} \gamma d_{ij} x_{ij}^v + \sum_{v \in V} \sum_{k \in H} \sum_{i \in I \cup J} f_v x_{ki}^v$$

and

$$Penaltycost(x) = \delta \sum_{k \in H} max\{0, \sum_{i \in I} z_{ik} O_i + \sum_{j \in J} z_{jk} D_j - 2\Gamma_k\} \tag{4.2}$$

in which δ is the penalty parameter for exceeding the capacity of hubs.

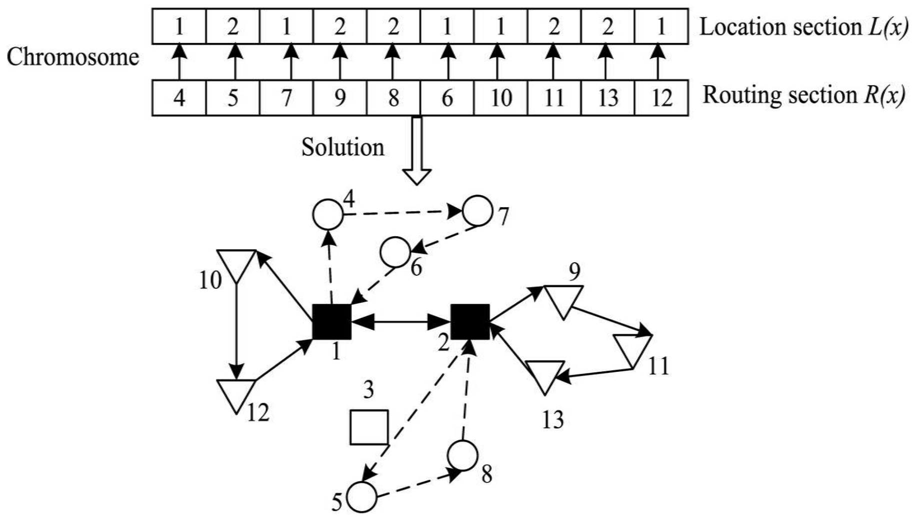


Figure 4.2: An example of chromosome coding for the hub location-routing problem

The initial population for our GA, and then MA algorithm, contains *PopSize* individuals and is generated randomly. The genes of the location section $L(x)$ are generated randomly from the set of potential hubs based on different seeds. The routing section $R(x)$ consists of one random permutation of non-hub nodes representing the order of a collection or delivery route. Here, a randomized procedure using the system time as the initial seed is used to produce a random permutation of non-hub nodes. Thus, this method guarantees that each supplier and client can be assigned to one hub at random and it can also produce many different visiting orders for the routes. As a result, this random approach can provide a diversity of genetic materials and possible improvements in the objective function for the next generations. In addition, in order to consider a trade-off between efficiency and effectiveness, a study to determine the parameter of population size *PopSize* will be conducted in section 4, because a small size population would not explore enough search space effectively, while a too large population would need much more time.

4.2.3 Genetic operators

After calculating $F_{eval}(x)$ for all individuals, the population is ordered in descending order based on the value of $F_{eval}(x)$. Then, some pairs of good chromosomes are selected from the population to keep them in the next generation. For the selection operator, many methods can be found in the literature. In our MA, the parents are selected with a roulette wheel process [178] according to the following ranking probability equation where k is the kth chromosome ordered in descending order of the fitness value in the population, and $PopSize$ is the population size. This probability is inversely proportional to the fitness value. Thus, an individual with good fitness has a higher probability of being selected.

$$P(|k|) = \frac{2k}{PopSize(PopSize + 1)} \tag{4.3}$$

In our GA, a one-point crossover operator is applied to the chromosome $C(x)$ both on the location section $L(x)$ and on the routing section $R(x)$ (Figure 4.3). After two parents have been selected, one point P_L of the $L(x)$ section is generated randomly. Then, the first part of the offspring comes from the code of parent 1 before this point P_L, and the rest comes from parent 2 after P_L. For the routing section $R(x)$, after the selection of a crossover point P_R, the first part of the offspring is the same as parent 1 before the point P_R. For the rest, the offspring takes the code of parent 2 sequentially, except for those already present in the offspring. Thus, the routing section of the new offspring is obtained. To favor the diversity of the chromosomes, the crossovers on the two sections are implemented in parallel.

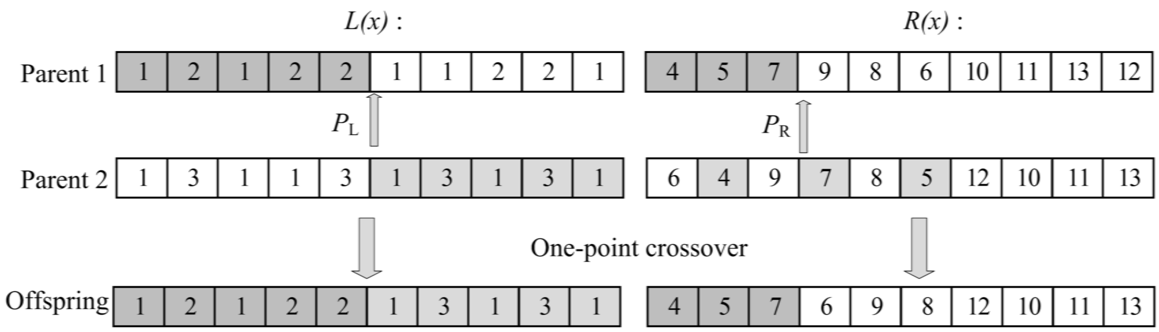


Figure 4.3: Crossover operator for location and routing sections

For the mutation operator (Figure 4.4), different methods are used for the two sections of the chromosome. In the location section $L(x)$, the hub assignment is modified by replacing some randomly chosen hubs by others also randomly chosen from the set of potential hubs. This can give rise to the possibility of opening a new hub or closing a hub. For example, in Figure 4.4, hub 2 will be closed after applying the mutation operator. For the routing section $R(x)$, two points are generated, and then the first point is removed from its current position and inserted into a new position after the second point. In Figure 4.4, nodes 7 and 10 are selected randomly. Then the mutation operator will insert node 7 after node 10, and the nodes between 7 and 10 (6, 9, 8, 12) will be shifted to the left one by one. After the Crossover and Mutation processes have been completed, a replacement strategy is executed, were each new generated offspring is compared to the worst individual in the population and replaces it if it is better. Thus, the best individuals of the population can be kept for the next generation.

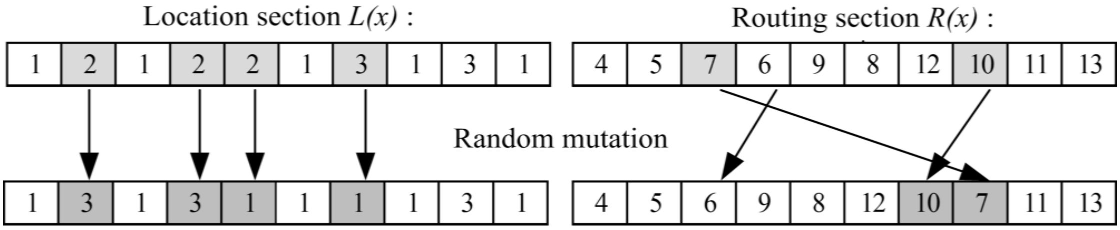


Figure 4.4: Mutation operator for location and routing sections

4.3 Iterated local search for the CSAHLRP

This section presents the design of the iterated local search to be part of the memetic algorithm for the CSAHLRP, including the generic framework, the local search on the routing and hub location processes.

4.3.1 Overview

Local search procedures are greedy algorithms applied to a feasible solution to further improvement of its quality [45]. It consists in generating a locally optimal solution by exploring the neighborhood of a given solution. The local search is characterized by its neighborhood structure. A neighborhood is achieved by modifying some components of the solution to create a new one. Therefore, identifying some effective neighborhood structures is very important.

The goal of the local search in the MA is to overcome the weaknesses of a genetic algorithm by increasing the population's diversity. For the HLRP, an iterated local search (ILS) is used on both location and routing sections. The framework of our ILS is presented in Algorithm 4. First, a local search on the routing section of the chromosomes is applied to the initial solution s_0 to find a better solution $s^{'}$. Then a local search on the location section, in the form of a perturbation operator, is performed on the solution $s^{'}$. A new neighborhood solution s is obtained, defining the location of the hubs and the assignment of non-hub nodes. Then, for each neighborhood s of the location section, a local search on the routing section is applied again to generate a new local optimum solution $s^{''}$. Finally, the solution $s^{''}$ is evaluated and replaces the solution $s^{'}$ if its fitness is better. The ILS process is repeated until the termination criterion is satisfied. In this ILS, four kinds of classic operators are used to generate new neighborhoods and improve the routing solution, including intra-route and inter-route operators. The intra-route operators used are the swap and the insert operators. 2-opt* and relocate moves are used between the different routes. For the location part, three perturbation operators are used to improve the location-allocation solution including hub closing, hub opening and hub swap operators. All of the local search procedures are performed sequentially.

4.3.2 Local search on the routing

The local search on the routing section aims to generate a local optimal solution for the collection and delivery tours. It can be achieved by modifying some components of the routing section according to some neighborhood methods. For the HLRP, when the hub locations are known, the problem becomes a multiple-depot vehicle routing problem (MDVRP). Many successful local search procedures applied to the VRP [30] and the LRP [134] can be used. Here, we have selected four local searches to construct the neighborhoods of the routing section: the swap procedure and the 1-insertion operator for intra-route improvement and the 2-opt* operator and relocate operator to modify several routes simultaneously. These local searches are

Algorithme 4 Iterated local search (ILS)

1: s_0 = initial solution after applying **genetic operators**;
2: $s^{'}$ = **Routing local search** (s_0);
3: **Repeat**
4: s = **Perturbation -location local search** $(s^{'})$;
5: $s^{''}$ = **Routing local search** (s);
6: **if** $F_{eval}(s^{''}) < F_{eval}(s^{'})$ **then**
7: $s^{'} \longleftarrow s^{''}$;
8: **endif**;
9: **Until** the stop criteria of ILS is satisfied;
10: Return $s^{'}$.

illustrated in Figure 4.5 and are described in the following. Index 0 in Figure 4.5 represents the hub. All of them are carried out on the same type of route (i.e. collection or delivery routes). In the routing local search process, the four methods are performed sequentially in the given order. An initial solution s_0 is improved by applying them one by one. If a new better solution is found by a given local search, its application is repeated until no improvement is found. Then, the next local search starts. The process is continued until none of the local searches produces a positive improvement. Moreover, in all cases, the capacity constraints on vehicles must be respected.

(1) The swap intra-route procedure is performed between two randomly selected nodes assigned to the same route which swap their positions. The corresponding route is modified but neither the number nor the assignment of non-hub nodes in the route is changed. In the repeated process, which considers all feasible better swaps of two nodes, a move is accepted if the resulting total cost is less than the current one. As Figure 4.5.a shows, nodes 2 and 4 will be swapped if $c(2,5) + c(1,4) > c(1,2) + c(4,5)$.

(2) The 1-insertion operator is carried out by removing one node from its position and inserting it elsewhere in the same route as shown in Figure 4.5.b. Under the same cycle criterion as with the swap operator, the new route will be kept if it is better than the current one. In Figure 4.5.b, node 2 is selected to insert between node 1 and node 4 because $c(1,2) + c(2,4) + c(3,5) < c(1,4) + c(3,2) + c(2,5)$.

(3) The 2-opt* operator is an extension of the 2-opt intra-route operator. The principle is to remove two edges from two different routes and then reconnect the corresponding nodes with other edges. The basic idea is to exchange the end part of the two routes after the cutting points, thus preserving the orientation of the routes. In Figure 4.5.c, the end parts of the routes after nodes 3 and 7, respectively, are exchanged. This is performed by replacing edges $(3,4)$ and $(7,8)$ with edges $(3,8)$ and $(7,4)$. This process is repeated until the stopping criterion is met.

(4) The relocate inter-route operator is performed by simply moving a node from one route to another. As seen in Figure 4.5.d, node 3 from the first route is relocated to the second route while preserving the orientation of the routes. Thereby, arcs $(2,3)$, $(3,4)$ and $(8,9)$ are removed and replaced by arcs $(2,4)$, $(3,9)$ and $(8,3)$.

4.3.3 Local search on hub location-perturbation

For the local search on the hub location part of the solution, three ways are applied sequentially to perturb the solution: hub closing, hub opening and hub swap operators (Figure 4.6). The three procedures can provide more opportunities to deal with a different type of solution by

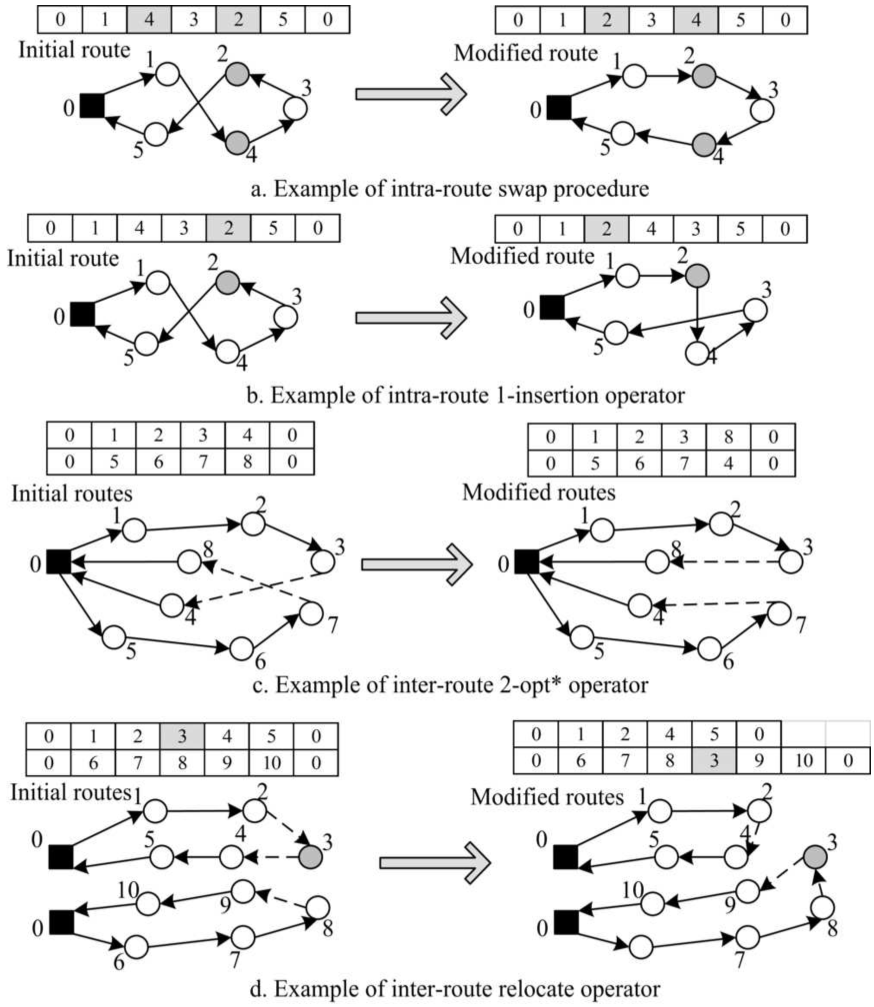


Figure 4.5: Illustration of four neighborhoods in routing local search

modifying the hub location and assignment solution. All three perturbations are performed one by one followed by the routing local search to find a local optimum for each location solution. The resulting solution of each perturbation is evaluated to replace the current one if it is better until the stopping criterion is met.

As shown in Figure 4.6, for the hub closing operator, one open hub is first randomly selected to be closed, then the suppliers and clients previously allocated to this hub are reassigned to the nearest open hub. In Figure 4.6.a, hub 1 is selected to be closed and all allocated nodes are reassigned to hub 2. The corresponding decoded solution is shown below the chromosome. This operator can produce new solutions by decreasing the number of open hubs. For the hub opening operator, a closed hub is randomly chosen and is opened. Then supplier and client nodes are reassigned to the nearest hub. For example, in Figure 4.6.b, hub 3 is selected to open and the two nearest nodes 5 and 8 are allocated to hub 3 from hub 2. Therefore, hub 3 will replace the corresponding position of hub 2 in the location section of the chromosome. This operator can perturb the solution by increasing the number of open hubs. For the hub swap operator, one open hub is selected randomly, and then an other open hub is chosen by a roulette wheel selection. The selection probabilities are inversely proportional to the distance to the first selected hub. Finally, the positions of the two selected hubs are exchanged in the chromosome. As seen in Figure 4.6.c, open hub 1 is exchanged with hub 2 and then all of the assignments are also changed. The corresponding solution is presented below the chromosome.

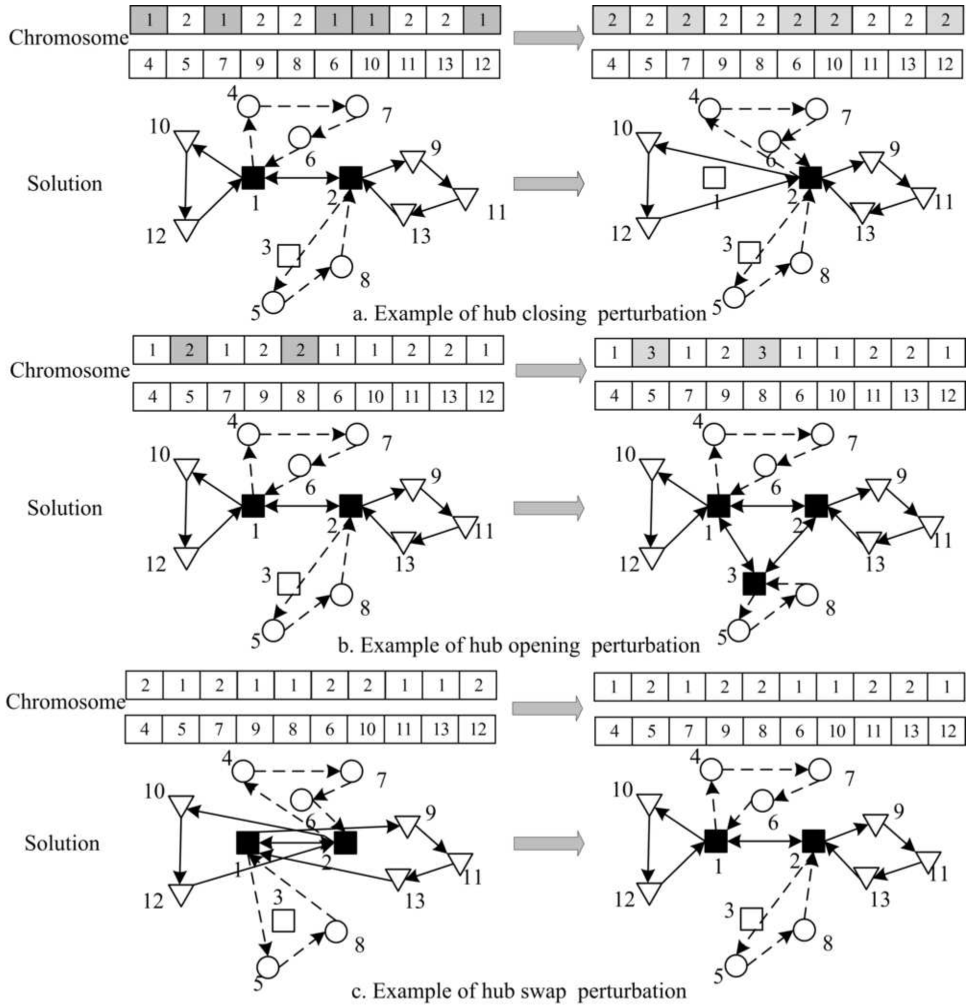


Figure 4.6: Perturbation operators-location local search

4.4 Computational experiments and results

In this section, we present the results of the computational experiments which we have carried out with the memetic algorithm described in the previous section, on all the generated instances introduced in Chapter 3. The proposed MA is implemented in the C++ language and all experiments were conducted on an Intel Core i3 CPU of 2.93 GHz and 6 GB of memory, running on the operational system Window 7. For each instance, the MA was run 10 times. To evaluate the quality of the MA, the results are compared to those obtained by solving our MILP model with CPLEX, presented in Chapter 3 on some small and medium instances of up to 25 suppliers and 25 clients. Firstly, some computational tunings are done to set the parameters of the MA. These results are discussed in section 4.4.1. In section 4.4.2, the results of the MA are compared to those obtained with CPLEX (chapter 3). Finally, all the results obtained by the MA including larger instances are detailed in section 4.4.3, followed by the performance analysis.

4.4.1 Parameters tuning

Several parameters are involved in the proposed MA to solve CSAHLRP, such as the size of the population $PopSize$, the number of iterations $MaxGen$, the crossover probability P_c and mutation probabilities P_{m1} and P_{m2}. These values can influence directly the quality of the generated solutions, so need to be tuned. Theoretically, all parameters should be tuned but it is complex and too time consuming. So based on general principles of MA and related reference [71], we mainly tuned three of them ($PopSize$, P_{m1} and P_{m2}) to find a better parameter combinations. For their possible values, we set $PopSize \in \{100, 200\}$, $(P_{m1}, P_{m2}) \in \{(0.7, 0.9), (0.5, 0.9), (0.07, 0.05)\}$. Then a total of 6 combinations of values (listed in Table 4.4) need to be tested. As it is time consuming, the parameter tunings are not necessarily performed on all instances. And considering the reliability and fairness, four group of problem instances are chosen for the parameter tuning, including instances 3-10-10, 3-15-15, 6-20-20 and 6-25-25. All of the tuning results are shown in Table 4.5 and 4.6 for the different combinations.

Table 4.4: Different combinations for parameter tuning and testing

Combination	$PopSize$	P_{m1}	P_{m2}	Combination	$PopSize$	P_{m1}	P_{m2}
Combination 1	100	0.7	0.9	Combination 4	200	0.7	0.9
Combination 2	100	0.5	0.9	Combination 5	200	0.5	0.9
Combination 3	100	0.07	0.05	Combination 6	200	0.07	0.05

In Tables 4.5 and 4.6, the first two columns declare the instance name (number of hubs, suppliers and clients), and also the hub capacity. In order to evaluate and compare the results of different combinations, we present for each one the best objective value found by MA (Z_{best}), the percentage between the average value of 10 runs and the best value(Aver gap), and the average running time (T_{aver}) in seconds. The bold values correspond to the best values in each row. From these results, it can be seen that when $PopSize = 100$, Combination 1 with the highest mutation probabilities gets the lowest average gap, which indicates the stability of this combination. However, Combination 2 finds most of the best solutions and Combination 3 takes less time. When the population size increases to 200 (Combinations 4, 5, 6), the overall performance of the MA is improved with a larger time. Especially Combination 4 not only can find the best solutions, but also obtains a lower gap. Therefore, it can be considered as the best combination to solve all instances and will be used for all computational experiments in the following.

Table 4.5: Parameter tuning results of the different combinations with $PopSize = 100$

Instance name		Combination 1			Combination 2			Combination 3		
H-I-J	Hub_Cap	Z_{best}	Aver gap	T_{aver}(s)	Z_{best}	Aver gap	T_{aver}(s)	Z_{best}	Aver gap	T_{aver}(s)
3-10-10	45	**7613.94**	2.07	0.93	7864.22	2.79	1.00	**7613.94**	2.25	0.92
	60	**6828.25**	1.47	0.93	**6828.25**	1.89	1.02	**6828.25**	1.49	0.96
	120	**6249.60**	0.00	0.79	**6249.60**	0.00	0.78	**6249.60**	0.00	0.87
3-15-15	45	10854.02	2.89	3.41	**10799.64**	2.96	3.07	11130.49	2.10	2.99
	75	9377.02	2.76	3.15	**8940.28**	3.87	2.86	9214.70	2.59	3.12
	135	8248.80	1.60	2.60	**8232.80**	1.24	2.84	**8232.80**	0.56	2.65
6-20-20	60	**10640.92**	2.91	29.95	11116.62	2.86	29.95	11034.85	3.02	21.78
	90	9826.03	4.57	29.95	**9793.74**	3.28	20.88	9834.03	4.10	20.98
	165	9337.60	2.76	25.11	**9336.80**	2.29	21.86	**9336.80**	3.97	19.44
6-25-25	75	12835.44	2.98	47.86	12860.53	3.83	49.23	**12772.27**	3.59	48.03
	105	**12096.43**	2.34	47.21	12175.21	2.78	48.13	12130.50	4.66	47.54
	195	11112.80	2.56	46.86	**11093.60**	2.89	50.73	11112.00	3.09	47.15
Average			**2.41**	19.90		2.56	19.36		2.62	**18.04**

Table 4.6: Parameter tuning results of the different combinations with $PopSize = 200$

Instance name		Combination 4			Combination 5			Combination 6		
H-I-J	Hub_Cap	Z_{best}	Aver gap	T_{aver}(s)	Z_{best}	Aver gap	T_{aver}(s)	Z_{best}	Aver gap	T_{aver}(s)
3-10-10	45	**7613.94**	0.83	1.32	**7613.94**	2.64	1.32	8053.43	2.77	1.14
	60	**6828.25**	0.00	1.66	**6828.25**	0.38	1.54	**6828.25**	2.64	1.11
	120	**6249.60**	0.00	0.78	**6249.60**	0.00	1.23	**6249.60**	0.00	1.03
3-15-15	45	**10622.13**	1.00	3.83	10687.50	2.22	3.83	10832.74	2.66	3.40
	75	**8940.28**	1.07	2.85	**8940.28**	1.55	3.78	9095.47	1.46	3.30
	135	**8232.80**	0.02	2.84	**8232.80**	0.71	3.74	**8232.80**	0.42	3.14
6-20-20	60	**10640.92**	2.71	29.95	10940.28	3.04	24.07	11253.13	2.77	21.63
	90	**9793.74**	2.05	20.88	9834.03	3.10	24.44	**9793.74**	3.82	21.18
	165	**9336.80**	1.13	20.12	**9336.80**	1.74	23.02	9352.80	2.52	23.99
6-25-25	75	**12621.48**	3.79	57.28	12803.37	2.83	56.90	12747.33	3.55	52.76
	105	**11854.31**	2.28	47.21	**11854.31**	2.85	56.48	12252.31	3.80	53.00
	195	**11093.60**	1.02	50.73	11135.20	2.72	60.07	**11093.60**	2.72	54.99
Average			**1.32**	**19.95**		1.98	21.70		2.43	20.06

4.4.2 Comparison with the results from CPLEX solver

Following the tuning of the main parameters as described above, the following values of the MA parameters are chosen as indicated thereafter to perform all computational experiments and results comparison. The size of the population $PopSize$ is set to 200 and the algorithm stops if the maximum number of iterations ($MaxGen$) reaches 200 or when the best individual remains unchanged after 100 consecutive iterations ($MaxNoImp$). The crossover probability (P_c) is set to 0.8. The mutation probabilities P_{m1} and P_{m2} for the location and routing sections are set to 0.7 and 0.9, respectively. The penalty parameter δ of the fitness function is set to 1000 to avoid infeasible solutions.

First, Tables 4.7 and 4.8 compare the results obtained by CPLEX and the proposed MA respectively, for small and medium instances with 3 potential hubs and more. The best gap between the two methods is indicated in boldface. With CPLEX, we report the optimal solution if it is obtained before the time limit, or the best feasible solution otherwise. The best lower bound found is also reported if the time limit is reached. In these two tables, the instance name is denoted by the number of potential hubs, the number of suppliers, the number of clients and the hub capacity. For the other columns of the tables, the following notations are used:

- LB: lower bound or best lower bound found by CPLEX in three hours;
- UB: upper bound (best objective value) found by CPLEX in three hours on each instance, marked "*" if the solution is optimal;
- $\%LB$: deviation in % of the upper bound from the lower bound found by CPLEX. Here, $\%LB = \frac{UB-LB}{UB} \times 100\%$;
- T_{UB}: CPU time in seconds used to obtain the upper bound value the first time by CPLEX;
- Time (s): CPU time in seconds used by the corresponding method;
- Z_{best}: best objective value found by the MA in 10 runs, marked "*" if the solution is optimal;
- $\%LB'$: deviation in % between the best objective value and the lower bound of CPLEX, Here, $\%LB' = \frac{Z_{best}-LB}{Z_{best}} \times 100\%$;
- $\%UB$: deviation in % between the best objective value and upper bound of CPLEX, Here, $\%UB = \frac{Z_{best}-UB}{UB} \times 100\%$;

- T_{best} (s): CPU time in seconds for the best objective obtained by the MA;
- T_{total} (s): total CPU time in seconds used by the MA for 10 runs.

The results in Table 4.7 show that the MA outperforms CPLEX on all 15 instances with 3 potential hubs. The MA can reach all optimal solutions (5 instances) obtained by CPLEX in a shorter computational time, which is less than 2 seconds for the best run and 16 seconds for 10 runs. Especially, when CPLEX cannot find the optimal solution within the time limit, the MA can find a better solution in a shorter time (see the value of $\%UB$). Here, on the 10 instances for which CPLEX did not find an optimal solution, the MA found 6 new best solutions. The gap with the lower bound found by CPLEX shows that the MA outperforms CPLEX with an average value of 12.56% versus 13.37% for CPLEX.

Table 4.7: Results comparison between CPLEX and MA with 3 potential hubs

Instance name		CPLEX-*CSAHLRP-F3*					MA				
H-I-J	Hub_Cap	LB	UB	$\%LB$	T_{UB}	Time(s)	Z_{best}	$\%LB'$	$\%UB$	T_{best}	T_{total}
3-5-5	15	3867.85	3867.85*	**0.00**	1.61	8.55	3867.85*	**0.00**	0.00	0.20	2.03
	30	3068.00	3068.00*	**0.00**	1.72	4.40	3068.00*	**0.00**	0.00	0.17	1.89
	45	3068.00	3068.00*	**0.00**	1.76	4.48	3068.00*	**0.00**	0.00	0.17	1.84
3-10-10	45	7280.72	7613.94	**4.38**	266.9	10800.00	7613.94	**4.38**	0.00	1.17	13.17
	60	6828.25	6828.25*	**0.00**	16.94	3632.55	6828.25*	**0.00**	0.00	1.64	16.59
	120	6249.60	6249.60*	**0.00**	1.12	5878.04	6249.60*	**0.00**	0.00	0.76	7.82
3-15-15	45	8758.11	10653.01	17.79	9761.03	10800.00	10622.13	**17.55**	-0.29	3.99	38.28
	75	7714.49	8940.28	**13.71**	9285.85	10800.00	8940.28	**13.71**	0.00	2.89	28.47
	135	7487.25	8232.80	**9.06**	626.94	10800.00	8232.80	**9.06**	0.00	2.47	28.38
3-20-20	60	8977.83	12186.05	26.33	9807.08	10800.00	11948.93	**24.86**	-2.26	9.42	96.64
	90	8312.03	10469.24	20.61	7685.36	10800.00	10348.40	**19.68**	-1.15	8.66	94.65
	165	6939.54	9336.80	**25.68**	3111.13	10800.00	9336.80	**25.68**	0.00	9.22	93.54
3-25-25	75	10099.80	13165.30	23.28	10421.32	10800.00	12828.40	**21.27**	-2.56	21.06	214.61
	105	8954.44	13121.13	31.76	10413	10800.00	12057.20	**25.73**	-9.39	24.41	259.54
	195	8155.36	11325.60	27.99	8105.95	10800.00	11093.60	**26.49**	-2.05	25.30	259.99
Average				13.37	4633.85			**12.56**	-1.18	**7.44**	77.16

The results for the instances with more hubs in Table 4.8 also show the good performance of the MA compared to CPLEX on all the 18 instances. For all of them, the MA reaches all the best solutions in a shorter computational time (16.10 seconds in average), and also it improves most of the solutions, especially for the medium instances, such as instance 6-20-20 and 10-20-20. Although for most instances, it is difficult to prove the optimality of the solutions. But from the CPU time to find the first feasible solution (upper bound value) by CPLEX, i.e. T_{UB}, it can be seen that the MA can find a better value than CPLEX in a shorter time (16.10s vs 7609.26s in average).

4.4.3 Detailed results of the memetic algorithm

Tables 4.9-4.11 present the detailed results obtained with all instances (small, medium and large ones) for different values of the number of potential hubs, and the details of the performance of the MA. The "Best run" represents the best solution found in 10 runs by the MA including the best objective value Z_{best}, the corresponding running time (T_{best} (s)) and the details of the best solution found by the MA. "Open hub" gives the index of the located hubs and "Max routes" represents the maximum number of routes assigned to a hub for collection and

Table 4.8: Results comparison between CPLEX and MA with more potential hubs

Instance name		CPLEX-*CSAHLRP-F3*					MA				
H-I-J	Hub_Cap	LB	UB	$\%LB$	T_{UB}	Time(s)	Z_{best}	$\%LB'$	$\%UB$	T_{best}	T_{total}
6-10-10	45	6744.25	7613.94	**11.42**	6173.93	10800.00	7613.94	**11.42**	0.00	2.59	25.88
	60	6585.81	6828.25	**3.55**	6193.96	10800.00	6828.25	**3.55**	0.00	2.36	24.49
	120	5877.90	6249.60	**5.95**	3.31	10800.00	6249.60	**5.95**	0.00	1.98	25.30
6-15-15	45	7986.42	9608.55	16.88	7924.74	10800.00	9581.90	**16.65**	-0.28	9.92	100.85
	75	7189.37	8940.28	**19.58**	10032.19	10800.00	8940.28	**19.58**	0.00	9.44	100.92
	135	7049.12	8232.80	**14.38**	3066.42	10800.00	8232.80	**14.38**	0.00	9.39	96.56
6-20-20	60	7823.82	10903.48	28.24	10307.5	10800.00	10640.92	**26.47**	-2.41	29.73	299.48
	90	7199.82	10469.24	31.23	9382.8	10800.00	9793.74	**26.49**	-6.45	20.16	208.76
	165	6706.20	9479.20	29.25	7802.76	10800.00	9336.80	**28.17**	-1.50	18.74	201.23
10-10-10	45	6080.37	7549.08	19.46	4794.89	10800.00	7366.08	**17.45**	-2.42	3.21	35.97
	60	5943.59	6896.10	13.81	10028.85	10800.00	6828.25	**12.96**	-0.98	3.29	32.15
	120	5898.25	6249.60	**5.62**	5347.42	10800.00	6249.60	**5.62**	0.00	2.98	33.07
10-15-15	45	7108.60	9600.12	25.95	9902.02	10800.00	9447.72	**24.76**	-1.59	19.97	198.47
	75	6214.67	8755.91	29.02	7792.37	10800.00	8594.36	**27.69**	-1.85	13.88	132.71
	135	6199.54	8333.60	25.61	10081.64	10800.00	8232.80	**24.70**	-1.21	12.20	141.65
10-20-20	60	7171.53	11466.44	37.46	10679.64	10800.00	10619.89	**32.47**	-7.71	43.43	406.98
	90	6465.97	10611.96	39.07	7488.22	10800.00	9793.74	**33.98**	-7.71	43.43	402.75
	165	6391.69	10469.24	38.95	9964.08	10800.00	9065.60	**29.50**	-13.41	46.08	488.25
Average				21.97	7609.26			**20.10**	-2.62	**16.10**	164.19

delivery together in the best solution. The "Average on 10 runs" shows the statistical result of 10 runs of the MA. The average objective value $\overline{Z}$ of the solutions found in 10 runs is shown in the column of "Average value" and then the following statistical indicators are used to evaluate the performance of the MA:

- Aver gap (%): the average deviation in % between each value obtained by the MA and the best value. Here, $Avergap = \frac{\overline{Z}-Z_{best}}{Z_{best}} \times 100\%$;
- $CV(\%)$: the coefficient of variance for the objective values of the 10 runs with the average value. Here, $CV = SD/\overline{Z} \times 100\%$, where SD is the standard deviation of all the objective values of the 10 runs;
- $CV'(\%)$: the coefficient of variance for all objective values of the 10 runs with the best objective value. Here, $CV' = SD/Z_{best} \times 100\%$;
- T_{aver} (s): the average running time for the 10 runs of the MA.

The results in Tables 4.9-4.11 show that the proposed MA can solve effectively all the instances, even the largest ones which include 100 nodes with 10 potential hubs. For all instances, the proposed MA can find feasible solutions for the CSAHLRP in less than 20 minutes for the instances with 3 and 6 potential hubs. And for the ones with 10 potential hubs, the proposed MA can solve all the problems in less than 50 minutes. Moreover, the small average resulting gaps (0.60% for the instances with 3 hubs, 2.38% for the instances with 6 hubs and 2.72% for the ones with 10 potential hubs) prove the robustness and usefulness of the memetic algorithm. Moreover, the coefficient of variance with the average objective value CV (0.45%, 1.50% and 1.62% in average, respectively) and with the best objective value CV' (0.45%, 1.54% and 1.67% in average, respectively), demonstrate the good stability of the MA. For the best solution, it can be seen that the assignment solution is changed or the number of open hubs is decreased when the capacity of the hubs increases, and the total cost for most instances decreases because

Table 4.9: Results from the MA for the instances with 3 potential hubs

Instance name		Best run				Average on 10 runs				
H-I-J	Hub_Cap	Z_{best}	T_{best}(s)	Open hub	Max routes	Average value	Aver gap	CV	CV'	T_{aver}(s)
3-5-5	15	3867.85*	0.20	1, 2	2	3867.85	0.00	0.00	0.00	0.20
	30	3068.00*	0.17	1	4	3068.00	0.00	0.00	0.00	0.19
	45	3068.00*	0.17	1	4	3068.00	0.00	0.00	0.00	0.18
3-10-10	45	7613.94	1.17	2, 3	8	7677.07	0.83	1.69	1.70	1.32
	60	6828.25*	1.64	2, 3	9	6828.25	0.00	0.00	0.00	1.66
	120	6249.60*	0.76	2	15	6249.60	0.00	0.00	0.00	0.78
3-15-15	45	10622.13	3.99	1, 2, 3	7	10727.99	1.00	0.76	0.77	3.83
	75	8940.28	2.89	1, 3	11	9035.75	1.07	0.92	0.93	2.85
	135	8232.80	2.47	1	18	8234.20	0.02	0.02	0.02	2.84
3-20-20	60	11948.93	9.42	1, 2, 3	10	12172.04	1.87	1.34	1.37	9.66
	90	10348.40	8.66	2, 3	12	10478.64	1.26	0.71	0.72	9.46
	165	9336.80	9.22	2	20	9350.88	0.15	0.18	0.18	9.35
3-25-25	75	12828.40	21.06	1, 2, 3	11	12994.59	1.30	0.55	0.56	21.46
	105	12057.20	24.41	2, 3	12	12227.03	1.41	0.47	0.48	25.95
	195	11093.60	25.30	3	24	11098.80	0.05	0.06	0.06	26.00
Average			7.44				0.60	0.45	0.45	7.72

less hubs may be operated to satisfy the total demand of suppliers and clients, and better route composition is formed.

Figure 4.7 and 4.8 show the relationship between the average gap or the coefficient of variance $CV'(\%)$ and the number of nodes, respectively, for the different values of the potential hubs number. They demonstrate the stability of the MA. It can be seen that the average gap between the best single run and the average run of the MA is less than 4.00%. In addition, for all instances, when the number of nodes exceeds 50, the average gap is relatively stable without significant fluctuation. And Figure 8 shows that the coefficient of variance of all the objective values from the best one is less than 2.50%. Even when the number of nodes exceeds 50, it is close to 2.00%. This shows that the solutions in all the 10 runs are close to the best one. Hence, it offers the decision maker several solutions near to the optimum.

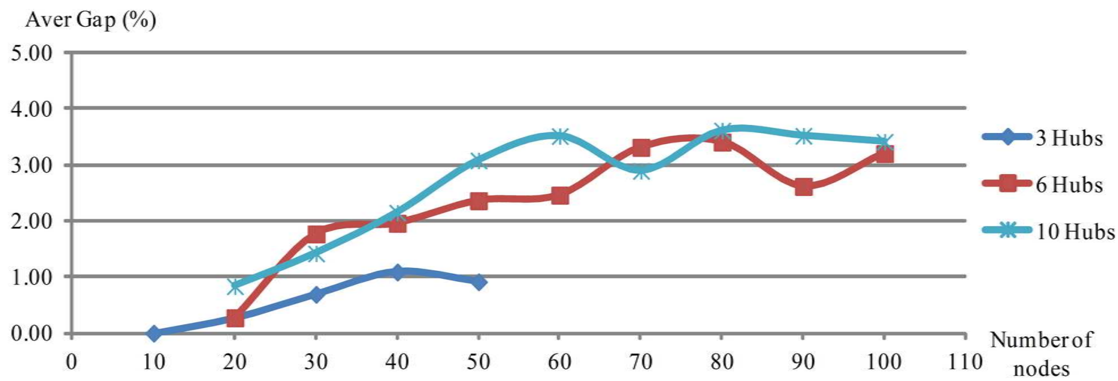


Figure 4.7: The average gap of the MA depending on the problem scale (number of nodes)

In addition, in order to compare the best objective value found by CPLEX and the proposed MA, Figures 4.9-4.11 give some insights for their tendencies as a function of CPU time based

Table 4.10: Results from the MA for the instances with 6 potential hubs

Instance name		Best run				Average on 10 runs				
H-I-J	Hub_Cap	Z_{best}	T_{best}(s)	Open hub	Max routes	Average value	Aver gap	CV	CV'	T_{aver}(s)
6-10-10	45	7613.94	2.59	2, 3	8	7676.63	0.82	0.73	0.73	2.59
	60	6828.25	2.36	2, 3	9	6828.25	0.00	0.00	0.00	2.45
	120	6249.60	1.98	2	15	6249.60	0.00	0.00	0.00	2.53
6-15-15	45	9581.90	9.92	1, 3, 5	7	9839.30	2.69	1.90	1.95	10.09
	75	8940.28	9.44	1, 3	11	9152.80	2.38	1.39	1.42	10.09
	135	8232.80	9.39	1	18	8253.52	0.25	0.44	0.44	9.66
6-20-20	60	10640.92	29.73	2, 4, 5	9	10929.50	2.71	1.54	1.58	29.95
	90	9793.74	20.16	2, 5	13	9994.26	2.05	1.48	1.51	20.88
	165	9336.80	18.74	2	20	9441.92	1.13	0.63	0.63	20.12
6-25-25	75	12621.48	57.41	2, 3, 4	10	13100.14	3.79	1.99	2.06	57.28
	105	11854.31	49.95	2, 4	14	12124.35	2.28	2.08	2.13	47.21
	195	11093.60	47.64	3	24	11207.24	1.02	2.14	2.17	50.73
6-30-30	90	14222.26	107.66	1, 3, 5	13	14781.56	3.93	1.37	1.42	102.94
	120	13207.59	118.26	1, 5	17	13508.79	2.28	1.14	1.16	130.13
	240	12453.60	110.79	5	32	12600.24	1.18	1.48	1.49	96.50
6-35-35	90	16775.89	204.97	1, 5, 6	14	17454.82	4.05	2.04	2.13	197.25
	135	16327.47	185.30	5, 6	19	16841.45	3.15	1.74	1.80	280.62
	270	16209.60	173.63	1	35	16656.50	2.76	1.65	1.70	180.54
6-40-40	90	16135.02	346.04	1, 4, 5	13	16956.13	5.09	2.53	2.65	349.14
	135	14998.09	365.96	1, 5	18	15382.23	2.56	1.11	1.13	383.99
	255	14891.20	295.06	1	34	15275.76	2.58	2.68	2.75	486.04
6-45-45	105	17180.81	569.05	1, 5, 6	16	17756.32	3.35	2.16	2.23	863.56
	150	16193.70	557.31	5, 6	22	16567.36	2.31	1.58	1.62	567.98
	285	15424.00	526.08	6	38	15763.46	2.20	2.20	2.24	622.21
6-50-50	105	18654.91	988.75	2, 4, 6	16	19455.80	4.29	2.12	2.21	940.49
	150	17162.17	968.44	4, 6	21	17248.99	3.36	1.27	1.31	953.66
	300	16458.40	955.55	4	41	16676.49	1.96	1.06	1.08	1020.99
Average			249.34				2.38	1.50	1.54	275.54

Table 4.11: Results from the MA for the instances with 10 potential hubs

Instance name		Best run				Average on 10 runs				
H-I-J	Hub_Cap	Z_{best}	T_{best}(s)	Open hub	Max routes	Average value	Aver gap	CV	CV'	T_{aver}(s)
10-10-10	45	7366.08	3.21	2, 7	8	7532.87	2.26	0.95	0.97	3.60
	60	6828.25	3.29	2, 3	9	6845.36	0.25	0.43	0.43	3.22
	120	6249.60	2.98	2	15	6249.60	0.00	0.00	0.0	3.31
10-15-15	45	9447.72	19.97	5, 7, 8	7	9671.40	2.37	1.23	1.25	19.85
	75	8594.36	13.88	7, 8	9	8713.79	1.39	0.98	1.00	13.27
	135	8232.80	12.20	1	8	8274.88	0.51	0.83	0.83	14.17
10-20-20	60	10619.89	40.53	5, 7, 10	9	10958.84	3.19	1.89	1.95	40.70
	90	9793.74	43.43	2, 5	13	9978.64	1.89	0.83	0.84	40.27
	165	9065.60	46.08	7	20	9189.68	1.37	1.01	1.03	48.83
10-25-25	75	12333.12	102.82	3, 4, 7	10	12842.05	4.13	2.60	2.71	97.88
	105	11797.13	120.82	3, 7	14	12097.21	2.54	1.15	1.18	134.55
	195	11093.60	111.84	3	24	11379.45	2.58	3.35	3.43	123.86
10-30-30	90	13965.64	262.53	1, 5, 9	12	14775.07	5.80	1.23	1.30	265.39
	120	13207.59	206.56	1, 5	17	13619.30	3.12	1.85	1.90	259.28
	240	12437.60	193.75	5	32	12641.52	1.64	2.03	2.07	198.56
10-35-35	90	16675.10	479.97	1, 5, 8	14	17380.77	4.23	2.14	2.24	480.26
	135	16240.32	390.13	5, 6	20	16686.17	2.75	1.82	1.87	408.04
	270	16113.60	364.06	9	36	16389.19	1.71	1.54	1.56	409.18
10-40-40	90	16135.02	807.88	1, 4, 5	13	16999.99	5.36	1.83	1.93	844.55
	135	14998.09	782.80	1, 5	18	15458.87	3.07	2.55	2.63	805.08
	255	14866.40	823.53	1	34	15225.29	2.41	1.57	1.61	804.17
10-45-45	105	17135.03	1461.31	1, 5, 6	16	17972.89	4.89	1.38	1.44	1482.06
	150	16200.24	1314.53	5, 6	20	16728.71	3.26	2.58	2.67	1381.46
	285	15414.40	1281.28	6	38	15787.08	2.42	2.55	2.62	1341.39
10-50-50	105	17491.90	2413.68	4, 6, 7	15	18401.05	5.20	1.93	2.04	2331.45
	150	17154.52	2086.29	4, 6	21	17669.33	3.00	1.88	1.94	2171.20
	300	16400.80	2056.53	4	41	16737.23	2.05	1.62	1.65	1844.59
Average			572.07				2.72	1.62	1.67	576.67

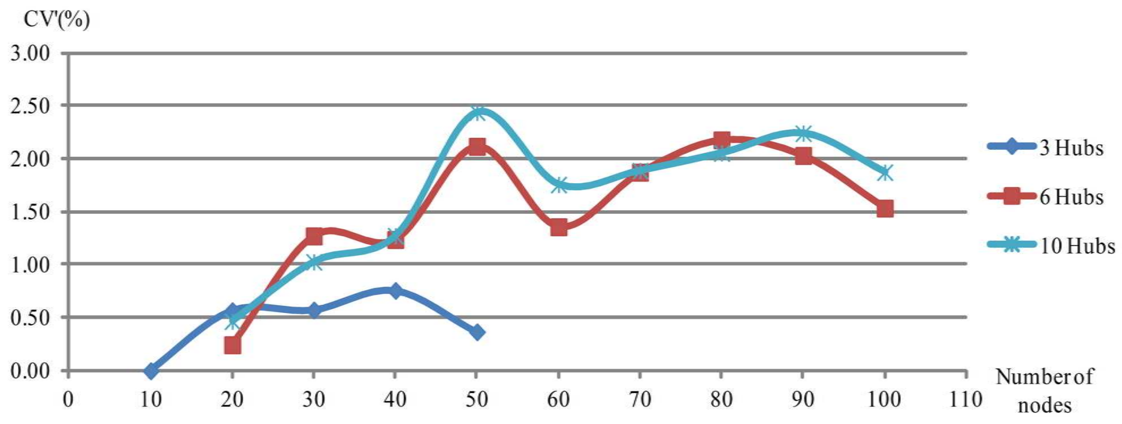


Figure 4.8: CV' changes of the MA depending on the problem scale (number of nodes)

on instance 3-5-5, 3-10-10 and 6-10-10. In these figures, the red lines show the evolution of the best objective values found by CPLEX, and the green ones are the tendency of fitness values in the best run of the MA. The blue lines show the lower bound found by CPLEX as a function of CPU time. From them, it can be seen that the MA can always find better objective values than CPLEX within the same time. And for the optimal solutions or best solutions, the MA always reaches them earlier than CPLEX. For example, for instance 3-5-5 with 15t hub capacity (shown in Figure 4.9.a), CPLEX reaches the optimal value 3867.85 in 1.61 seconds, while the MA can reach it within 0.5 seconds. For the instance 3-10-10 with 60t hub capacity (shown in Figure 4.10.b), the MA can obtain the optimal value 6828.25 within 2 seconds compared to 16.94 seconds for CPLEX. For the best value 7613.94 of instance 6-10-10 with 45t hub capacity, CPLEX needs 6200 seconds to find it, while MA just needs less than 1 second. These figures outline the difficulty for CPLEX to improve the lower bound and prove the optimality of the solution. Moreover, Figures 4.12-4.14 present the evolution of the fitness value at each generation of the MA according to the number of generations. In particular, Figure 4.12 shows the average best fitness value at each generation for the 10 runs based on instance 3-10-10 with different hub capacities. The other figures show the best fitness values at each generation in the best run of the MA based on instances 6-10-10 and 6-15-15, respectively. All of them demonstrate a good convergence of the fitness values of our MA.

All of these observations let us believe that the results of the MA for large instances are near the optimum values. However this property should be validated through further research either by determining more efficient lower bounds or by comparing the results to those of another metaheuristic.

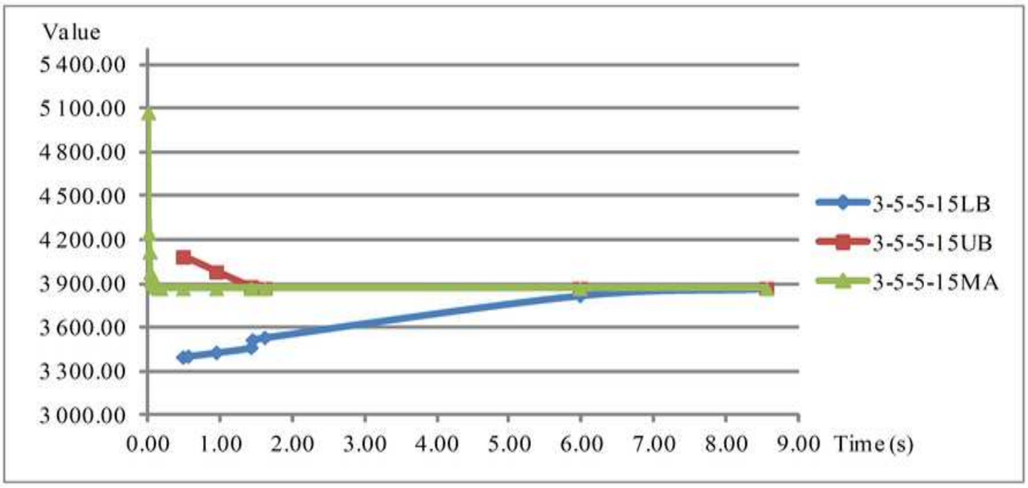
In his recent work, Campbell [32] points out the necessity to validate the consolidation role of the hubs in a hub-and-spoke organization, by comparing the inter-hub flows with the routing flows. In order to do so, we have evaluated the average flow transfered between hubs $Flow_{hub}$ and the average flow routed by each local route $Flow_{route}$ in the best solutions obtained by the MA on some large instances and we have reported them in Table 4.12. The first two columns present the instance name and hub capacity. Column 3 and 4 provide respectively the number of hub arcs and the local routes in the best solution for each instance. In addition, it also reports the minimum flow between hubs $Minflow_{hub}$ and the maximum flow in local routes $Maxflow_{route}$ in column 7 and 8, respectively. The last column provides an average percentage (%) of the number of non-hub nodes with flow larger than the hub arc flow. It can be observed from this table that the average hub arc flow is always larger than the average route flow for these instances. There are no local route with a flow larger than the hub arc flow. In addition, there are also no spokes (non-hub nodes) with flow larger than the hub arc flow. All these observations demonstrate the interest of hub terminals to aggregate flows in the LTL shipment network and the efficiency of the inter-hub transportation. They also justify the underlying hypothesis of the problem.

Table 4.12: The comparison of flow on hub arcs and local routes

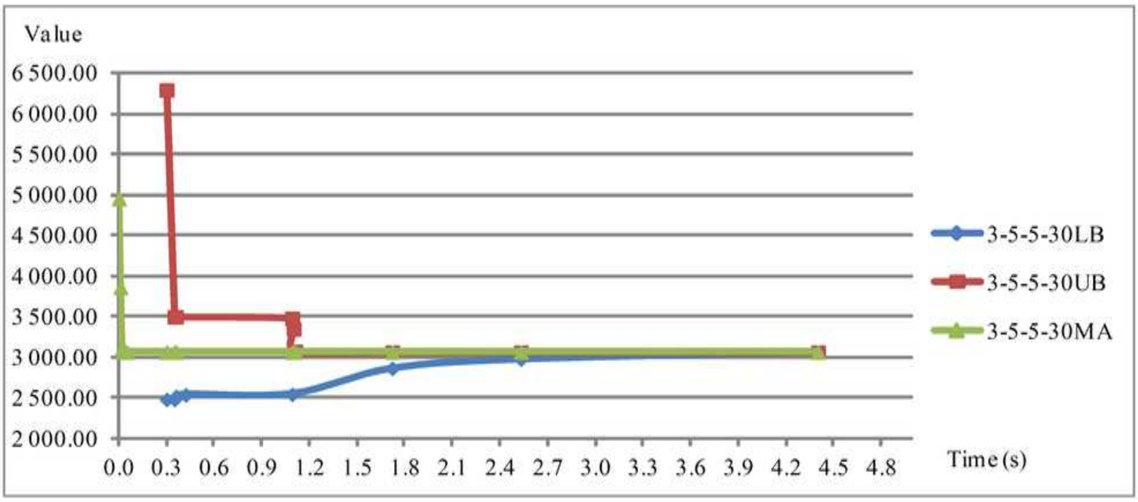
Instance	Γ_k	Hub arcs	Routes	$Flow_{hub}$	$Flow_{route}$	$Minflow_{hub}$	$Maxflow_{route}$	Average %
6-40-40	90	6	36	26.58	13.03	22.36	14.97	0%
	135	2	36	53.23	13.06	42.21	14.97	0%
6-50-50	105	6	42	25.93	12.88	19.08	14.91	0%
	150	2	42	65.54	12.87	56.37	14.98	0%
10-40-40	90	6	36	26.58	13.03	22.36	14.97	0%
	135	2	36	53.23	13.06	42.21	14.97	0%
10-50-50	105	6	43	29.07	12.60	21.71	14.96	0%
	150	2	41	60.98	13.19	55.03	14.95	0%

4.5 Conclusion

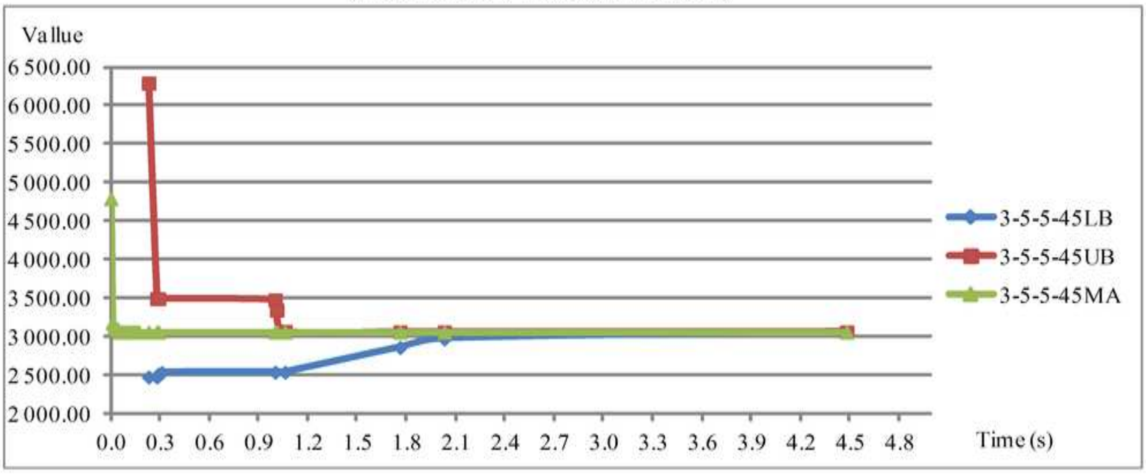
In this chapter, we have proposed a memetic algorithm, combining a genetic algorithm and an iterative local search, in order to solve medium and large instances of the CSAHLRP. After presenting an overview of memetic algorithms, especially the literature review of researches related to ours, the detailed operators of this algorithm have been presented. For the coding of the chromosome, we used a two-dimension array to represent the location and routing informations. For the genetic part, we have described the chromosome selection process, and the crossover and mutation operators as well as the replacement strategy. For the iterative local search, different operators for the hub location and vehicle routing sections have been implemented sequentially. In order to evaluate the performance of the proposed MA, many computational experiments have been conducted on the instances presented in Chapter 3, including tuning of the main parameters. A comparison with the results obtained by solving our model with CPLEX on small and medium size instances has shown that our MA could determine the optimal solutions for small instances within a shorter time. In addition, it can provide good and stable solutions for medium and large instances efficiently and reliably, as it has been shown through many computational experiments. In the next chapter, a branch-and-cut algorithm will be presented in order to solve our problem exactly and further evaluate the results obtained by this metaheuristics.



a. Instance 3-5-5 with 15t hub capacity

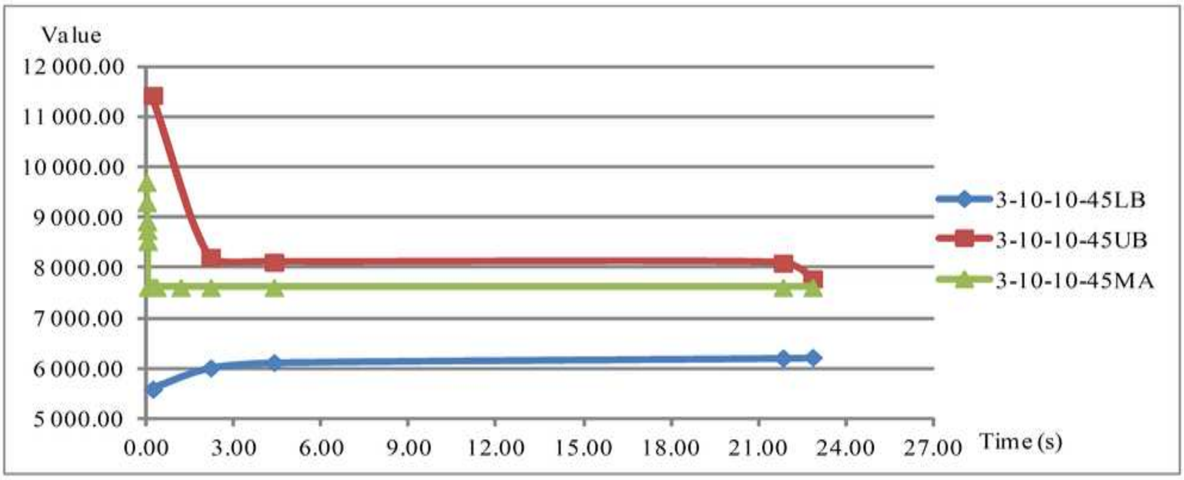


b. Instance 3-5-5 with 30t hub capacity

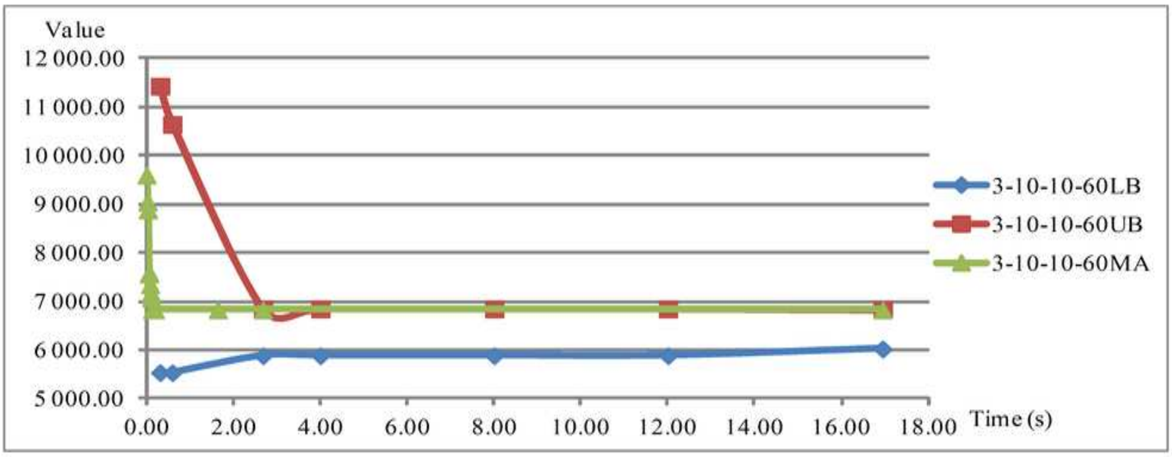


c. Instance 3-5-5 with 45t hub capacity

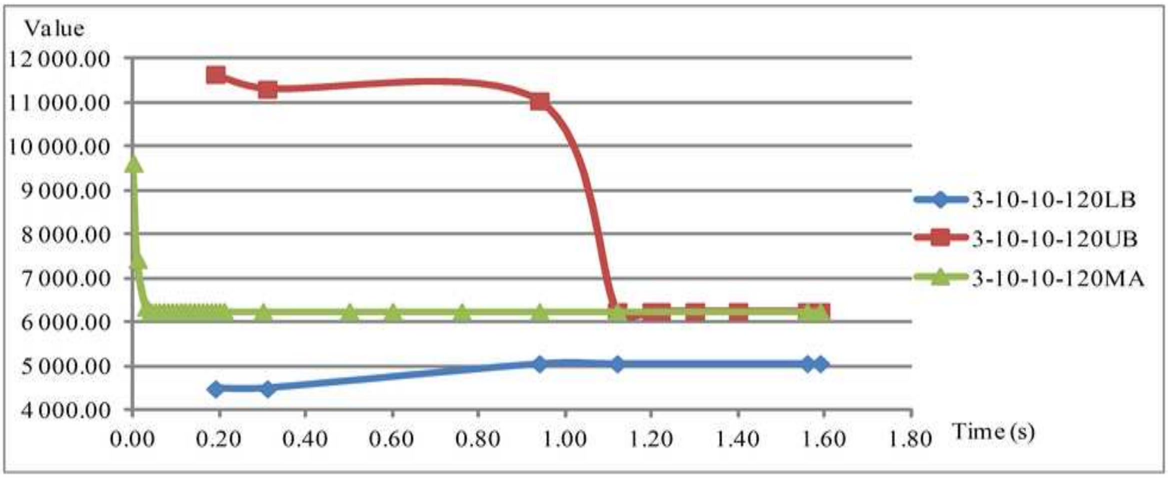
Figure 4.9: Comparison of the solution evolution between CPLEX and the MA for instance 3-5-5



a. Instance 3-10-10 with 45t hub capacity



b. Instance 3-10-10 with 60t hub capacity



c. Instance 3-10-10 with 120t hub capacity

Figure 4.10: Comparison of the solution evolution between CPLEX and the MA for instance 3-10-10

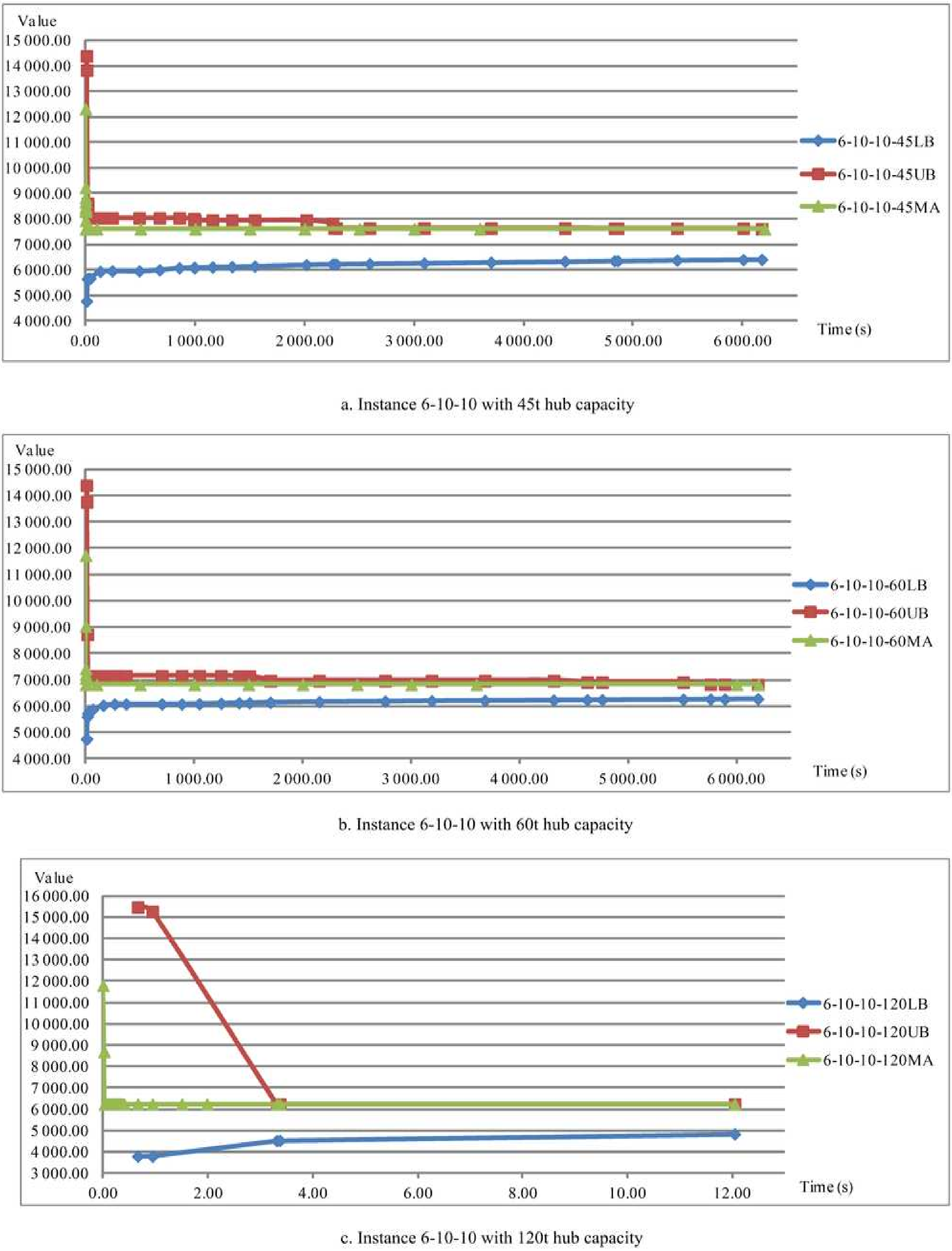


Figure 4.11: Comparison of the solution evolution between CPLEX and the MA for instance 6-10-10

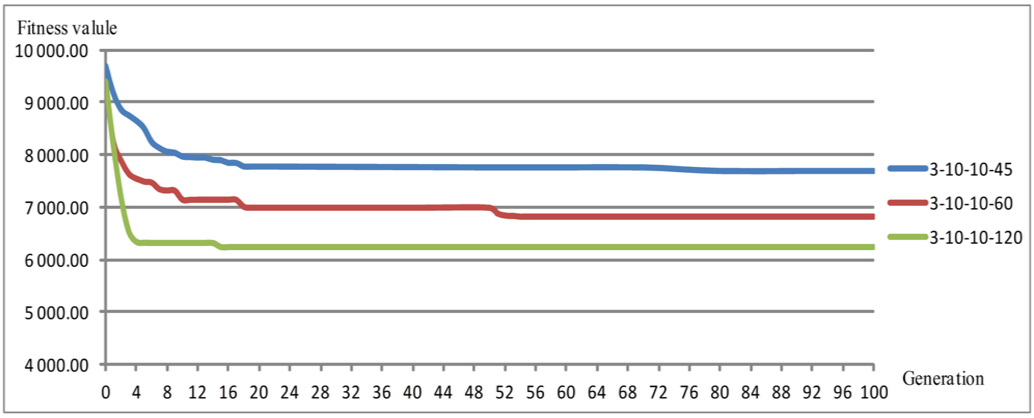


Figure 4.12: The convergence of the average fitness value in 10 runs for instance 3-10-10

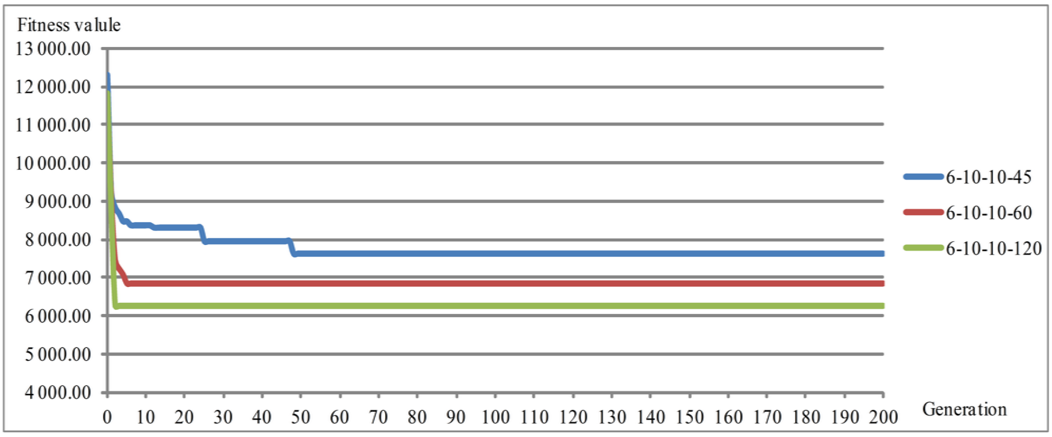


Figure 4.13: The convergence of the fitness value in the best run for instance 6-10-10

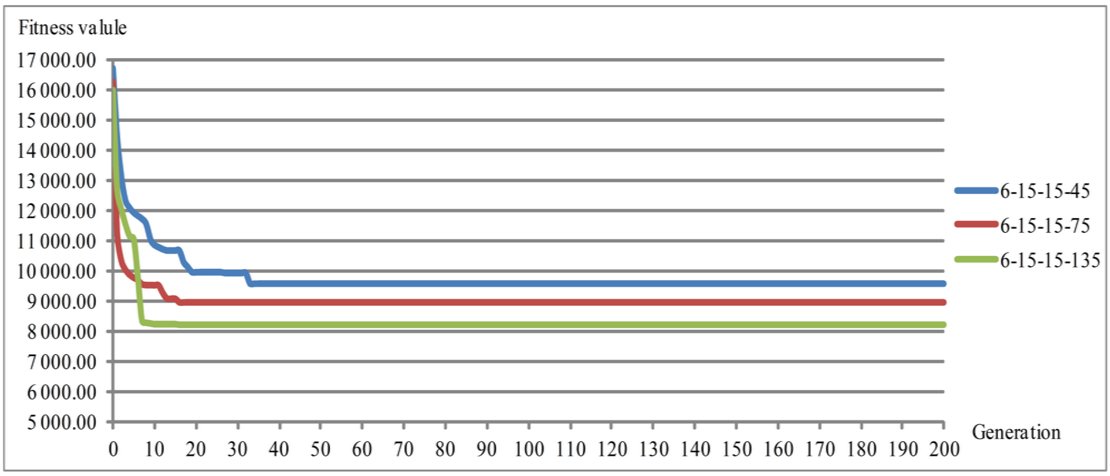


Figure 4.14: The convergence of the fitness value in the best run for instance 6-15-15

5

Branch-and-cut algorithm for the CSAHLRP

In this chapter, an exact method based on the branch-and-cut scheme is proposed to solve the capacitated single allocation hub location-routing problem (CSAHLRP) for the LTL shipments. Firstly, we proposed a mathematical formulation for this problem based on a three-index vehicle-flow model of the LRP [173]. Then some valid inequalities are introduced to strengthen this formulation. Finally, a branch-and-cut algorithm is developed and some computational experiments are implemented based on the instances generated in Chapter 3.

5.1 Introduction

The Branch-and-cut (B&C) algorithm is a widely used exact method to solve integer linear programming problems (ILPs). As a divide-and-conquer approach, it combines the branch-and-bound (B&B) technique and the cutting plane procedure. The B&B method solves a sequence of linear programming (LP) relaxation of ILPs to generate fractional or integer solutions. And the cutting plane method is used to tighten the LP relaxations by dynamically identifying the valid inequalities violated by the current fractional solution. Thus, at each step of the B&C algorithm, the violated inequalities are added to the LP relaxation that is solved again to yield different solutions.

The framework of the B&C method (shown in Algorithm 5) can be visualized by constructing an enumeration tree where each node represents a subproblem. After setting the initial upper bound z_{UB} and lower bound z_{LB} for the ILP, the set of unexplored nodes of the enumeration tree L needs to be initialized, typically with the subproblem at the root node ILP^0. Then a node i is selected from the set L and its LP relaxation is first solved to obtain the current solution S_i and the objective value z_i. If S_i is infeasible, then the ILP is also infeasible. The algorithm will prune the current node i and stop, as well as if $z_i \geq z_{UB}$. However, if the current solution is integral feasible, then the upper bound is updated with the minimum objective value z_i. Otherwise, some violated inequalities generated by the cutting plane procedure are added to the ILP^i and the LP relaxation is re-optimized in Step 4. If no additional cut is found, the

Algorithme 5 Branch-and-cut algorithm

Input: An integer linear programming problem ILP.

Output: Optimal solution S_{opt} and the optimal objective value z_{opt}.

1. *Initialization*:
   Set the upper bound $z_{UB} = +\infty$ or to the value of a known feasible solution;
   Set the lower bound $z_{LB} = -\infty$;
   Initialize the candidate list of unexplored nodes denoted $L \leftarrow \{ILP^0\}$.
2. *Termination*:
   If ($z_{LB} \geq z_{UB}$) or ($L = \varnothing$) or (other termination criterion is met), then
   $z_{opt} = z_{UB}$, and report the corresponding solution as optimal solution S_{opt}. Stop.
   else,
   go to the next step.
3. *Node selection*:
   Select an enumeration node $i \in L$ and delete it from L, i.e. $L \leftarrow L \setminus \{i\}$.
4. *Solve relaxation*:
   Solve the relaxation of subproblem ILP^i at node i;
   Obtain current solution S_i and its objective value z_i, update $z_{LB} = z_i$.
   - If (S_i is infeasible), then
     set $z_{LB} = +\infty$ and go to Step 2.
   - If ($z_i \geq z_{UB}$), then
     go to Step 2.
   - If (S_i is integral feasible) and ($z_i < z_{UB}$), then
     set $z_{UB} = z_i$, and go to Step 2.
   - else
     go to next step.
5. *Add cutting plane*:
   If desired, search for valid inequalities violated by solution S_i, denoted as K.
   - If ($K \neq \varnothing$), then
     add the set K to the problem ILP^i and return to Step 4.
   - else
     proceed.
6. *Branching*:
   Branch based on a non-integer variable and create two new nodes i_1, i_2;
   Add new nodes to L, i.e. $L \leftarrow L \cup \{i_1, i_2\}$.
   Return to Step 2.

branching is executed and adds two new nodes to the set L while node i is deleted from L. This process is repeated until an optimal solution is found or other termination criterion is satisfied. This algorithm has been used to solve exactly many combinatorial optimization problems and in various application areas, as, for example, the vehicle routing problem [15, 16, 133] and the location-routing problem [26, 48, 110] as mentioned in Chapter 2. In addition, this algorithm is also a good approach to provide bounds for large and/or hard problems when it is not possible to prove the optimality of solutions.

In the development of a B&C algorithm, the main challenges are the determination of the classes of valid inequalities and the separation routines to generate them. In most cases, heuristic algorithms are applied for the separation procedure. In the following sections, some valid inequalities for the CSAHLRP and the corresponding separation methods are presented based on a new mathematical formulation.

5.2 A new mathematical formulation for the CSAHLRP

In order to exactly solve the CSAHLRP or provide good bounds using the B&C, a new mathematical formulation is proposed based on a *three-index vehicle-flow* model of the LRP [45]. The collection and delivery routing parts of the original 4-index mathematical model *CSAHLRP-F4* presented in Chapter 3 are changed according to this formulation. In this new model, the activities of collection from suppliers and delivery to clients remain separated, so a supplier and a client can not be allocated to the same local tour. Keeping the same hypotheses and notations as those introduced in Chapter 3, the new model is also defined in a complete graph $G = (N, E)$ with a vertex set $N = H \cup I \cup J$. Thus in order to determine the optimal locations of hubs, the routing of each flow between suppliers and clients, as well as the local collection and delivery tours, some additional notations are used in the new model.

$S_I -$ subset of I, $\forall S_I \subseteq I$, and $\overline{S_I} = \{I \bigcup H\} \setminus \overline{S_I}$ is the complementary set of S_I;

$S_J -$ subset of J, $\forall S_J \subseteq J$, and $\overline{S_J} = \{J \bigcup H\} \setminus \overline{S_J}$ be the complementary set of S_J;

$\delta(S) -$ the set of edges with exactly one end-vertex in S, i.e., $\delta(S) = \{\{i, j\} \in E : i < j, i \in S, j \in \overline{S}$ or $i \in \overline{S}, j \in S\}$;

$\delta^{'}(S : S') -$ the set of edges with one end-node in S and the other one in S', $\forall S \subseteq N, \forall S' \subseteq N$;

$r(S, k) -$ the minimum number of vehicles needed to service the suppliers or clients from hub k in subset S;

$r(S) -$ the minimum number of vehicles needed to service the supplier or clients from all hubs in subset S ;

$q(S, k) -$ the total quantity of flow from the suppliers or to clients in subset S served by hub k;

$q(S) -$ the total quantity of flow from the suppliers or to clients in subset S.

For the decision variables, z_{ik} is used as the allocation variable to represent if a node i is assigned to a hub k. Y_{ijkl} represents the fraction of flow from supplier i to client j via hub k to hub l. For the routing part, the following variables are defined in the new model:

$x_{ij}^k (i < j) -$ an integer variable to represent the number of times that edge $\{i, j\}$ is traversed by one vehicle from hub k. It may take the values $\{0, 1\}$, $\forall \{i, j\} \in E, i, j \notin H$ and the values $\{0, 1, 2\}$, $\forall \{i, j\} \in E, i \in H, j \notin H$. Note that $x_{ij}^k = 2$ corresponds to a return trip between the hub k and node j.

$M_k^c \geqslant 1, M_k^d \geqslant 1-$ integer variables, corresponding to the number of vehicles used at hub k to collect and deliver, respectively. Note that the two variables can also be known constants based on specific conditions.

Then, based on the above notations and variables, the new mathematical formulation for the CSAHLRP (*CSAHLRP-B&C*) is presented as follows:

CSAHLRP-B&C

$$\text{Min} \quad \sum_{k\in H} F_k z_{kk} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in H}\sum_{l\in H} \alpha d_{kl} q_{ij} Y_{ijkl} + \sum_{k\in H}\sum_{i\in I\cup H, i<j}\sum_{j\in I} \beta d_{ij} x_{ij}^k + \sum_{k\in H}\sum_{i\in J\cup H, i<j}\sum_{j\in J} \gamma d_{ij} x_{ij}^k + f_v \sum_{k\in H}(M_k^c + M_k^d) \tag{5.1}$$

subject to

- hub location constraints (3.2)-(3.5), (3.38) and (3.39);
- collection and delivery routing constraints including:

$$\sum_{(i,j)\in\delta(S_I)} x_{ij}^k \geq 2r(S_I, k) \qquad \forall k \in H, \forall S_I \subseteq I, |S_I| \geq 2 \tag{5.2}$$

$$\sum_{u\in I\cup k, u<i} x_{ui}^k + \sum_{l\in I\cup k, l>i} x_{il}^k = 2z_{ik} \qquad \forall i \in I, \forall k \in H \tag{5.3}$$

$$\sum_{i\in I} x_{ki}^k = 2M_k^c \qquad \forall k \in H \tag{5.4}$$

$$\sum_{(i,j)\in\delta(S_J)} x_{ij}^k \geq 2r(S_J, k) \qquad \forall k \in H, \forall S_J \subseteq J, |S_J| \geq 2 \tag{5.5}$$

$$\sum_{u\in J\cup k, u<j} x_{uj}^k + \sum_{l\in J\cup k, l>j} x_{jl}^k = 2z_{jk} \qquad \forall j \in J, \forall k \in H \tag{5.6}$$

$$\sum_{j\in J} x_{kj}^k = 2M_k^d \qquad \forall k \in H \tag{5.7}$$

- variable values constraints (3.27), (3.28) and

$$x_{ij}^k \in \{0,1\} \qquad \forall k \in H, \forall i \notin H, \forall j \notin H \tag{5.8}$$

$$x_{ij}^k \in \{0,1,2\} \qquad \forall k \in H, \forall i \in H, \forall j \notin H \tag{5.9}$$

The objective function (5.1) minimizes the sum of the establishing hub costs, transportation cost between hubs, collection routing costs, delivery routing costs and fixed costs of operating vehicles. The hub location constraints keep the same implications as the corresponding ones in Chapter 3. For the local routing part, constraints (5.2)-(5.4) are collection routing constraints. Constraints (5.2) play a dual role: they prevent the formation of sub-tours in each collection route and they ensure that route capacity constraints are not violated. These inequalities, for any subset S_I of suppliers from hub k not including the hub, impose that $r(S_I, k)$ vehicles enter and leave S_I from hub k. Constraints (5.3) are not only the flow conservation constraints which ensure that two edges are incident to each supplier, but also specify that a supplier can be assigned to a hub only if there is a route from that hub going through that supplier. They can be called as the degree constraints for each supplier. Formulas (5.4) define the degree constraint of each hub and also connect the routing variable x_{ij}^k and the number of vehicles M_k^c used in collection phase. Equations (5.5)-(5.7) are the delivery routing constraints which have the similar meanings to the collection routing constraints. Constraints (5.8)-(5.9) are the variables value constraints.

5.3 Valid inequalities

In this section, some valid inequalities are proposed to strengthen the linear relaxation of the new formulation by eliminating some fractional solutions from the solution space.

5.3.1 Known valid inequalities

As mentioned in Chapter 3, the two efficient polynomial-size valid inequalities for the formulation *CSAHLRP-F4*, are obviously valid for the new formulation. The first one is the *open hub limitation* inequality (3.46) which can be added directly into the linear relaxation of the new formulation. It provides the minimum number of hubs to open.

The second class of known polynomial-size valid inequalities (3.47) and (3.48) give the minimum number of routes used for the collection and delivery from the hubs. Based on the new variable definitions, this family of valid inequalities can be derived from the *hub degree* constraints (5.4) and (5.7), respectively. They limit the number of edges entering and leaving the hubs and denote that each vehicle must enter and leave the same hub if it passes this hub. Obviously, the variables M_k^c and M_k^d have the following lower bound limitation to serve the demands of all suppliers or clients allocated to hub k:

$$M_k^c \geq \frac{\sum_{i\in I} z_{ik} O_i}{Q} \qquad \forall k \in H \tag{5.10}$$

$$M_k^d \geq \frac{\sum_{j\in J} z_{jk} D_j}{Q} \qquad \forall k \in H \tag{5.11}$$

So the constraints (5.4) and (5.7) can be replaced by the following inequalities:

$$\sum_{i\in I} x_{ki}^k \geq 2\frac{\sum_{i\in I} z_{ik} O_i}{Q} \qquad \forall k \in H \tag{5.12}$$

$$\sum_{j\in J} x_{kj}^k \geq 2\frac{\sum_{j\in J} z_{jk} D_j}{Q} \qquad \forall k \in H \tag{5.13}$$

In order to simplify the above two groups of inequalities, all the hubs are grouped and can be viewed as a large fictive depot. Thus the two hub degree constraints on each hub can be transformed into the following two single allocation constraints.

$$\sum_{k\in H}\sum_{i\in I} x_{ki}^k \geq 2\frac{\sum_{i\in I} O_i}{Q} \tag{5.14}$$

$$\sum_{k\in H}\sum_{j\in J} x_{kj}^k \geq 2\frac{\sum_{j\in J} D_j}{Q} \tag{5.15}$$

Because the left sides of above inequalities are integer values, the rounded lower bounds can replace the right sides. Therefore, the valid inequalities providing the minimum number of routes associated to the hubs, called as *hub degree* valid inequalities are given as follows:

$$\sum_{k \in H} \sum_{i \in I} x_{ki}^k \geq 2 \lceil \frac{\sum_{i \in I} O_i}{Q} \rceil \tag{5.16}$$

$$\sum_{k \in H} \sum_{j \in J} x_{kj}^k \geq 2 \lceil \frac{\sum_{j \in J} D_j}{Q} \rceil \tag{5.17}$$

where $\lceil x \rceil$ denotes the smallest integer not less than x.

5.3.2 Simple valid inequalities between variables

Some other simple and efficient polynomial-size inequalities can be used to improve the lower bounds of the linear relaxation by a straightforward way. They are given as follows:

$$x_{ij}^k \leq z_{ik} \qquad \forall k \in H, \forall i \in I, \forall j \in I \tag{5.18}$$

$$x_{ij}^k \leq z_{jk} \qquad \forall k \in H, \forall i \in I, \forall j \in I \tag{5.19}$$

$$x_{ij}^k \leq z_{ik} \qquad \forall k \in H, \forall i \in J, \forall j \in J \tag{5.20}$$

$$x_{ij}^k \leq z_{jk} \qquad \forall k \in H, \forall i \in J, \forall j \in J \tag{5.21}$$

$$x_{ki}^k \leq 2z_{kk} \qquad \forall k \in H, \forall i \in I \cup J \tag{5.22}$$

These valid inequalities impose a strong relationship between the routing variables and the allocation variables. They show that, if a supplier i or a client j is not allocated to a open hub k, then all routing variables from the hub k and linked to that supplier i or client j should be set to 0. And it is also valid for the routing variable starting from hub k as in inequality (5.22). It shows that there is no link between a closed hub k and any supplier or client.

Except the previous straightforward valid inequalities, the following exponential-size inequalities derived from the CVRP [16, 133] or the CLRP [26, 45] can also be adapted to our problem. They can be applied in the cutting plane procedure to improve the efficiency of the B&C and obtain better solutions or lower bounds.

5.3.3 Rounded route capacity constraints

Let us consider the constraints (5.2) and (5.5), they are the capacity constraints for the routes (also called generalized subtour elimination constraints) which represent that at least $r(S, k)$ vehicles enter and leave S from hub k, for any subset of suppliers or clients. They are derived from similar constraints of the CVRP and have been proved to be NP-hard to compute $r(S, k)$ [16]. However, they remain valid if $r(S, k)$ is replaced by a lower bound on its value, such as follows:

$$r(S_I, k) \geq \frac{\sum_{i \in S_I} O_i z_{ik}}{Q} \qquad \forall k \in H, \forall S_I \subseteq I_k, |S_I| \geq 2 \tag{5.23}$$

$$r(S_J, k) \geq \frac{\sum_{i \in S_J} D_j z_{jk}}{Q} \qquad \forall k \in H, \forall S_J \subseteq J_k, |S_J| \geq 2 \tag{5.24}$$

Moreover, if we group all hubs into one large fictive hub like in the *hub degree* valid inequality, the collection and delivery process can be considered as two capacitated vehicle routing problems (CVRP). Thus, with a rounded lower bound of the right side, the *rounded route capacity* (RRC) constraints for the CSAHLRP can be presented as follows:

$$\sum_{k\in H}\sum_{(i,j)\in\delta(S_I)} x_{ij}^k \geq 2\lceil\frac{\sum_{i\in S_I} O_i}{Q}\rceil \qquad \forall S_I \subseteq I, |S_I| \geq 2 \tag{5.25}$$

$$\sum_{k\in H}\sum_{(i,j)\in\delta(S_J)} x_{ij}^k \geq 2\lceil\frac{\sum_{j\in S_J} D_j}{Q}\rceil \qquad \forall S_J \subseteq J, |S_J| \geq 2 \tag{5.26}$$

They guarantee that the number of routes serving a set of suppliers or clients is not less than the corresponding lower bound for collection and delivery process, respectively. They can be also expressed as in the capacitated location-routing problem [110] as follows:

$$\sum_{k\in H}\sum_{(i,j)\in\delta'(S_I:S_I)} x_{ij}^k \leq |S_I| - \lceil\frac{\sum_{i\in S_I} O_i}{Q}\rceil \qquad \forall S_I \subseteq I, |S_I| \geq 2 \tag{5.27}$$

$$\sum_{k\in H}\sum_{(i,j)\in\delta'(S_J:S_J)} x_{ij}^k \leq |S_J| - \lceil\frac{\sum_{j\in S_J} D_j}{Q}\rceil \qquad \forall S_J \subseteq J, |S_J| \geq 2 \tag{5.28}$$

5.3.4 Hub capacity valid inequalities

These valid inequalities are based on the *depot capacity constraints* which were originally proposed for the CLRP [26]. Based on the relationship between the HLRP and the LRP, we have adapted these valid inequalities for the CSAHLRP, called as *hub capacity* (HC) valid inequalities as follows:

$$\sum_{k\in H k\neq h}\sum_{(i,j)\in\delta'(S_I:I\setminus S_I)} x_{ij}^k + \sum_{k\in H,k\neq h}\sum_{i\in S_I} x_{ki}^k \geq 2 \quad \forall h \in H, \forall S_I \subseteq I, q(S_I) > \Gamma_h \tag{5.29}$$

$$\sum_{k\in H,k\neq h}\sum_{(i,j)\in\delta'(S_J:J\setminus S_J)} x_{ij}^k + \sum_{k\in H,k\neq h}\sum_{j\in S_J} x_{kj}^k \geq 2 \quad \forall h \in H, \forall S_J \subseteq J, q(S_J) > \Gamma_h \tag{5.30}$$

They are valid for the CSAHLRP and emphasize the capacity restriction for hubs in the collection and delivery processes, respectively. Given any feasible CSAHLRP solution, when the total quantity of flow originating from the supplier subset S_I is larger than the capacity of hub h, i.e. $q(S_I) > \Gamma_h$, the suppliers in S_I can't be completely served by the vehicles from the only hub h. There should exist at least one supplier in S_I allocated to another hub. Therefore, it needs at least two links from suppliers in subset S_I to another hub than h or to another supplier $j \notin S_I$ served by another hub. Similar inequalities (5.30) are also valid for the delivery process when the total amount of flow sent to the client subset S_J exceeds the capacity of hub h.

5.3.5 Strengthened hub degree valid inequalities and disaggregated co-circuit constraints

Inspired from the *depot degree constraints* of Belenguer et al. [26] and the *disaggregated facility degree constraints* of Contardo [45] for the capacitated location-routing problem, we developed the *strengthened hub degree* (SHD) inequalities for the CSAHLRP, and named them SHD in order to distinguish them from the *hub degree* constraints (5.16) and (5.17). They are valid for the CSAHLRP when the edge distances satisfy the triangle inequality. The SHD valid inequalities for the CSAHLRP are given as follows:

$$\sum_{i \in S_I} x_{ki}^k + \sum_{i \in S_I} \sum_{j \in S_I} x_{ij}^k \leq 2z_{kk} + |S_I| - 1 \qquad \forall S_I \subseteq I, q(S_I) \leq Q, \forall k \in H \tag{5.31}$$

$$\sum_{j \in S_J} x_{kj}^k + \sum_{i \in S_J} \sum_{j \in S_J} x_{ij}^k \leq 2z_{kk} + |S_J| - 1 \qquad \forall S_J \subseteq J, q(S_J) \leq Q, \forall k \in H \tag{5.32}$$

Inequalities (5.31) are for the collection process and (5.32) are for the delivery process. They are valid for the subsets S for which the total quantity of flow $q(S)$ doesn't exceed the vehicle capacity Q.

Proposition 5.3.1 *Inequalities (5.31) and (5.32) are valid for the CSAHLRP.*

Proof. We consider the inequalities (5.31) as an example. There are two cases based on the value of z_{kk} to verify them. Firstly when $z_{kk} = 0$, it means that the hub k will not be open. Then the left side of (5.31) will be zero, i.e. $\sum_{i \in S_I} x_{ki}^k + \sum_{i \in S_I} \sum_{j \in S_I} x_{ij}^k = 0$ and $0 \leq |S_I| - 1$ always holds. In another case where $z_{kk} = 1$, the right side of (5.31) will be $|S_I| + 1$. If $q(S_I) \leq Q$, all the suppliers in S_I can be at most served by one vehicle starting from the hub k under the assumption that edge distances hold the triangular inequalities. Indeed, if there are more than one vehicle from hub k visiting these suppliers, they can always be merged into one vehicle aiming to saving the distance cost and the vehicle number used by this hub. Therefore, inequality $\sum_{i \in S_I} x_{ki}^k + \sum_{i \in S_I} \sum_{j \in S_I} x_{ij}^k \leq |S_I| + 1$ holds. Similar proof can be extended to the inequalities (5.32).

The other family of valid inequalities are called as *disaggregated co-circuit* (DCoCC) constraints and are based on the ones proposed by Contardo [45] for a three-index formulation of the CLRP. We have adapted them for the CSAHLRP as follows:

$$\sum_{(i,j) \in \{\delta(S_I) \setminus F_I\}} x_{ij}^k \geq \sum_{(i,j) \in F_I} x_{ij}^k - |F_I| + 1 \quad \forall k \in H, \forall S_I \subseteq I, \forall F_I \subset \delta'(S_I : I \setminus S_I), |F_I| \text{is odd} \tag{5.33}$$

$$\sum_{(i,j) \in \{\delta(S_J) \setminus F_J\}} x_{ij}^k \geq \sum_{(i,j) \in F_J} x_{ij}^k - |F_J| + 1 \quad \forall k \in H, \forall S_J \subseteq J, \forall F_J \subset \delta'(S_J : J \setminus S_J), |F_J| \text{is odd} \tag{5.34}$$

where F with odd size is a subset of edges with one end-node in S_I or S_J and the other in $I \setminus S_I$ or $J \setminus S_J$. These constraints are valid if the return trips are not considered in F. In this case, the number of other edges used in the feasible solution must still be an even due to the circuit vehicle routes. The return trip is the vehicle route serving only one supplier or one client.

Proposition 5.3.2 *Inequalities (5.33) and (5.34) are valid for the CSAHLRP.*

Proof. Let use the constraints (5.33) as an example where $\sum_{(i,j) \in F_I} x_{ij}^k \leq |F_I|$ for $F_I \subset \delta'(S_I : I \setminus S_I)$ always holds.

If $\sum_{(i,j) \in F_I} x_{ij}^k < |F_I|$, constraints (5.33) are obviously satisfied because the right side is non-positive.

If $\sum_{(i,j) \in F_I} x_{ij}^k = |F_I|$, then the right side will be 1. In this case, the left side $\sum_{(i,j) \in \{\delta(S_I) \setminus F_I\}} x_{ij}^k$ has two possibilities. If the set $\delta(S_I) \setminus F_I$ involve return trips from hub k, then $\sum_{(i,j) \in \{\delta(S_I) \setminus F_I\}} x_{ij}^k \geq 1$ always holds. Otherwise, because $\sum_{(i,j) \in F_I} x_{ij}^k = |F_I|$ indicates that all the edges in F_I are visited by vehicles starting from hub k, it follows that there must be at least one edge existing in $\delta(S_I) \setminus F_I$ linked to hub k because $|F_I|$ is odd. Therefore, $\sum_{(i,j) \in \{\delta(S_I) \setminus F_I\}} x_{ij}^k \geq 1$.

Similarly, constraints (5.34) are valid for the delivery process of the CSAHLRP.

5.3.6 Generalized large multistar inequalities

The last exponential-size inequalities are derived from the *generalized large multistar* (GLM) inequalities, which were presented for the CVRP [133] and have been utilized for the LRP [110]. The original form for the CVRP is given as follows:

$$Q^{'}x^{'}(E^{'}(S^{'})) + \sum_{j\in V_c\setminus S^{'}} q_j^{'}x^{'}(E^{'}(S^{'}:\{j\})) \leq Q^{'}|S| - q^{'}(S) \tag{5.35}$$

where V_c denotes the set of customers in CVRP; $S^{'} \subset V_c, |S^{'}| \geq 2$ is a subset of customer set called as a *nucleus*; $E^{'}(S^{'})$ is the set of edges with both end-nodes in $S^{'}$; while $E^{'}(S^{'}:\{j\})$ denotes the set of edges with one end-vertex in $S^{'}$ and the other one is j called as a *satellite* [128]. In addition, $Q^{'}$ is the vehicle capacity, $q_j^{'}$ is the demand for each customer j and $q^{'}(S^{'}) = \sum_{j\in S^{'}} q_j^{'}$ denotes the total demand of customers in subset $S^{'}$. The variable $x_{ij}^{'}$ represents the number of times the edge (i,j) is used in the CVRP solution. And $x^{'}(E^{'}(N)) = \sum_{(i,j)\in E^{'}(N)} x_{ij}^{'}$.

As mentioned above, if all hubs are shrunk into one large fictive hub, the collection process of the CSAHLRP corresponds to a capacitated vehicle routing problem. Then the relationship between the variable $x_{ij}^{'}$ of the CVRP and the variable x_{ij}^k in formulation *CSAHLRP-B&C* is expressed as $x_{ij}^{'} = \sum_{k\in H} x_{ij}^k$, for $(i,j) \in E, i < j$. Then based on this relationship as well as between corresponding parameters, the adapted GLM inequality for the collection and delivery processes of the CSAHLRP are given as follows, respectively:

$$Q\sum_{k\in H}\sum_{i\in S_I}\sum_{j\in S_I, i<j} x_{ij}^k + \sum_{i\in I\setminus S_I} O_i \sum_{k\in H}\sum_{(i,j)\in\delta^{'}(S_I:\{i\})} x_{ij}^k \leq Q|S_I| - q(S_I) \quad \forall S_I \subset I, |S_I| \geq 2 \tag{5.36}$$

$$Q\sum_{k\in H}\sum_{i\in S_J}\sum_{j\in S_J, i<j} x_{ij}^k + \sum_{j\in J\setminus S_J} D_j \sum_{k\in H}\sum_{(i,j)\in\delta^{'}(S_J:\{j\})} x_{ij}^k \leq Q|S_J| - q(S_J) \quad \forall S_J \subset J, |S_J| \geq 2 \tag{5.37}$$

5.4 Branch-and-cut algorithm

In this section, our branch-and-cut algorithm for the CSAHLRP is described based on the framework presented in Algorithm 5. Initially, a linear program (LP) of the formulation *CSAHLRP-B&C* is built, containing the objective function and some constraints described later. Then, at each node of the search tree, we solve the current LP and look for a set of violated inequalities among those presented in section 5.3 by solving the separation problem. The violated constraints are added to the current LP, which is optimized again. This process stops when no more violated cuts can be found. If there are also fractional values for integer variables, the branching is implemented. If all the integer constraints are satisfied, a new node is explored until an optimal solution is found or the limited time is arrived. The main implementation features of our B&C algorithm are discussed below.

5.4.1 Initial linear relaxation

To initialize the LP model of the formulation *CSAHLRP-B&C*, the complex constraints (5.2) and (5.5) are relaxed as well as the integer constraints on the variables of the original formulation to reduce the size of the LP model and efficiently solve the model. In addition, we relax the constraints (5.4) and (5.7), which are replaced by the *hub degree* valid inequalities (5.16) and (5.17), respectively. The *open hub limitation* inequality (3.46) is also contained in the initial LP model to limit the number of open hubs at the root node. Hence, the initial LP model LF is defined as follows:

$$\text{Min} \sum_{k\in H} F_k z_{kk} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in H}\sum_{l\in H} \alpha d_{kl} q_{ij} Y_{ijkl} + \sum_{k\in H}\sum_{i\in I\cup H, i<j}\sum_{j\in I} \beta d_{ij} x_{ij}^k + \sum_{k\in H}\sum_{i\in J\cup H, i<j}\sum_{j\in J} \gamma d_{ij} x_{ij}^k + \frac{f_v}{2}\sum_{k\in H}\sum_{i\in i\cup J} x_{ki}^k \tag{5.38}$$

subject to constraints (3.2)-(3.5), (3.38), (3.39), (5.3), (5.6) and continuous relaxation of decision variables, together with valid inequalities (3.46), (5.16) and (5.17).

5.4.2 Separation procedures and branching strategy

An important step in the B&C algorithm is the design of the separation procedures which decide how to identify violated inequalities in the cutting plane scheme. For the constraints presented in section 5.3, several algorithms are applied to find the violated ones from the current LP solution. Most of them are heuristic methods which have been used in the CVRP or the CLRP for some similar inequalities.

Separation of the *simple* valid inequalities

The polynomial-size inequalities, i.e. *simple* valid inequalities (5.18)-(5.22), are separated by a complete enumeration and the violated cuts are added into the LP model at each node.

Separation of the RRC constraints

To separate the RRC constraints (5.27)-(5.28), a *greedy randomized algorithm* is applied, as in the CVRP [3, 16] and in the vehicle covering tour problem [95]. It is an iterative procedure that is applied to a number of subsets $S_I \subseteq I$ and $S_J \subseteq J$ generated a priori. Here, we use the collection process as an example to illustrate this heuristic. Given an initial subset of suppliers S_I, at each iteration, it is updated with a new supplier that maximizes $x(\delta(S_I : i))$ over all $i \in I \setminus S_I$. Let $i^* \in I \setminus S_I$ be the new added supplier who should satisfy

$$\sum_{k\in H}\Big(\sum_{u\in S_I, u<i^*} x_{ui^*}^k + \sum_{u\in S_I, u>i^*} x_{i^*u}^k\Big) = \max_{i\in I\setminus S_I}\left[\sum_{k\in H}\Big(\sum_{u\in S_I, u<i} x_{ui}^k + \sum_{u\in S_I, u>i} x_{iu}^k\Big)\right] \tag{5.39}$$

Then the constraint (5.27) is checked for the subset $S_I \bigcup i^*$. If the current solution violates this constraint, it is added to the LP model, and S_I is updated.

In this separation algorithm (shown in Algorithm 6), the initial subset S_I is generated with a single supplier from a seed set Υ_I which includes half of the supplier nodes selected randomly [110]. After the initialization of S_I, it is iteratively expanded by a new supplier node i^* based on the above criteria. Then the constraint (5.27) violation is checked for $S_I \cup i^*$. The violated ones are stored into the identified inequality set K and added to the model. This process is repeated until all suppliers are contained into the set S_I. Then the next seed from Υ is considered as the initial subset S_I and the above steps are repeated until the Υ_I is empty. During this process, a checking duplicate policy is implemented in the set K to avoid the generation of previously identified subset. At the same time, the constraint (5.28) is also checked based on the set of clients with a similar method. The detailed steps of this separation algorithm for the RRC constraints are presented in Algorithm 6.

Algorithme 6 Separation algorithm for the RRC constraints (5.27) and (5.28)

1. Generate a supplier seed set Υ_I which randomly includes half of the supplier set nodes;
   Generate a client seed set Υ_J which randomly includes half of the client set nodes.
2. **Repeat**
   2.1. Initialize randomly S_I with a supplier $i \in \Upsilon_I$ as well as S_J with a client $j \in \Upsilon_J$;
   2.2. $S_I \leftarrow i$, $S_J \leftarrow j$, $\Upsilon_I \leftarrow \Upsilon_I \setminus i$ and $\Upsilon_J \leftarrow \Upsilon_J \setminus j$;
   2.3. **Repeat**
   - Select next supplier i^* and next client j^* such that

$$\sum_{k\in H}\Big(\sum_{u\in S_I,u<i^*} x^k_{ui^*} + \sum_{u\in S_I,u>i^*} x^k_{i^*u}\Big) = \max_{i\in I\setminus S_I}\left[\sum_{k\in H}\Big(\sum_{u\in S_I,u<i} x^k_{ui} + \sum_{u\in S_I,u>i} x^k_{iu}\Big)\right]$$

$$\sum_{k\in H}\Big(\sum_{l\in S_J,l<j^*} x^k_{lj^*} + \sum_{l\in S_J,l>j^*} x^k_{j^*l}\Big) = \max_{j\in J\setminus S_J}\left[\sum_{k\in H}\Big(\sum_{l\in S_J,l<j} x^k_{lj} + \sum_{l\in S_J,l>j} x^k_{jl}\Big)\right]$$

   - $S_I \leftarrow S_I \cup i^*$, $S_J \leftarrow S_I \cup j^*$;
     Check the violation of constraint (5.27) for S_I and constraint (5.28) for S_J.
   - Add the violated ones to the LP model and store them in set K after repetitive inspection.

   Until $S_I = I$ and $S_J = J$.

   Until $\Upsilon_I = \varnothing$ and $\Upsilon_J = \varnothing$.

To separate the hub capacity valid inequalities (5.29)-(5.30), the SHD inequalities (5.31)-(5.32) and the GLM inequalities (5.36)-(5.37), similar heuristics as Algorithm 6 are implemented.

Separation of the hub capacity valid inequalities

The separation of the hub capacity valid inequalities (5.29)-(5.30) is performed as in Algorithm 6, but the procedure only considers subsets $S_I \subseteq I$ and $S_J \subseteq J$ for each hub $h \in H$, such that $q(S_I) > \Gamma_h$ and $q(S_J) > \Gamma_h$.

Separation of the SHD inequalities

The violation of SHD inequalities (5.31) and (5.32) are checked for each hub and the same subsets S_I and S_J as generated in Algorithm 6, respectively. But they should satisfy $q(S_I) \leq Q$ and $q(S_J) \leq Q$.

Separation of the GLM inequalities

The GLM inequalities are checked for the shrunk fictive depot and each generated subsets S_I and S_J but not including all suppliers and clients. Therefore, in order to reduce the computational time, the seed sets Υ_I and Υ_J for them are generated randomly with only one supplier and one client, respectively.

Separation algorithm for the DCoCC constraints

About the separation algorithm for the DCoCC constraints (5.33) and (5.34), the generation of sets S_I and S_j keeps the same as the Algorithm 6 for the SHD inequalities. Then for these sets, it needs to look for subsets $F_I \subset \delta(S_I)$ and $F_J \subset \delta(S_J)$ that define the violated constraints (5.33) and (5.34) for each hub $k \in H$, respectively. Here, a heuristic procedure, as for the CLRP [45], is applied to delimit the edge sets F_I and F_J and to minimize the left side of constraints (5.33) and (5.34). Given a subset S_I and hub k, the generation of the subset F_I can be executed in a linear time by defining $F_I = \{(i,j) \in \delta(S_I) : x^k_{ij} \leq 1/2\}$. If $|F|$ is even, then either add to or remove from F_I the edge in $\delta(S_I)$ minimizing the variety of the left side of (5.33). The similar process is implemented to generate subset F_J. Then, for each supplier set S_I and generated set F_I, and also for each client set S_J and edge set F_J, the corresponding DCoCC constraint is checked for each hub until all suppliers and clients are included in S_I and S_J, respectively. Finally, violated identified inequalities are stored in the set K.

Strategies to generate the cuts

Based on the preliminary experiments presented in Section 5.5, the following strategies are utilized to decide when and where the above separation algorithms are used to identify the corresponding violated inequalities at each node of the complete B&C algorithm. Firstly, the separation approach for the *simple* valid inequalities (5.18)-(5.22) is called to generate a set of violated cuts whose maximum size is limited to 100. Then the separation algorithm for the RRC constraints (5.27)-(5.28) is called to generate corresponding violated cuts until no one is found. Finally, the separation routines for the SHD valid inequalities (5.31)-(5.32) and the GLM inequalities (5.36)-(5.37) are applied sequentially to generate the sets of corresponding violated. The maximum number of cuts for the SHD inequalities is limited to 20 and to 50 for the GLM inequalities. These values have been fixed after some preliminary tests. If no violated cuts for the two valid inequalities (the SHD and the GLM) are found, the separation algorithm for the *hub capacity* valid inequalities (5.29)-(5.30) is called to identify a set of corresponding violated cuts whose maximum size is limited to 10. The separation method for the DCoCC constraints (5.33)-(5.34) is called finally if no above violated cuts are found and the maximum size is limited to 10. All above violated inequalities found by the separation strategy are stored in the set K and added to the LP model if K is not empty. Then the model is optimized again until an optimal solution is obtained or the limited computational time is reached.

Our branch-and-cut algorithm is built in C++ using the Concert Technology framework of CPLEX 12.5, where some default cuts are used to improve the lower bounds. In order to turn off some useless CPLEX cuts and reduce the computational time, some tests were done by activating each CPLEX cut one by one in the B&C algorithm. We observed that the four CPLEX cuts (the flow path cut, the disjunctive cut, the GUB cut and the flow cover cut) were useless for our problem and then were turned off in the complete B&C algorithm. Moreover, in order to avoid errors due to floating point arithmetic, a certain tolerance $\varepsilon > 0$ should be used for checking the violation of each cut [45]. After several tests, we use $\varepsilon = 0.1$ for all of the cuts except for the generation of the RRC cut where the tolerance is set to $\varepsilon = 0.01$.

Branching techniques

We have tested several branching strategies such as branching on the allocation variables z before the routing variables x, branching on the flow variables Y before the routing variables x and branching on the allocation variables z before the flow variables Y. But none of these strategies outperform the CPLEX branching, so we let CPLEX make the branching decisions. In addition, at the root node of the search tree, we introduce the solution provided by the MA as an initial feasible solution of the B&C algorithm, and the best objective value obtained by the MA for each instance is imported as an upper bound of the optimal solution value, at the root node of the branching tree.

5.5 Computational experiments and results

In this section, we describe the computational experiments and results of the branch-and-cut algorithm based on the instances generated in Chapter 3. In order to give a comprehensive evaluation of the algorithm, we firstly investigate how each family of valid inequalities affects the linear relaxation. Then the complete B&C algorithm, with all the efficient cuts, is evaluated by comparing the results with the ones obtained by the solver CPLEX (Chapter 3) and the memetic algorithm (Chapter 4), respectively. In addition, all experiments have been conducted on a computer with Intel Core i3 CPU of 2.93 GHz and 6 GB of memory, under the Window 7 Operating System.

5.5.1 Effect evaluations of valid inequalities

Following the experimental study of Karaolan et al. [110], a first experiment has been conducted to study the effects of each family of valid inequalities on strengthening the LP relaxation formulation. These experiments are based on all the small and medium instances generated in Chapter 3. Table 5.1 and 5.2 report the results obtained by solving the LP relaxation of the original formulation (without cuts) and the enlarged formulation by adding one family of valid inequalities in the cutting plane at a time. In the two tables, the computational results are analyzed using a lower bound percentage gap $\%LB_0$ and the number of corresponding valid inequalities Num found by the corresponding formulation, to compare the effects of each family of valid inequalities. Here, the gap $\%LB_0$ is calculated as $\%LB_0 = (UB - LB_0)/UB * 100\%$, where UB is the upper bound provided by the best solution value obtained by the proposed B&C algorithm, and LB_0 is the LP relaxation bound given by the studied formulation. Except these notations, the first two columns in each row give the name of each test instance. And the third column give the value of the LP relaxation bound LB_0 obtained by the original formulation.

Table 5.1 shows the results for the simple (Sim) valid inequalities (5.18)-(5.22), the rounded route capacity (RRC) constraints (5.27)-(5.28) and the hub capacity (HC) valid inequalities (5.29)-(5.30), and Table 5.2 for the strengthened hub degree (SHD) valid inequalities (5.31)-(5.32), the disaggregated co-circuit (DCoCC) inequalities (5.33)-(5.34) and the generalized large multistar (GLM) inequalities (5.36)-(5.37).

As seen in the two tables, the original formulation without additional cuts has large lower bound percentage gaps (22.32% on average) for all test instances. The worst case is more than 29% for the instances with 40 non-hub nodes. Obviously, this performance is improved by adding a family of valid inequality at a time to the original formulation. Among all of the valid inequalities, the RRC constraints give the best improvement and reduce the average gap to 16.32% for all the test instances with almost 6% improvement on average. Depending on the number of potential hubs (3, 6 and 10), the reduction is about 8%, 5% and 4% on average, respectively. In addition, the GLM and SHD valid inequalities are also efficient and reduce the average percentage gaps by around 2.0% and 1.3% for all the test instances, respectively. Finally, the simple valid inequalities and hub capacity valid inequalities lead to slight improvements. However, the DCoCC inequalities don't improve the lower bounds.

Moreover, from the column labeled Num, it can be noted that the family of the RRC constraints is the most used, following by the simple and the GLM valid inequalities. The other three families of valid inequalities are rarely used, especially the DCoCC constraints which have not been identified in the cutting plane. Thus we decided not to include the DCoCC valid inequalities in the implementation of the complete B&C algorithm.

5.5.2 Results of the complete branch-and-cut algorithm

The complete branch-and-cut algorithm has been run on all instances generated in Chapter 3 with a time limit of 10800s (3 hours). In order to evaluate the performance of our B&C algorithm, we compare the results with those obtained by the solver CPLEX, for the small and medium instances (shown in Table 5.3). We also test the branch-and-cut algorithm with and without an initial solution provided by the memetic algorithm (MA). The B&C algorithm without initial solution is named $B\&C_0$. However, for the large instances, the B&C results are compared to the ones obtained by the MA in Chapter 4 (shown in Table 5.4) because CPLEX and the $B\&C_0$ didn't find solutions within the time limit or for out of memory.

Table 5.3 depicts the computational results obtained by the complete B&C, the $B\&C_0$ and CPLEX based on the *CSAHLRP-F4* formulation presented in Chapter 3 for all small and medium instances. The first two columns in this table give the information about the test in-

Table 5.1: Effects of the valid inequalities (Sim, RRC and HC)

Instance name		UB	Original		Sim		RRC		HC	
H-I-J	Γ_k		LB_0	$\%LB_0$	$\%LB_0$	Num	$\%LB_0$	Num	$\%LB_0$	Num
3-5-5	15	3867.85	3558.54	8.00	4.22	44	8.00	4	6.55	1
	30	3068.00	2709.60	11.68	11.68	0	0.00	19	11.68	0
	45	3068.00	2709.60	11.68	11.68	0	0.00	19	11.68	0
3-10-10	45	7613.94	5800.59	23.82	23.57	79	19.88	136	23.82	0
	60	6828.25	5720.15	16.23	16.15	44	6.41	191	16.23	0
	120	6249.60	5057.60	19.07	19.04	18	3.14	253	19.07	0
3-15-15	45	10614.20	8302.15	21.78	21.64	74	18.91	160	21.78	0
	75	8940.28	7289.83	18.46	18.41	76	14.31	272	18.46	0
	135	8232.80	6584.80	20.02	19.97	32	10.18	319	20.02	0
3-20-20	60	11737.60	8608.86	26.66	26.63	103	19.81	419	25.62	3
	90	10348.40	7603.83	26.52	26.46	95	19.85	298	26.52	0
	165	9336.80	6612.00	29.18	29.18	0	12.08	667	29.18	0
3-25-25	75	12828.40	9551.64	25.54	25.49	116	18.90	544	25.54	0
	105	12057.20	8675.37	28.05	28.03	122	24.78	416	28.05	0
	195	11093.60	8011.20	27.79	27.75	65	17.03	1028	27.79	0
6-10-10	45	7613.94	5751.31	24.46	24.41	122	19.76	65	24.46	0
	60	6828.25	5699.56	16.53	15.13	92	8.95	121	16.53	0
	120	6249.60	5057.60	19.07	19.04	36	6.16	264	19.07	0
6-15-15	45	9581.90	7835.25	18.23	18.18	152	16.27	190	18.23	0
	75	8940.28	7135.00	20.19	20.18	91	18.26	198	20.19	0
	135	8232.80	6584.80	20.02	19.94	38	14.00	288	20.02	0
6-20-20	60	10640.92	8162.53	23.29	23.26	223	22.47	310	23.18	2
	90	9793.74	7289.49	25.57	25.36	202	23.13	270	23.22	2
	165	9336.80	6612.00	29.18	29.18	0	20.60	386	29.18	0
6-25-25	75	12621.48	9473.83	24.94	24.92	214	22.30	448	24.94	0
	105	11854.31	8649.27	27.04	27.02	231	23.10	437	27.04	0
	195	11093.60	8011.20	27.79	27.75	67	19.68	644	27.79	0
10-10-10	45	7366.06	5637.44	23.47	23.45	125	20.75	101	23.47	0
	60	6828.25	5614.15	17.78	17.70	176	16.67	68	17.78	0
	120	6249.60	5057.60	19.07	19.04	34	8.30	212	19.07	0
10-15-15	45	9447.72	7582.80	19.74	19.72	226	19.47	11	19.74	0
	75	8594.36	6845.63	20.35	20.28	201	16.31	158	20.35	0
	135	8232.80	6322.40	23.20	23.20	0	15.06	240	23.20	0
10-20-20	60	10619.89	8009.58	24.58	24.45	306	23.37	303	24.11	1
	90	9793.74	7217.20	26.31	26.27	220	23.26	207	26.31	0
	165	9065.60	6563.20	27.60	25.40	42	19.50	282	27.60	0
10-25-25	75	12226.42	9396.93	23.14	22.31	397	21.38	441	23.14	0
	105	11692.22	8591.11	26.52	25.64	311	23.42	503	26.52	0
	195	11093.60	8011.20	27.79	27.70	99	21.14	549	27.79	0
Average				22.32	22.04	115	**16.32**	293	22.18	1
Average in 3-hub instances				20.97	20.66	58	**12.88**	316	20.87	0
Average in 6-hub instances				23.03	22.87	122	**17.89**	302	22.82	1
Average in 10-hub instances				23.30	22.93	178	**19.05**	256	23.26	0

Table 5.2: Effects of valid inequalities (SHD, DCoCC and GLM) for the original formulation

Instance name		UB	Original		SHD		DCoCC		GLM	
H-I-J	Γ_k		LB_0	$\%LB_0$	$\%LB_0$	Num	$\%LB_0$	Num	$\%LB_0$	Num
3-5-5	15	3867.85	3558.54	8.00	7.76	2	8.00	0	5.13	2
	30	3068.00	2709.60	11.68	5.11	6	11.68	0	11.68	0
	45	3068.00	2709.60	11.68	5.11	6	11.68	0	11.68	0
3-10-10	45	7613.94	5800.59	23.82	22.30	2	23.82	0	22.46	13
	60	6828.25	5720.15	16.23	14.88	9	16.23	0	14.40	11
	120	6249.60	5057.60	19.07	19.07	0	19.07	0	10.66	29
3-15-15	45	10614.20	8302.15	21.78	21.19	4	21.78	0	20.46	25
	75	8940.28	7289.83	18.46	15.27	19	18.46	0	16.33	47
	135	8232.80	6584.80	20.02	13.38	26	20.02	0	19.60	32
3-20-20	60	11737.60	8608.86	26.66	26.49	2	26.66	0	22.06	269
	90	10348.40	7603.83	26.52	26.52	0	26.52	0	21.57	238
	165	9336.80	6612.00	29.18	29.18	0	29.18	0	29.18	0
3-25-25	75	12828.40	9551.64	25.54	25.37	2	25.54	0	21.34	356
	105	12057.20	8675.37	28.05	27.44	16	28.05	0	24.14	236
	195	11093.60	8011.20	27.79	27.79	0	27.79	0	23.05	175
6-10-10	45	7613.94	5751.31	24.46	24.04	1	24.46	0	22.19	15
	60	6828.25	5699.56	16.53	14.36	8	16.53	0	14.10	16
	120	6249.60	5057.60	19.07	18.23	2	19.07	0	15.64	24
6-15-15	45	9581.90	7835.25	18.23	16.39	6	18.23	0	17.25	27
	75	8940.28	7135.00	20.19	17.80	4	20.19	0	19.30	20
	135	8232.80	6584.80	20.02	16.62	12	20.02	0	17.77	23
6-20-20	60	10640.92	8162.53	23.29	22.13	12	23.29	0	22.07	120
	90	9793.74	7289.49	25.57	23.00	17	25.57	0	23.69	95
	165	9336.80	6612.00	29.18	29.18	0	29.18	0	29.18	0
6-25-25	75	12621.48	9473.83	24.94	24.89	2	24.94	0	24.25	130
	105	11854.31	8649.27	27.04	26.71	4	27.04	0	26.29	114
	195	11093.60	8011.20	27.79	27.79	0	27.79	0	24.85	118
10-10-10	45	7366.06	5637.44	23.47	23.09	2	23.47	0	22.86	5
	60	6828.25	5614.15	17.78	16.85	2	17.78	0	17.68	2
	120	6249.60	5057.60	19.07	18.23	3	19.07	0	18.23	10
10-15-15	45	9447.72	7582.80	19.74	18.45	5	19.74	0	18.97	14
	75	8594.36	6845.63	20.35	17.69	17	20.35	0	19.35	13
	135	8232.80	6322.40	23.20	23.20	0	23.20	0	23.20	0
10-20-20	60	10619.89	8009.58	24.58	24.57	1	24.58	0	23.54	69
	90	9793.74	7217.20	26.31	25.74	5	26.31	0	24.71	63
	165	9065.60	6563.20	27.60	27.07	12	27.60	0	23.02	100
10-25-25	75	12226.42	9396.93	23.14	23.08	4	23.14	0	23.09	24
	105	11692.22	8591.11	26.52	25.67	1	26.52	0	26.34	31
	195	11093.60	8011.20	27.79	27.17	14	27.79	0	26.99	23
Average				22.32	20.99	6	22.32	0	**20.47**	64
Average in 3-hub instances				20.97	19.12	6	20.97	0	**18.25**	96
Average in 6-hub instances				23.03	21.76	6	23.03	0	**21.38**	59
Average in 10-hub instances				23.30	22.57	6	23.30	0	**22.33**	30

stances as in the previous tables. The column labeled UB displays the upper bound for each instance provided with the best-known solution value obtained by all methods. Then successive columns compare the performance of the three methods. The first one denoted as $Gap\%$ is the deviation percentage of the lower bound LB found from the upper bound, i.e. $Gap\% = (UB - LB)/UB \times 100\%$. The second one is the total number of nodes inspected during the branching tree of each method. And the last one is the total running time in seconds.

Table 5.3 shows that the B&C algorithm (with or without initial solution) can always provide a feasible solution, even when Cplex cannot obtain one as for instance 6-25-25. Moreover, 17 instances out of the 36 are solved to optimality by the complete B&C algorithm within 10 minutes on average, including one instance with 30 non-hub nodes and 10 potential hubs. In the same time, Cplex can only find 3 optimal solutions. For the 19 instances unsolved optimally, the gaps between the lower bound and the upper bound is between 0.57% and 8.23% with an average value of 3.84%. Meantime, 34 instances have a gap lower than 6%. With respect to the instances with 3 potential hubs, 8 out of 15 instances are optimally solved within 6 minutes and the average gap is 1.15% for all considered instances. For the instances with 6 or 10 potential hubs, the average gaps are around 3% and 2%, respectively.

Comparing the results obtained with the introduction of the initial solution provided by the MA and without ($B\&C_0$), we can notice the important impact of the initial solution, reducing the average gap (2.03% versus 3.10%) for all the small and medium instances. The initial solution also greatly reduces the number of nodes in the search tree for 30 instances among the 36 (4904 versus 44601, on average). Moreover, all the best solution values reported in column UB have been found by the complete B&C. These results indicate the good effectiveness of the initial solutions on the performance of the B&C algorithm. Finally, the comparative results with those obtained by CPLEX show that the B&C outperforms CPLEX for the test instances in terms of the number of reached optimal solutions, the average gap (2.03% versus 16.82%), the number of explored nodes in the search tree (4904 versus 82237) and the average running time (5913.81s versus 9818.72s).

Table 5.4 reports the results obtained by the complete B&C algorithm for the large instances and compares them with those obtained by the MA in Chapter 4. In this table, Z^* is the value of the final solution found by the B&C algorithm or the value of the initial solution if the B&C fails to improve a solution. The values in bold indicate the new solution found by B&C. Except the instance name and Z^*, the following notations are used in Table 5.4:

- $UB\%$: the gap between the best value UB found by the B&C within 3 hours and the value of Z^*, i.e. $UB\% = (UB - Z^*)/Z^* \times 100\%$.
- $LB\%$: the deviation in % of the upper bound from the lower bound LB found by the complete B&C. Here, $LB\% = (UB - LB)/UB \times 100\%$.
- Time (s): CPU time in seconds used by the corresponding method.
- Open hubs: the index of the located hubs in the best solution found by the corresponding method.
- BestObj: the best objective value found by the MA in 10 runs;
- $UB'\%$: the percentage gap between the best value $BestObj$ obtained by the proposed MA and the value of Z^*, Here, $UB'\% = (BestObj - Z^*)/Z^* \times 100\%$.
- $LB'\%$: the deviation in % between the $BestObj$ of the MA and the lower bound LB obtained by the complete B&C, i.e. $LB'\% = (BestObj - LB)/BestObj \times 100\%$.

From Table 5.4, it can be observed that the completed B&C has not solved the large instances to optimality but finds 18 new best solutions out of the 33 instances including three 6-50-50 instances and two 10-50-50 instances. The average lower bound gap $LB\%$ for all large instances varies between 5.32% and 25.70% and 15 instances have $LB\%$ gaps less than

Table 5.3: Results comparison for the small and medium instances with $B\&C_0$ and CPLEX

Instance name		UB	Complete B&C			$B\&C_0$			CPLEX-*F4*		
H-I-J	Γ_k		$Gap\%$	Nodes	Time (s)	$Gap\%$	Nodes	Time(s)	$Gap\%$	Nodes	Time(s)
3-5-5	15	3867.85	0.00	0	0.11	0.00	0	0.09	0.00	3457	8.03
	30	3068.00	0.00	0	0.14	0.00	0	0.14	0.00	2381	4.87
	45	3068.00	0.00	0	0.09	0.00	0	0.14	0.00	2381	5.01
3-10-10	45	7613.94	0.00	1318	15.94	0.00	3453	16.69	4.93	448951	10800.00
	60	6828.25	0.00	127	3.49	0.00	254	3.71	0.98	127031	10800.00
	120	6249.60	0.00	0	3.62	0.00	20	4.51	1.60	355489	10800.00
3-15-15	45	10614.20	1.68	29023	10800.00	2.19	475519	10800.00	18.03	177464	10800.00
	75	8940.28	0.00	23178	356.95	0.00	39893	434.24	16.20	236187	10800.00
	135	8232.80	0.00	5932	202.92	0.00	6883	200.06	11.21	143788	10800.00
3-20-20	60	11737.60	5.12	9677	10800.00	6.46	124760	10800.00	25.49	100179	10800.00
	90	10348.40	1.74	10411	10800.00	2.29	253800	10800.00	27.45	101564	10800.00
	165	9336.80	0.57	13497	10800.00	1.96	175915	10800.00	27.37	104298	10800.00
3-25-25	75	12828.40	5.69	2404	10800.00	6.81	65200	10800.00	22.73	24677	10800.00
	105	12057.20	1.66	3117	10800.00	2.25	35514	10800.00	25.95	20281	10800.00
	195	11093.60	0.81	3154	10800.00	2.55	56714	10800.00	25.67	22034	10800.00
6-10-10	45	7613.94	0.00	6895	111.58	0.00	8084	172.21	14.25	125311	10800.00
	60	6828.25	0.00	147	16.72	0.00	194	15.19	6.93	164351	10800.00
	120	6249.60	0.00	30	9.56	0.00	31	11.70	6.36	186270	10800.00
6-15-15	45	9581.90	1.84	14685	10800.00	2.79	65485	10800.00	15.42	68436	10800.00
	75	8940.28	0.00	6665	3057.11	0.00	78417	4773.01	17.91	40620	10800.00
	135	8232.80	0.00	6079	1039.90	0.00	22397	1152.57	15.34	21913	10800.00
6-20-20	60	10640.92	5.31	1040	10800.00	8.50	16556	10800.00	25.27	16710	10800.00
	90	9793.74	4.49	1370	10800.00	7.58	20648	10800.00	27.51	19861	10800.00
	165	9336.80	3.07	1768	10800.00	3.55	34500	10800.00	32.35	15900	10800.00
6-25-25	75	12621.48	8.23	181	10800.00	13.98	7444	10800.00		NFS	
	105	11854.31	8.05	240	10800.00	10.45	6852	10800.00		NFS	
	195	11093.60	5.26	271	10800.00	7.02	8042	10800.00		NFS	
10-10-10	45	7366.08	0.00	1208	209.712	0.00	16276	1026.35	16.62	22942	10800.00
	60	6828.25	0.00	451	146.05	0.00	2338	197.248	11.79	53737	10800.00
	120	6249.60	0.00	51	54.887	0.00	225	68.3596	7.72	77734	10800.00
10-15-15	45	9447.72	3.86	979	10800.00	5.13	19931	10800.00	18.65	6297	10800.00
	75	8594.36	1.90	1090	10800.00	3.65	22209	10800.00	21.99	10080	10800.00
	135	8232.80	0.00	1533	2468.28	0.00	20109	4286.28	22.44	3161	10800.00
10-20-20	60	10619.89	5.48	11300	10800.00	9.59	6705	10800.00	26.51	1764	10800.00
	90	9793.74	4.89	8972	10800.00	8.82	6688	10800.00	29.28	4465	10800.00
	165	9065.60	3.34	9742	10800.00	5.98	4567	10800.00	31.26	4114	10800.00
Average			**2.03**	4904	**5913.81**	3.10	44601	6043.40	16.82	82237	9818.72
Average in 3-hub instances			**1.15**	6789	**5078.88**	1.63	82528	5083.97	13.84	124677	8641.19
Average in 6-hub instances			**3.02**	3281	**6652.91**	4.49	22388	6810.39	17.93	73264	10800.00
Average in 10-hub instances			**2.16**	3925	**6319.88**	3.69	11005	6619.80	20.70	20477	10800.00

Table 5.4: Results comparison for the large instances with the memetic algorithm

Instance name		Z^*	Completed B&C				MA			
H-I-J	Γ_k		$UB\%$	$LB\%$	Time(s)	Open hub	$UB'\%$	$LB'\%$	Time(s)	Open hub
6-30-30	90	14222.26	0.00	9.45	10800.00	1, 3, 5	0.00	9.45	107.66	1, 3, 5
	120	13207.59	0.00	9.00	10800.00	1, 5	0.00	9.00	118.26	1, 5
	240	**12437.00**	0.00	5.76	10800.00	5	0.13	5.89	110.79	5
6-35-35	90	16775.89	0.00	9.38	10800.00	1, 5, 6	0.00	9.38	204.97	1, 5, 6
	135	**16111.84**	0.00	8.62	10800.00	5, 6	1.34	9.83	185.30	5, 6
	270	**16037.30**	0.00	6.01	10800.00	5, 6	1.07	7.01	173.63	1
6-40-40	90	16135.02	0.00	14.00	10800.00	1, 4, 5	0.00	14.00	346.04	1, 4, 5
	135	14998.09	0.00	10.72	10800.00	1, 5	0.00	10.72	365.96	1, 5
	255	**14866.40**	0.00	9.85	10800.00	1	0.17	10.00	295.06	1
6-45-45	105	**16791.97**	0.00	19.36	10800.00	1, 2, 5	2.32	21.18	569.05	1, 5, 6
	150	16193.70	0.00	12.41	10800.00	5, 6	0.00	12.41	557.31	5, 6
	285	**15414.40**	0.00	9.78	10800.00	6	0.06	9.84	526.08	6
6-50-50	105	**18453.88**	0.00	22.64	10800.00	2, 4, 6	1.09	23.47	988.75	2, 4, 6
	150	**16688.49**	0.00	17.02	10800.00	4, 6	2.84	19.31	968.44	4, 6
	300	**16355.20**	0.00	10.07	10800.00	4	0.63	10.63	955.55	4
10-25-25	75	**12226.42**	0.00	9.52	10800.00	3, 4, 7	0.87	10.30	102.82	3, 4, 7
	105	**11692.22**	0.00	6.26	10800.00	3, 7	0.90	7.10	120.82	3, 7
	195	11093.60	0.00	5.32	10800.00	3	0.00	5.32	111.84	3
10-30-30	90	13965.64	0.00	10.25	10800.00	1, 5, 9	0.00	10.25	262.53	1, 5, 9
	120	13207.59	0.00	9.10	10800.00	1, 5	0.00	9.10	206.56	1, 5
	240	12437.60	0.00	6.35	10800.00	5	0.00	6.35	193.75	5
10-35-35	90	**16332.42**	0.00	16.67	10800.00	5, 6, 9	2.10	18.38	479.97	1, 5, 8
	135	**16097.96**	0.00	12.57	10800.00	5, 6	0.88	13.34	390.13	5, 6
	270	**16037.30**	0.00	8.75	10800.00	5, 6	0.48	9.19	364.06	9
10-40-40	90	16135.02	0.00	16.00	10800.00	1, 4, 5	0.00	16.00	807.88	1, 4, 5
	135	14998.09	0.00	13.49	10800.00	1, 5	0.00	13.49	782.80	1, 5
	255	14866.40	0.00	9.03	10800.00	1	0.00	9.03	823.53	1
10-45-45	105	17135.03	0.00	20.72	10800.00	1, 5, 6	0.00	20.72	1461.31	1, 5, 6
	150	**16193.70**	0.00	17.57	10800.00	5, 6	0.04	17.61	1314.53	5, 6
	285	**15389.60**	0.00	11.76	10800.00	6	0.16	11.90	1281.28	6
10-50-50	105	17491.90	0.00	25.70	10800.00	4, 6, 7	0.00	25.70	2413.68	4, 6, 7
	150	**16814.98**	0.00	23.71	10800.00	4, 6	2.02	25.22	2086.29	4, 6
	300	**16283.20**	0.00	12.76	10800.00	4	0.72	13.39	2056.53	4
Average			0.00	**12.41**	10800.00		0.54	12.86	**658.58**	
Average in 6-hub instances			0	11.60	10800.00		0.64	12.14	431.52	
Average in 10-hub instances			0	13.08	10800.00		0.45	13.43	847.80	

10%. Compared to the results obtained by the MA, it can be seen that the completed B&C has a smaller lower bound gap (12.41% versus 12.86%, on average) but a larger running time (10800s versus 658.58s on average). It is also interesting to note that the B&C algorithm improves the best solution for the instance 6-35-35 with 270t hub capacity, in increasing the number of open hubs.

Computational results and details on the number of valid inequalities detected by our algorithm and added to the LP during the complete B&C are given in Table 5.5-5.7. They show the results for all generated instances with 3, 6 and 10 potential hubs, respectively. In these tables, following the "instance name" column, columns 3-5 give the computational results obtained in 3 hours for each instance including the upper bound UB, the deviation between the lower bounds and upper bound $Gap\%$ and the total number of nodes in the branch-and-cut tree. Then the columns 6-10 show the numbers of corresponding cuts found in our algorithm including the simple valid inequalities (5.18)-(5.22), the rounded route capacity (RRC) constraints (5.27)-(5.28), the hub capacity (HC) valid inequalities (5.29)-(5.30), the strengthened hub degree (SHD) valid inequalities (5.31)-(5.32) and the generalized large multistar (GLM) inequalities. The last two columns in these tables provide the information on the best solutions obtained by the B&C algorithm, in terms of the index of open hubs and the maximum number of routes operated by one hub. On average for all the test instances, almost 76.1% of these valid inequalities correspond to the RRC constraints, which are the most frequent ones. And the GLM valid inequalities and simple valid inequalities are the second most frequent ones with a percentage around 11.6% and 7.8% of all found cuts, respectively. This conclusion is consistent with the one summarized in the first experiment to inspect the effects of each family of valid inequalities. With respect to the instances with different number of potential hubs, the average lower bound gaps are 1.15%, 7.79% and 9.44% for 3, 6 and 10 potential hubs, respectively. In addition, it can be observed from each table that the number of non-hub nodes gives a greater impact on the difficulty of solving this problem exactly, especially when the non-hub nodes exceed 50.

Table 5.5: Number of valid equalities found by the complete B&C for instances with 3 hubs

Instance name		UB	$Gap\%$	Nodes	Cuts					Solution	
H-I-J	Γ_k				Sim	RRC	HC	SHD	GLM	Open hub	Max of routes
3-5-5	15	3867.85	0.00	0	0	13	0	0	8	1, 2	2
	30	3068.00	0.00	0	0	20	0	4	0	1	4
	45	3068.00	0.00	0	0	20	0	4	0	1	4
3-10-10	45	7613.94	0.00	1318	11	288	52	4	188	2, 3	8
	60	6828.25	0.00	127	7	266	8	10	39	2, 3	9
	120	6249.60	0.00	0	1	295	0	3	24	2	15
3-15-15	45	10614.20	1.68	29023	27	1889	276	8	731	1, 2, 3	7
	75	8940.28	0.00	23178	11	1703	37	1	476	1, 3	11
	135	8232.80	0.00	5932	7	1962	0	28	71	1	18
3-20-20	60	11737.60	5.12	9677	46	2523	168	11	657	1, 2, 3	9
	90	10348.40	1.74	10411	23	2187	29	4	424	2, 3	12
	165	9336.80	0.57	13497	3	3404	0	48	88	2	20
3-25-25	75	12828.40	5.69	2404	64	3184	92	8	536	1, 2, 3	11
	105	12057.20	1.66	3117	55	2844	18	6	482	2, 3	12
	195	11093.60	0.81	3154	50	2634	0	12	580	3	24
Average			1.15	6789	20	1549	45	10	287		

Table 5.6: Number of valid equalities found by the complete B&C for the instances with 6 hubs

Instance name		UB	$Gap\%$	Nodes	Cuts					Solution	
H-I-J	Γ_k				Simple	RRC	HC	SHD	GLM	Open hub	Max of routes
6-10-10	45	7613.94	0.00	6895	10	290	168	4	334	2, 3	8
	60	6828.25	0.00	147	11	273	2	5	153	2, 3	9
	120	6249.60	0.00	30	0	320	0	5	57	2	15
6-15-15	45	9581.90	1.84	14685	6	1286	83	2	102	1, 3, 5	7
	75	8940.28	0.00	6665	22	845	91	1	201	1, 3	11
	135	8232.80	0.00	6079	32	882	0	7	663	1	18
6-20-20	60	10640.92	5.31	1040	41	670	190	3	427	2, 4, 5	9
	90	9793.74	4.49	1370	34	1048	40	5	324	2, 5	13
	165	9336.80	3.07	1768	28	2360	0	7	302	2	20
6-25-25	75	12621.48	8.23	181	69	1184	10	11	514	2, 3, 4	10
	105	11854.31	8.05	240	57	1124	48	4	448	2, 4	14
	195	11093.60	5.26	271	50	2070	0	9	540	3	24
6-30-30	90	14222.26	9.45	13235	145	517	84	5	324	1, 3, 5	13
	120	13207.59	9.00	6319	137	1218	5	7	131	1, 5	17
	240	12437.00	5.76	4576	64	3310	0	17	64	5	32
6-35-35	90	16775.89	9.38	3869	200	881	2	8	146	1, 5, 6	14
	135	16111.84	8.62	2941	186	1947	12	24	274	5, 6	19
	270	16037.30	6.01	2168	164	2078	0	41	107	5, 6	19
6-40-40	90	16135.02	14.01	3582	281	1211	3	4	224	1, 4, 5	13
	135	14998.09	10.72	4617	291	1352	4	3	146	1, 5	18
	255	14866.40	9.85	3956	141	1504	0	25	301	1	34
6-45-45	105	16791.97	19.36	157	430	2654	8	71	167	1, 2, 5	16
	150	16193.70	12.41	213	415	1495	1	4	222	5, 6	22
	285	15414.40	9.78	178	222	1347	0	80	114	6	38
6-50-50	105	18453.88	22.64	16	239	1381	0	2	149	2, 4, 6	16
	150	16688.49	17.02	21	255	2114	0	15	135	4, 6	21
	300	16355.20	10.07	186	186	2129	0	19	101	4	41
Average			7.79	3163	138	1389	28	14	147		

Table 5.7: Number of valid equalities found by the complete B&C for the instances with 10 hubs

Instance name		UB	$Gap\%$	Nodes	Cuts					Solution	
H-I-J	Γ_k				Simple	RRC	HC	SHD	GLM	Open hub	Max of routes
10-10-10	45	7366.08	0.00	1208	30	340	216	4	400	2, 7	8
	60	6828.25	0.00	451	18	406	12	1	234	2, 3	9
	120	6249.60	0.00	51	6	317	0	4	168	2	15
10-15-15	45	9447.72	3.86	979	32	611	150	0	224	5, 7, 8	7
	75	8594.36	1.90	1090	47	807	0	2	282	7, 8	9
	135	8232.80	0.00	1533	28	1446	0	5	128	1	8
10-20-20	60	10619.89	5.48	11300	2	6440	0	61	94	5, 7, 10	9
	90	9793.74	4.89	8972	7	3645	0	73	173	2, 5	13
	165	9065.60	3.34	9742	68	5045	0	37	412	7	20
10-25-25	75	12226.42	9.52	3837	49	527	19	4	268	3, 4, 7	10
	105	11692.22	6.26	5137	211	432	0	1	131	3, 7	14
	195	11093.60	5.32	3715	99	1026	0	14	89	3	24
10-30-30	90	13965.64	10.25	2539	148	656	7	4	180	1, 5, 9	12
	120	13207.59	9.10	938	148	1039	3	5	159	1, 5	17
	240	12437.60	6.35	1496	64	749	0	3	93	5	32
10-35-35	90	16332.42	16.67	854	403	1130	0	4	49	5, 6, 9	14
	135	16097.96	12.57	2106	317	997	12	4	64	5, 6	20
	270	16037.30	8.75	2736	268	1155	0	9	58	5, 6	19
10-40-40	90	16135.02	16.00	321	518	1231	0	2	97	1, 4, 5	13
	135	14998.09	13.49	480	400	1368	14	9	102	1, 5	18
	255	14866.40	9.03	692	141	723	0	2	146	1	34
10-45-45	105	17135.03	20.72	106	655	1368	1	7	119	1, 5, 6	16
	150	16193.70	17.57	95	620	788	1	1	168	5, 6	20
	285	15389.60	11.76	146	311	1209	0	1	188	6	38
10-50-50	105	17491.90	25.70	28	659	2114	9	6	52	4, 6, 7	15
	150	16814.98	23.71	37	573	2066	3	0	85	4, 6	21
	300	16283.20	12.76	56	401	1630	0	4	55	4	41
Average			9.44	2246	230	1454	17	10	156		

5.6 Conclusion

In this chapter, we presented a branch-and-cut (B&C) algorithm to solve the capacitated single allocation hub location-routing problem (CSAHLRP) based on a new formulation. In the proposed B&C algorithm, some families of valid inequalities were introduced to strengthen the LP relaxation of the new formulation, as well as their separation algorithms and the branching strategy. In addition, the best solutions from the memetic algorithm of Chapter 4 were imported as the initial solutions at the root node of the B&C algorithm and the upper bound of the optimal solutions. Computational experiments have been conducted on all instances generated in Chapter 3. The results demonstrate a good performance of our B&C algorithm for the small and medium instances both in terms of quality and computing time, compared with the ones obtained by solver CPLEX. Furthermore, the computational results reveal that a good initial solution, as the one obtained by the MA, helps the branch-and-cut algorithm in finding high quality solutions by exploring fewer nodes of the search tree. For large instances, the comparison was done with the MA and the complete B&C algorithm found some new best known solutions although with lager lower bound gaps. We believe that the large gaps for large instances are due to the poor quality of the lower bounds. In fact, we have tested some instances based on 6-50-50 solved in 12 hours. However, it is found that the best solutions have not been improved and the lower bounds get a smaller reduction (3.37%, on average for three hub capacity levels). Therefore there is still a wide space for developing more valid inequalities and improving the cutting plane procedure.

6

Model and solution method for the HLRP in postal service system

In this chapter, we focus on an application of the hub location-routing problem in the postal service system, where the collection and delivery operations can be done simultaneously in the same route. Based on the main features of this application, a mathematical formulation is firstly described which is derived from the model proposed in Chapter 3. Then the memetic algorithm (MA) presented in Chapter 4 is adapted to solve the HLRP in postal service systems. Finally, the computational experiments are implemented based on the instances inspired from the AP data set [77] to investigate the solutions with different parameters and evaluate the performance of the MA for solving the postal system cases.

6.1 Introduction

Besides the less-than-truckload (LTL) shipments, the private express industry or public postal system represents another application area of the hub location-routing problem (HLRP) . In this application, each node location represent a post code district which has both a collection and a delivery flow. These flows correspond to parcel or mail volumes and can be exchanged between any two node locations. The hub nodes play not only a consolidation role but also a sorting function. In this hub location-routing network of postal systems, instead of direct connection between two nodes, the parcels are consolidated, sorted, regrouped at hub facilities and then are sent to the different destinations. In order to reduce the number of vehicles used in the system, local tours are operated between non-hub nodes to collect and deliver the parcels or mails instead of direct links between non-hub nodes and hubs . The network is similar to the network of LTL shipment for general goods except for local tours. However, we can notice the following significant differences between the two applications:

- In postal service systems, non-hub nodes usually send flow to themselves. This indicates that a parcel may be sent to a hub node where it is sorted and then be returned to the same post district (same node).

- Collection and delivery unit costs are different in postal systems due to quite different processes and transportation modes [36], and consequently unit routing costs are different from the LTL application .
- Collection and delivery processes happen simultaneously in the same route. Therefore, each homogeneous non-hub node location can be visited once by only one vehicle route.

Based on the above description, the network of HLRP in postal systems can be represented in Figure 6.1 where the squares and circles represent the established hub nodes and non-hub nodes, respectively. And the bold lines are inter-hub arcs while dotted lines denote local tour arcs. So in order to optimize this hub location-routing network for a postal system, it needs to determine the location of hub nodes, the allocation of non-hub nodes, as well as the local tours visiting each node location for the collection and delivery. To the best our knowledge, most of the HLRP literature focus on this application and considers particular constraints in their models [40, 66, 180, 203] as the limitation of the driving time for the local tours, the multiple allocation of non-hub nodes or the possibility of direct connection between two non-hub nodes. For more details and a state of the art about the HLRP in postal systems, one can refer to the section 2.3 in Chapter 2. The assumptions used in the postal systems are as follows:

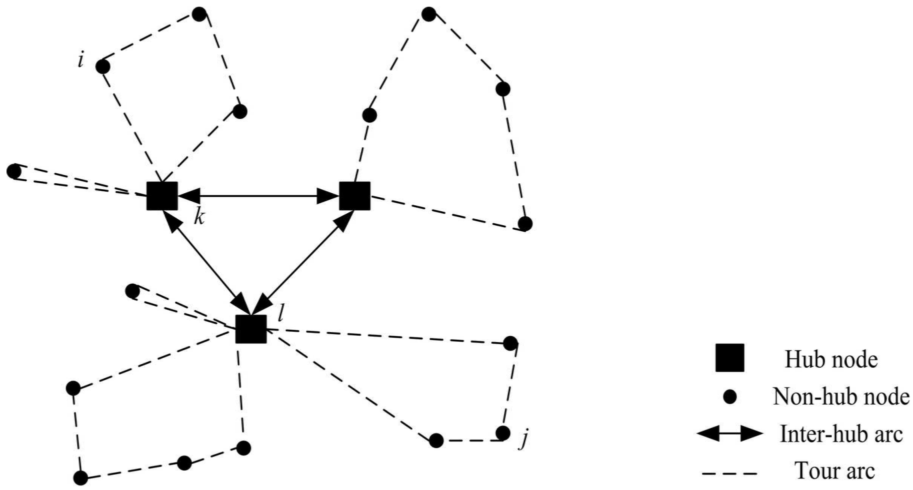


Figure 6.1: The network of the HLRP in postal systems

(1) Each non-hub node can be assigned to one hub and can be served by only one local tour, i.e. single allocation problem;
(2) The total quantity of flow assigned to one hub can not exceed its capacity, including the collection flow and also delivery flow, i.e. capacitated problem;
(3) Each local tour is limited to a maximum number of visited nodes q, including the hub node;
(4) Each local tour must begin at a hub and return to the same one;
(5) The flow between any two O-D nodes can pass through two hubs at most.

6.2 Mathematical formulation

Let $G^{'} = (N^{'}, A)$ be a complete graph and represent the HLRP network in a postal system. $N^{'} = H \cup N$ is the set of all nodes, where H and N represent the candidate hub nodes and non-hub nodes, respectively. Usually $H \subseteq N$, however, the decisions maker can always select some potential hub nodes from all known node locations based on the economic, public transportation and environmental factors. Thus it will be assumed that $H \subset N$ and they belong to different sets in the mathematical formulation even they may have the same geographical sites. For the case $H = N$, it can be modeled by duplicating the set of non-hub nodes N into H [26]. In this network, each arc $(i, j) \in A : i, j \in N^{'}, i \neq j$ has a nonnegative cost, which can be given by the Euclidean distance d_{ij} or a shortest path in the road network between node i and j. In addition, a flow matrix with the elements $w_{ij} : i, j \in N$ is considered to represent the parcel volume from node i to node j through one or two installed hubs. Normally $w_{ij} \neq w_{ji}$ and the demand in the same tour is not exchanged directly, but via a hub.

Furthermore, all candidate hub nodes have a capacity restriction $\Gamma^{'}_{k}$ and are associated with a fixed installing cost $F^{'}_{k}$. For the vehicles operated in the local tours, a homogeneous fleet V is considered with a fixed cost f. So in order to carry out the determination of the HLRP network in postal systems, we use the same decision variables as the ones presented in Chapter 3:

- $Y_{ijkl}-$ the fraction of flow from node i to node j via hubs k and l. It is a classical decision variable for the HLP and it indicates that the flow between two nodes is routed along the path $i \to k \to l \to j$. Here, the hubs k and l can be located at the same node;
- $z_{ik}-$ the allocation variable of a node i to a hub k. It is equal to 1 if the node i is allocated to the hub k, 0 otherwise; specially, $z_{kk} = 1$ if the hub k is open. It is also a classical variable for the HLP;
- $x^{v}_{ij}-$ the three-index vehicle flow variable, it equals 1 if the arc (i, j) is served by vehicle v, 0 otherwise. It defines the visiting order of each local tour;

  $U_{iv}-$ Auxiliary variables for sub-tours eliminations.

The objective function minimizes the total cost, including the fixed cost of establishing the hubs, the inter-hubs transportation cost, the transportation cost of local tours and the fixed cost of the vehicles, and a fixed charge for assigning the non-hub nodes to the hubs, called as the handling cost $c^{'}_{ik}$ for the incoming and outgoing volumes of node $i \in N$ by hub $k \in H$, as proposed in [66]. In addition, the transportation cost between hubs depends on the arc distance and the parcel volumes transferred, which is reduced by a factor $\alpha^{'}$ to reflect the economies of scale. The local tour cost is determined by the distance of the arcs traversed as it is usually assumed. We also add a factor λ to change the relative weight of the local routing cost component in the objective value [180]. Compared to the other cost elements, the local tour costs are very small and don't permit the optimization of the routes if they aren't enhanced. Then with the aforementioned definition, in order to minimize the total cost and meet the service requirements, the capacitated single allocation HLRP in postal systems (*CSAHLRP-pos*) can be modeled as follows:

CSAHLRP-pos

$$\text{Min} \sum_{k\in H} F^{'}_{k} z_{kk} + \sum_{i\in N}\sum_{k\in H} c^{'}_{ik} z_{ik} + \sum_{i\in N}\sum_{j\in N}\sum_{k\in H}\sum_{l\in H} \alpha^{'} d_{kl} w_{ij} Y_{ijkl} + \lambda \sum_{v\in V}\sum_{i\in N'}\sum_{j\in N', j\neq i} d_{ij} x^{v}_{ij} + \sum_{v\in V}\sum_{k\in H}\sum_{i\in N} f x^{v}_{ki} \tag{6.1}$$

subject to

$$z_{ik} \leq z_{kk} \qquad \forall i \in N^{'}, \forall k \in H \tag{6.2}$$

$$\sum_{k\in H} z_{ik} = 1 \qquad \forall i \in N^{'} \tag{6.3}$$

$$\sum_{l\in H} Y_{ijkl} = z_{ik} \qquad \forall i \in N, \forall j \in N, \forall k \in H \tag{6.4}$$

$$\sum_{k\in H} Y_{ijkl} = z_{jl} \qquad \forall i \in N, \forall j \in N, \forall l \in H \tag{6.5}$$

$$\sum_{i\in N}\sum_{j\in N}\sum_{l\in H} w_{ij} Y_{ijkl} \leq \Gamma^{'}_{k} z_{kk} \qquad \forall k \in H \tag{6.6}$$

$$\sum_{i\in N}\sum_{j\in N}\sum_{k\in H} w_{ij} Y_{ijkl} \leq \Gamma^{'}_{l} z_{ll} \qquad \forall l \in H \tag{6.7}$$

$$\sum_{i\in N}\sum_{j\in N', j\neq i} x^v_{ij} \leq q \qquad \forall v \in V \tag{6.8}$$

$$\sum_{i\in N'} x^v_{ij} - \sum_{i\in N'} x^v_{ji} = 0 \qquad \forall v \in V, \forall j \in N^{'} \tag{6.9}$$

$$\sum_{u\in N'} (x^v_{ku} + x^v_{ui}) \leq 1 + z_{ik} \qquad \forall i \in N, \forall k \in H, \forall v \in V \tag{6.10}$$

$$\sum_{v\in V}\sum_{j\in N'} x^v_{ij} = 1 \qquad \forall i \in N \tag{6.11}$$

$$\sum_{i\in H}\sum_{j\in N} x^v_{ij} \leq 1 \qquad \forall v \in V \tag{6.12}$$

$$U_{iv} - U_{jv} + |N| x^v_{ij} \leq |N| - 1 \qquad \forall v \in V, \forall i \in N, \forall j \in N \tag{6.13}$$

$$\sum_{i\in H}\sum_{j\in H} x^v_{ij} = 0 \qquad \forall v \in V \tag{6.14}$$

$$0 \leq Y_{ijkl} \leq 1 \qquad \forall i \in N, \forall j \in N, \forall k \in H, \forall l \in H \tag{6.15}$$

$$z_{ik} \in \{0, 1\} \qquad \forall i \in N^{'}, \forall k \in H \tag{6.16}$$

$$x^v_{ij} \in \{0, 1\} \qquad \forall i \in N^{'}, \forall j \in N^{'}, \forall v \in V \tag{6.17}$$

$$U_{iv} \geq 0 \qquad \forall i \in N, \forall v \in V \tag{6.18}$$

In this model, the objective function (6.1) minimizes the sum of the fixed hub operating costs, handling cost of nodes (assigning cost of nodes to hubs), transportation cost between hubs, local tours transportation cost and fixed costs of vehicles. Constraints (6.2)-(6.7) are the location constraints. Constraints (6.2) and constraints (6.3) impose that a non-hub node can be allocated to only one open hub. Constraints (6.4) and (6.5) represent the coherence between the allocation variables and the flow variables. They impose that if a non-hub node is allocated to a hub, then all the flows from or to this non-hub node should pass through the same hub. Constraints (6.6) and (6.7) are hub capacity constraints for the incoming and outgoing volumes, respectively. In the local routing part, constraints (6.8) restrict the total number of nodes served by each tour. Constraints (6.9) are the flow conservation constraints which ensure the continuity of every node visited by one vehicle. Constraints (6.10) are the connection between the location variables and the local routing variables. They specify that a node can be assigned to a hub only if there is a vehicle from that hub going through that node. Constraints (6.11) represent that a node can only be served by a single vehicle. Constraints (6.12) represent that each vehicle can be used at most once in a hub. Constraints (6.13) are sub-tour elimination constraints inspired from the TSP [118]. Equations (6.14) indicate that local tours can not happen between two hub nodes. Constraints (6.15)-(6.18) define the variable values.

6.3 The memetic algorithm for the HLRP in postal system

Based on the relationship between model (*CSAHLRP-pos*) for postal systems and model (*CSAHLRP-F4*) for general goods LTL shipments, we have adapted the memetic algorithm (MA) proposed in Chapter 4 to solve the HLRP for postal systems. Following the basic framework of MA algorithm (shown in Algorithm 2) and all operators (genetic operators and iterated local search) proposed in Chapter 4, main components of the adapted MA here are illustrated as follows:

1. Solution representation and evaluation: the chromosome $C^{'}(x)$ of the HLRP solution in postal systems is also represented with natural numbers including the location section and the routing section. The location section $L^{'}(x)$ consists in a string of hub index where each gene is generated randomly from the set of potential hubs. The routing section $R^{'}(x)$ is construct with only one random permutation of all non-hub nodes representing their visiting orders in the route. The process of chromosome decoding remains the same as the one presented in section 4.2.2 of Chapter 4 except that the routing part includes both suppliers and clients in the same route. Any non-hub node can be serviced by any vehicle route as long as all constraints are met. More clearly, consider the example shown in Figure 6.2 where 10 non-hub nodes and 3 candidate hub nodes are considered. To distinguish the two kinds of sets, they are named with ascending integers. So, $H = \{1, 2, 3\}$ is the set of candidate hub nodes; $N = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ is the set of non-hub nodes. In this example, the 3 candidate hubs are generated randomly and located at the same geographic place as the non-hub nodes 4, 13 and 12, respectively. Thus based on the method introduced in Chapter 4, nodes 4, 7, 6, 10 and 12 in Figure 6.2 are allocated to hub node 1 according to the correspondence between location and routing sections. A local tour serving these nodes is obtained based on their permutation order in $R^{'}(x)$, i.e. 1-4-7-6-10-12-1 if the maximum number of nodes in one route is set to 5. Meantime, another vehicle route 2-5-9-8-11-13-2 is constructed for the hub node 2. In each decoding procedure, when the node limitation is violated by the insertion of a new node, this node is assigned to a new route.

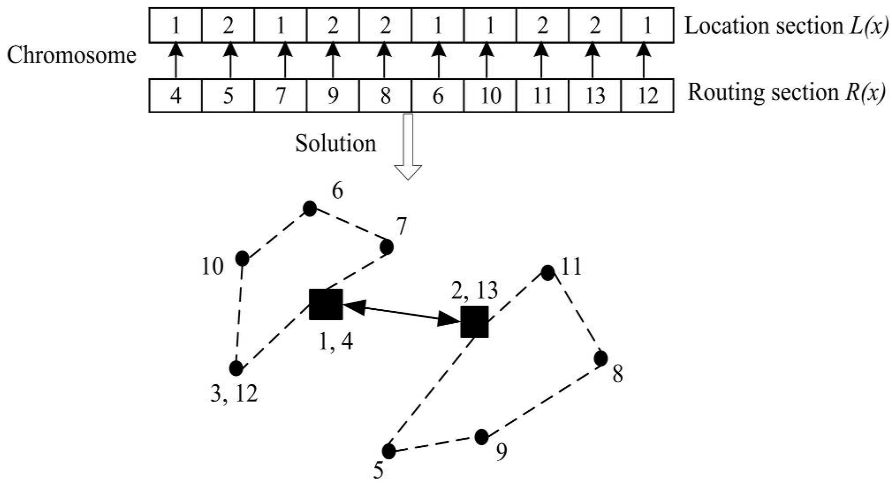


Figure 6.2: An example of a chromosome for the HLRP solution of postal systems

In addition, the hub capacity is checked by the fitness function of each solution s, consisting of the objective function and the penalty function also given as follows:

$$F_{eval}(s) = Objectivevalue(s) + Penaltycost(s) \tag{6.19}$$

where

$$Objectivevalue(s) = \sum_{k\in H} F_k^{'} z_{kk} + \sum_{i\in N}\sum_{k\in H} c_{ik}^{'} z_{ik} + \sum_{i\in N}\sum_{j\in N}\sum_{k\in H}\sum_{l\in H} \alpha^{'} d_{kl} w_{ij} Y_{ijkl} + \lambda \sum_{v\in V}\sum_{i\in N'}\sum_{j\in N', j\neq i} d_{ij} x_{ij}^v + \sum_{v\in V}\sum_{k\in H}\sum_{i\in N} f x_{ki}^v$$

and

$$Penaltycost(s) = \delta^{'} \sum_{k\in H} max\{0, \sum_{i\in I} z_{ik} \sum_{j\in N} w_{ij} + \sum_{j\in J} z_{jk} \sum_{i\in N} w_{ij} - 2\Gamma_k^{'}\}$$

in which $\delta^{'}$ is the penalty parameter for exceeding the capacity of hubs in postal instances.

2. Genetic operators: All genetic operators for the postal cases remain the same as the ones presented in section 4.2.3, including the *roulette wheel* based selection scheme, one-point crossover operation on location and routing sections and the mutation operators of two sections. The initial population is generated randomly and contains $PopSize$ individuals. The stopping criterion is based on the maximum number of generations or when the best fitness value can't be improved in a continuous iteration number.

3. Iterative local search: In this iterative local search procedure, all the neighborhood searches described in section 4.3 are applied including the local search on the hub location and vehicle routing. The implementation details keep consistent with the previous case except the capacity constraint on routes (maximum number of visited nodes) which must be respected by each neighborhood search.

6.4 Computational experiments and results

In this section, the computational experiments and results are introduced based on the instances inspired from the Australian Post (AP) standard data set obtained in *OR-Library* [1]. All tests are carried on a computer with Intel Core i3 CPU of 2.93 GHz and 6 GB of memory, under the Window 7 Operating System.

6.4.1 Instances and parameter values

The standard AP data set can produce instance sizes ranging from 10 up to 200 nodes with a generator. Here, the coordinates of nodes and the flow demand w_{ij} between any two non-hub nodes of each initial AP instance are used directly in our model. And for sake of simplicity, the sites of potential hubs H with size 3, 6 and 10 are generated randomly from the non-hub nodes N. The new cost item $c_{ik}^{'}$ is assumed to depend on the distance d_{ik} and total volume of incoming and outgoing flow at node i. In AP data set, it is defined as $c_{ik}^{'} = d_{ik}(aO_i^{'} + bD_i^{'})$, where $O_i^{'} = \sum_{j\in N} w_{ij}$, $D_i^{'} = \sum_{i\in N} w_{ij}$ is the total volume of parcels originating at node i and to node i as destination, respectively.

After some preliminary tests, the main parameter values used in the computational experiments are summarized in Table 6.1. For the hub capacities, we assumed them homogeneous. Based on the total flow of all nodes, the capacity is set to $\Gamma_k^{'} \in \{6000, 3000, 1500\}$ to ensure that 1, 2 or 3 hubs at least should open to process the total quantity, respectively. The maximum

1. http://people.brunel.ac.uk/ mastjjb/jeb/orlib/phubinfo.html

number of nodes in each route is set to $q = 20$. Finally, the vehicle fixed cost is set to $f = 1000$ as in [66]. The hub fixed cost is inspired from the AP data set, providing fixed costs for the first 50 hub nodes. In our experiments, we used the average value of all hub fixed costs as the homogeneous hub fixed cost $F'_k = 28000$.

Table 6.1: Values of the different parameters used in the experiments

Name	Value	Name	Value	
Discount factor	$\alpha' = 0.75$	Hub capacity Γ'_k	L	6000
Handling cost	$a = 3$		M	3000
	$b = 2$		S	1500
Routing percentage	$\lambda \in \{1, 100, 500, 1000\}$	Vehicle fixed cost	$f = 1000$	
Route nodes	$q = 20$	Hub fixed cost	$F'_k = 28000$	

6.4.2 Parameters and solutions analysis

The first computational experiments with different values of $\lambda \in \{1, 100, 500, 1000\}$ are carried on the instances with 10, 20, 25 non-hub nodes and 3, 6, 10 potential hubs, respectively. The values of parameter λ are chosen to obtain optimal or best solutions with an increasing percentage of the routing cost in the total cost and to measure the impact of this parameter on solutions. All 9 instances are solved by CPLEX with the three hub capacity levels $\Gamma'_k \in \{L, M, S\}$. The results obtained by CPLEX in three hours are summarized in Table 6.2 to 6.4 for the test instances with 3, 6 and 10 potential hubs, respectively. These tables present the cost components and computational information of the solutions. In these tables, the first two columns introduce the instance information including the number of non-hub nodes and the hub capacity Γ'_k. Column 3 shows the value of parameter λ. In the other columns, the following notations are used to investigate each solution:

- TC: the total cost of each solution found by CPLEX in three hours. The mark '*' indicates an optimal solution;
- VC: the total variable cost of each solution without the fixed hub and vehicle costs;
- FixedH%: the percentage of establishing hub costs on the total cost;
- InterH%: the percentage of inter-hub transportation costs on the total cost;
- Routing%: the percentage of local routing costs on the total cost;
- Alloca%: the percentage of allocation costs (handling costs) on the total cost;
- FixedV%: the percentage of fixed operating vehicle costs on the total cost;
- Open hub: the index of the open hubs in each solution;
- $Gap\%$: the gap of the best solution with the lower bound found by CPLEX in 3 hours ;
- Time(s): the CPU time used by CPLEX.

It can be observed from Table 6.2-6.4 that all instances are solved by CPLEX within the time limit with tight gaps (1.78%, 2.29%, 4.46%, on average for instances with 3, 6, 10 potential hubs, respectively). All instances with 10 non-hub nodes are solved to optimality except two instances with 6 potential hubs and two instances with 10 potential hubs when $\lambda = 1$. The gaps with lower bounds show that, as the hub capacity gets tighter, the problem becomes more difficult to solve, more hubs are open to satisfy the needs of customers and more possibilities of allocation strategies are enumerated to reach the optimality. Also the instances with large values of λ are usually more difficult to solve.

Table 6.2: Computational results with CPLEX for the instances with 3 potential hubs and different λ

Nodes	$\Gamma_k^{'}$	λ	TC	VC	FixedH%	InterH%	Routing%	Alloca%	FixedV%	Open hub	$Gap\%$	Time (s)
10	L	1	247623.40*	189623.40	22.61	9.62	0.05	66.91	0.81	1, 2	0.00	5102.86
		100	259821.39*	201821.39	21.55	9.17	4.74	63.77	0.77	1, 2	0.00	163.94
		500	309106.20*	251106.20	18.12	7.71	19.93	53.60	0.65	1, 2	0.00	302.58
		1000	363443.92*	276443.92	23.11	8.31	29.65	38.11	0.83	1, 2, 3	0.00	209.34
	M	1	247623.40*	189623.40	22.61	9.62	0.05	66.91	0.81	1, 2	0.00	5340.01
		100	259821.39*	201821.39	21.55	9.17	4.74	63.77	0.77	1, 2	0.01	117.14
		500	309106.20*	251106.20	18.12	7.71	19.93	53.60	0.65	1, 2	0.00	177.47
		1000	363443.92*	276443.92	23.11	8.31	29.65	38.11	0.83	1, 2, 3	0.00	251.85
	S	1	349561.39*	262561.39	24.03	12.11	0.04	62.96	0.86	1, 2, 3	0.00	10648.50
		100	364904.19*	277904.19	23.02	11.60	4.25	60.31	0.82	1, 2, 3	0.01	7978.17
		500	426895.31*	339895.31	19.68	9.91	18.15	51.56	0.70	1, 2, 3	0.00	3000.91
		1000	501491.90*	414491.90	16.75	8.16	29.31	45.18	0.60	1, 2, 3	0.00	3775.51
20	L	1	251538.20	193538.20	22.26	8.41	0.08	68.46	0.80	1, 3	0.40	10800.00
		100	270518.13	212518.13	20.70	7.82	7.09	63.66	0.74	1, 3	0.76	10800.00
		500	345205.48	287205.48	16.22	6.54	26.19	50.48	0.58	1, 3	1.78	10800.00
		1000	435598.05	377598.05	12.86	5.18	41.50	40.00	0.46	1, 3	1.43	10800.00
	M	1	251540.41	193540.41	22.26	8.41	0.08	68.46	0.80	1, 3	0.40	10800.00
		100	270518.13	212518.13	20.70	7.82	7.09	63.66	0.74	1, 3	0.65	10800.00
		500	345205.48	287205.48	16.22	6.54	26.19	50.48	0.58	1, 3	1.47	10800.00
		1000	435598.05	377598.05	12.86	5.18	41.50	40.00	0.46	1, 3	1.41	10800.00
	S	1	317729.55	230729.55	26.44	10.47	0.07	62.07	0.94	1, 2, 3	1.84	10800.00
		100	334800.12	247800.12	25.09	9.90	6.30	57.81	0.90	1, 2, 3	1.82	10800.00
		500	420514.51	333514.51	19.98	7.74	24.96	46.62	0.71	1, 2, 3	5.93	10800.00
		1000	531003.89	444003.89	15.82	6.32	39.35	37.95	0.56	1, 2, 3	9.37	10800.00
25	L	1	279858.93	221858.93	20.01	8.20	0.08	71.00	0.71	2, 3	2.31	10800.00
		100	297496.91	210496.91	28.24	9.38	7.49	53.88	1.01	1, 2, 3	1.41	10800.00
		500	386654.92	299654.92	21.72	7.22	28.82	41.46	0.78	1, 2, 3	3.09	10800.00
		1000	492144.79	405144.79	17.07	6.05	42.26	34.01	0.61	1, 2, 3	3.09	10800.00
	M	1	279861.01	221861.01	20.01	8.20	0.08	71.00	0.71	2, 3	2.30	10800.00
		100	302342.11	244342.11	18.52	7.50	7.45	65.87	0.66	2, 3	2.98	10800.00
		500	386654.92	299654.92	21.72	7.22	28.82	41.46	0.78	1, 2, 3	3.35	10800.00
		1000	492144.79	405144.79	17.07	6.05	42.26	34.01	0.61	1, 2, 3	2.93	10800.00
	S	1	286849.00	199849.00	29.28	10.31	0.09	59.27	1.05	1, 2, 3	0.71	10800.00
		100	310030.89	223030.89	27.09	9.54	7.56	54.84	0.97	1, 2, 3	1.86	10800.00
		500	403284.36	316284.36	20.83	7.33	28.93	42.16	0.74	1, 2, 3	4.84	10800.00
		1000	520901.89	433901.89	16.13	5.68	44.98	32.64	0.58	1, 2, 3	7.76	10800.00
Average											1.78	

Table 6.3: Computational results with CPLEX for the instances with 6 potential hubs and different λ

Nodes	Γ_k'	λ	TC	VC	FixedH%	InterH%	Routing%	Alloca%	FixedV%	Open hub	$Gap\%$	Time (s)
10	L	1	223127.15*	136127.15	37.65	9.80	0.05	51.15	1.34	2, 5, 6	0.00	10512.10
		100	234910.84*	147910.84	35.76	9.31	5.07	48.59	1.28	2, 5, 6	0.00	317.38
		500	274511.42*	158511.42	40.80	10.10	15.96	31.68	1.46	2, 3, 5, 6	0.00	44.46
		1000	318329.81*	202329.81	35.18	8.71	27.53	27.32	1.26	2, 3, 5, 6	0.00	60.98
	M	1	223127.15	136127.15	37.65	9.80	0.05	51.15	1.34	2, 5, 6	0.88	10800.00
		100	234910.84*	147910.84	35.76	9.31	5.07	48.59	1.28	2, 5, 6	0.00	403.39
		500	274511.42*	158511.42	40.80	10.10	15.96	31.68	1.46	2, 3, 5, 6	0.00	41.26
		1000	318329.81*	202329.81	35.18	8.71	27.53	27.32	1.26	2, 3, 5, 6	0.00	61.68
	S	1	234130.74	118130.74	47.84	12.11	0.05	38.30	1.71	1, 2, 5, 6	1.69	10800.00
		100	245500.25*	129500.25	45.62	11.55	4.68	36.53	1.63	1, 2, 5, 6	0.00	3814.15
		500	283408.62*	138408.62	49.40	12.06	14.73	22.05	1.76	1, 2, 3, 5, 6	0.00	39.23
		1000	325142.90*	180142.90	43.06	10.51	25.67	19.22	1.54	1, 2, 3, 5, 6	0.00	53.31
20	L	1	251538.20	193538.20	22.26	8.41	0.08	68.46	0.80	1, 3	0.40	10800.00
		100	270518.13	212518.13	20.70	7.82	7.09	63.66	0.74	1, 3	0.77	10800.00
		500	345205.48	287205.48	16.22	6.54	26.19	50.48	0.58	1, 3	1.96	10800.00
		1000	435598.05	377598.05	12.86	5.18	41.50	40.00	0.46	1, 3	2.10	10800.00
	M	1	251539.91	193539.91	22.26	8.41	0.08	68.46	0.80	1, 3	0.40	10800.00
		100	270518.13	212518.13	20.70	7.82	7.09	63.66	0.74	1, 3	0.65	10800.00
		500	345205.48	287205.48	16.22	6.54	26.19	50.48	0.58	1, 3	2.08	10800.00
		1000	435598.05	377598.05	12.86	5.18	41.50	40.00	0.46	1, 3	2.10	10800.00
	S	1	277133.39	190133.39	30.31	10.01	0.08	58.52	1.08	1, 3, 4	0.74	10800.00
		100	297178.45	210178.45	28.27	9.23	6.75	54.74	1.01	1, 3, 4	1.62	10800.00
		500	380983.87	293983.87	22.05	7.20	27.27	42.70	0.79	1, 3, 4	5.13	10800.00
		1000	486255.94	370255.94	23.03	6.87	40.15	29.12	0.82	1, 2, 3, 4	7.98	10800.00
25	L	1	246168.53	188168.53	22.75	6.14	0.09	70.21	0.81	2, 5	0.41	10800.00
		100	267789.09	209789.09	20.91	5.64	8.16	64.54	0.75	2, 5	0.72	10800.00
		500	372723.32	313723.32	15.02	4.46	32.79	46.92	0.80	2, 5	7.42	10800.00
		1000	477443.70	419443.70	11.73	2.53	46.68	38.64	0.42	2, 5	6.34	10800.00
	M	1	246168.53	188168.53	22.75	6.14	0.09	70.21	0.81	2, 5	0.41	10800.00
		100	267789.09	209789.09	20.91	5.64	8.16	64.54	0.75	2, 5	0.95	10800.00
		500	355144.88	297144.88	15.77	4.25	30.75	48.67	0.56	2, 5	1.96	10800.00
		1000	471730.57	413730.57	11.87	4.02	45.68	38.00	0.42	2, 5	5.67	10800.00
	S	1	283047.43	167047.43	39.57	11.68	0.09	47.25	1.41	1, 2, 4, 5	5.62	10800.00
		100	295568.71	208568.71	28.42	8.55	7.92	54.10	1.01	1, 2, 5	3.03	10800.00
		500	410486.60	323486.60	20.46	6.07	33.85	38.89	0.73	1, 2, 5	10.92	10800.00
		1000	516793.43	399793.43	21.67	6.38	44.98	26.00	0.97	1, 2, 4, 5	10.62	10800.00
Average											2.29	

These tables highlight that the allocation cost represents a large part in the total cost and it decreases when more hubs are opened. When the value of the parameter λ increases, the contribution of routing costs to the total cost obviously increases while the weights of inter-hub transportation cost and allocation costs on the total cost decrease. For example, for the instances with 10 non-hub nodes and 6 potential hubs (shown in Table 6.3), when $\Gamma_k^{'} = 6000$ (uncapacitated), the proportion of routing costs increases from 0.05% to 27.53% as λ increases from 1 to 1000, while the contribution of the inter-hub transportation costs decreases from 9.80% to 8.71%, as well as the contribution of allocation costs decreases from 51.15% to 27.32%. For the effect of the different hub capacity levels, we observe that the solutions have no obvious changes when $\Gamma_k^{'}$ decreases from 6000 to 3000 through the comparison of the total costs and cost structure with the same λ value. However, when $\Gamma_k^{'}$ decreases to 1500, all the solutions indicate more open hubs and operated routes to minimize the total cost. More closer insights into this effect can be seen in Table 6.5.

Table 6.4: Computational results with CPLEX for the instances with 10 potential hubs and different λ

Nodes	$\Gamma_k^{'}$	λ	TC	VC	FixedH%	InterH%	Routing%	Alloca%	FixedV%	Open hub	$Gap\%$	Time (s)
10	L	1	223127.15	136127.15	37.65	9.80	0.05	51.15	1.34	3, 4, 7	0.89	10800.00
		100	234910.84*	147910.84	35.76	9.31	5.07	48.59	1.28	3, 4, 7	0.00	1438.69
		500	271741.11*	126741.11	51.52	11.83	13.07	21.75	1.84	1, 2, 3, 7, 8	0.00	302.38
		1000	307247.09*	162247.09	45.57	10.46	23.11	19.23	1.63	1, 2, 3, 7, 8	0.00	168.68
	M	1	223127.15	136127.15	37.65	9.80	0.05	51.15	1.34	3, 4, 7	1.24	10800.00
		100	234910.84*	147910.84	35.76	9.31	5.07	48.59	1.28	3, 4, 7	0.00	1040.76
		500	271741.11*	126741.11	51.52	11.83	13.07	21.75	1.84	1, 2, 3, 7, 8	0.00	221.72
		1000	307247.09*	162247.09	45.57	10.46	23.11	19.23	1.63	1, 2, 3, 7, 8	0.00	219.85
	S	1	224359.71	137359.71	37.44	10.61	0.06	50.55	1.34	3, 7, 8	1.33	10800.00
		100	236779.79*	149779.79	35.48	10.06	5.30	47.90	1.27	3, 7, 8	0.00	1227.46
		500	273209.22*	128209.22	51.24	12.63	13.29	21.01	1.83	1, 2, 3, 7, 8	0.00	84.58
		1000	309515.25*	164515.25	45.23	11.15	23.46	18.54	1.62	1, 2, 3, 7, 8	0.00	3072.00
20	L	1	233829.92	175829.92	23.95	8.10	0.08	67.01	0.86	1, 7	0.43	10800.00
		100	252809.85	194809.85	22.15	7.50	7.58	61.98	0.79	1, 7	0.91	10800.00
		500	326030.83	268030.83	17.18	6.20	27.73	48.28	0.61	1, 7	2.10	10800.00
		1000	416423.40	358423.40	13.45	4.86	43.41	37.80	0.48	1, 7	2.56	10800.00
	M	1	233838.01	175838.01	23.95	8.10	0.09	67.01	0.86	1, 7	0.44	10800.00
		100	252809.85	194809.85	22.15	7.50	7.58	61.98	0.79	1, 7	1.04	10800.00
		500	326030.83	268030.83	17.18	6.20	27.73	48.28	0.61	1, 7	2.08	10800.00
		1000	416423.40	358423.40	13.45	4.86	43.41	37.80	0.48	1, 7	2.43	10800.00
	S	1	267182.34	151182.34	41.92	9.11	0.08	47.39	1.50	2, 7, 8, 9	6.83	10800.00
		100	270183.11	183183.11	31.09	7.00	7.19	53.61	1.11	1, 7, 8	1.39	10800.00
		500	369024.72	281024.72	22.76	5.04	30.53	40.59	1.08	1, 7, 8	9.08	10800.00
		1000	440217.74	353217.74	19.08	4.17	43.02	33.04	0.68	1, 7, 8	4.98	10800.00
25	L	1	233776.92	175776.92	23.95	7.36	0.10	67.73	0.86	6, 9	0.44	10800.00
		100	256352.73	198352.73	21.84	6.49	8.52	62.37	0.78	6, 9	1.28	10800.00
		500	348412.80	290412.80	16.07	4.94	32.97	45.45	0.57	6, 9	4.47	10800.00
		1000	452101.98	394101.98	12.39	3.53	47.76	35.88	0.44	6, 9	3.38	10800.00
	M	1	238045.43	180045.43	23.52	7.93	0.13	67.58	0.84	6, 9	2.23	10800.00
		100	257822.19	199822.19	21.72	7.10	9.17	61.23	0.78	6, 9	2.03	10800.00
		500	351358.39	293358.39	15.94	5.50	31.61	46.39	0.57	6, 9	5.32	10800.00
		1000	455231.39	397231.39	12.30	4.59	47.36	35.31	0.44	6, 9	4.75	10800.00
	S	1	338782.74	221782.74	33.06	6.82	0.09	58.56	1.48	4, 5, 6, 9	24.88	10800.00
		100	367590.64	250590.64	30.47	6.29	7.92	53.97	1.36	4, 5, 6, 9	25.48	10800.00
		500	437234.34	320234.34	25.62	5.82	30.59	36.82	1.14	1, 5, 6, 9	20.54	10800.00
		1000	629480.65	512480.65	17.79	3.67	46.23	31.52	0.79	4, 5, 6, 9	28.01	10800.00
Average											4.46	

Table 6.5 studies the optimal solutions of the instances with 10 non-hub nodes and different number of potential hubs. In this table, we present the detailed values of the total costs TC, the fixed establishing hub costs $FixedH$, the inter-hub transportation costs $InterH$, the local rout-

ing costs $Rouring$, the allocation costs $Alloca$ and the fixed operating vehicle costs $FixedV$ of the optimal or best solution obtained by CPLEX. In addition, the first three columns introduce the information on each test instance including the number of potential hub nodes, the value of λ and the hub capacity level. The last two columns present the index of the open hubs and the total number of operating vehicles in each solution.

Table 6.5: Results with different capacities and λ values for the instances with 10 non-hub nodes

Potential hubs	λ	Γ'_k	TC	$FixedH$	$InterH$	$Routing$	$Alloca$	$FixedV$	Open hubs	NV
3	1	L	247623.40	56000.00	23824.44	123.21	165675.75	2000.00	1, 2	2
		M	247623.40	56000.00	23824.44	123.21	165675.75	2000.00	1, 2	2
		S	349561.39	84000.00	42317.04	154.98	220089.37	3000.00	1, 2, 3	3
	100	L	259821.39	56000.00	23824.44	12321.20	165675.75	2000.00	1, 2	2
		M	259821.39	56000.00	23824.44	12321.20	165675.75	2000.00	1, 2	2
		S	364904.19	84000.00	42317.04	15497.78	220089.37	3000.00	1, 2, 3	3
	500	L	309106.20	56000.00	23824.44	61606.01	165675.75	2000.00	1, 2	2
		M	309106.20	56000.00	23824.44	61606.01	165675.75	2000.00	1, 2	2
		S	426895.31	84000.00	42317.04	77488.90	220089.37	3000.00	1, 2, 3	3
	1000	L	363443.92	84000.00	30188.26	107747.10	138508.55	3000.00	1, 2, 3	3
		M	363443.92	84000.00	30188.26	107747.10	138508.55	3000.00	1, 2, 3	3
		S	501491.90	84000.00	40921.60	147005.99	226564.32	3000.00	1, 2, 3	3
6	1	L	223127.15	84000.00	21870.53	119.03	114137.59	3000.00	2, 5, 6	3
		M	223127.15	84000.00	21870.53	119.03	114137.59	3000.00	2, 5, 6	3
		S	234130.74	112000.00	28346.91	114.84	89668.99	4000.00	1, 2, 5, 6	4
	100	L	234910.84	84000.00	21870.53	11902.72	114137.59	3000.00	2, 5, 6	3
		M	234910.84	84000.00	21870.53	11902.72	114137.59	3000.00	2, 5, 6	3
		S	245500.25	112000.00	28346.91	11484.35	89668.99	4000.00	1, 2, 5, 6	4
	500	L	274511.42	112000.00	27722.63	43818.40	86970.39	4000.00	2, 3, 5, 6	4
		M	274511.42	112000.00	27722.63	43818.40	86970.39	4000.00	2, 3, 5, 6	4
		S	283408.62	140000.00	34172.55	41734.28	62501.79	5000.00	1, 2, 3, 5, 6	5
	1000	L	318329.81	112000.00	27722.63	87636.79	86970.39	4000.00	2, 3, 5, 6	4
		M	318329.81	112000.00	27722.63	87636.79	86970.39	4000.00	2, 3, 5, 6	4
		S	325142.90	140000.00	34172.55	83468.56	62501.79	5000.00	1, 2, 3, 5, 6	5
10	1	L	223127.15	84000.00	21870.53	119.03	114137.59	3000.00	3, 4, 7	3
		M	223127.15	84000.00	21870.53	119.03	114137.59	3000.00	3, 4, 7	3
		S	224359.71	84000.00	23813.64	125.46	113420.62	3000.00	3, 7, 8	3
	100	L	234910.84	84000.00	21870.53	11902.72	114137.59	3000.00	3, 4, 7	3
		M	234910.84	84000.00	21870.53	11902.72	114137.59	3000.00	3, 4, 7	3
		S	236779.79	84000.00	23813.64	12545.53	113420.62	3000.00	3, 7, 8	3
	500	L	271741.11	140000.00	32136.66	35505.97	59098.48	5000.00	1, 2, 3, 7, 8	5
		M	271741.11	140000.00	32136.66	35505.97	59098.48	5000.00	1, 2, 3, 7, 8	5
		S	273209.22	140000.00	34515.22	36306.03	57387.97	5000.00	1, 2, 3, 7, 8	5
	1000	L	307247.09	140000.00	32136.66	71011.95	59098.48	5000.00	1, 2, 3, 7, 8	5
		M	307247.09	140000.00	32136.66	71011.95	59098.48	5000.00	1, 2, 3, 7, 8	5
		S	309515.25	140000.00	34515.22	72612.06	57387.97	5000.00	1, 2, 3, 7, 8	5

The effect of the different hub capacity levels on the network design solutions can be demonstrated by Table 6.5. For example with the instances with 10 non-hub nodes and 10 potential hubs, when $\lambda = 100$, the optimal solutions for $\Gamma'_k \in \{6000, 3000\}$ establish 3 hubs which are located at sites 3, 4 and 7 to process the total demand with a total cost of 234910.84. When $\Gamma'_k = 1500$, the optimal solution also opens 3 hubs but one hub is changed from site 4 to site 8. Then the allocation strategy and routing design are changed to adapt tighter capacity constraint with a slightly greater total cost 236779.79. The corresponding optimal network for this example is shown in Figure 6.3. In order to give more insights for the HLRP network in postal systems, Figure 6.4 display another example of the best solutions obtained for the instances with 20 non-hub nodes and 10 potential hubs, when $\lambda = 100$. In this figure, the 10 potential hubs are generated from 20 non-hub nodes. For example the candidate hub 1 is located at the same geography site as node 17, as well as candidate hubs 3, 7 corresponding to nodes 29, 14, respectively. In this test instance, the best solutions obtained for $\Gamma_k^{'} \in \{6000, 3000\}$ operate two local tours starting at hub 1 and hub 7 respectively. The total cost for this case is 252809.85. When $\Gamma_k^{'}$ decreases to 1500, the best solution obtained (with cost 270183.11) opens three hubs (1, 7, 8), each one with a local tour.

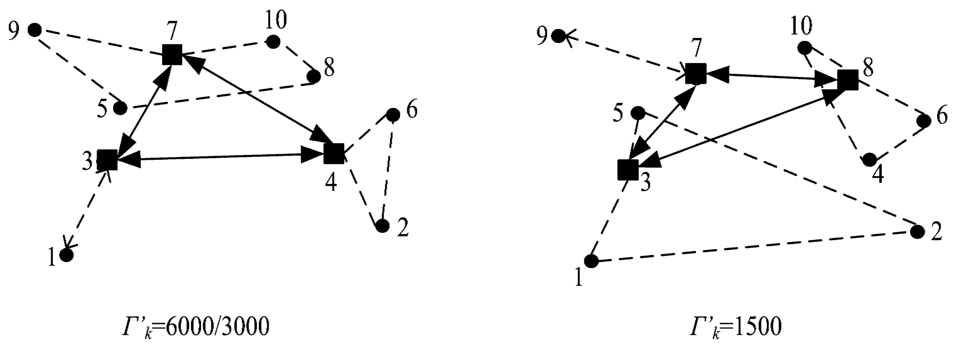


Figure 6.3: Optimal solutions for instance 10-10 with different capacities $\lambda = 100$

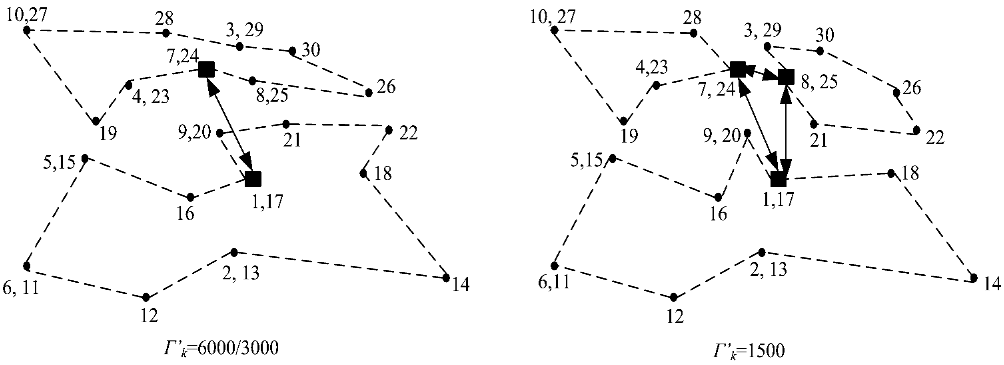


Figure 6.4: Best solutions for instance 10-20 with different capacities $\lambda = 100$

The effect of λ on the hub locations is presented in Table 6.5. When $\lambda = 1$ and $\lambda = 100$, with 6 or 10 potential hubs under the same capacity level, the open hubs remain the same. When λ increases to 500 or 1000, the number of open hubs increases. However, the location of hubs doesn't change for $\lambda = 500$ and $\lambda = 1000$. In addition, we note that for instances with 3 potential hubs, the optimal solutions don't change when $\lambda \in \{1, 100, 500\}$ except the increase of the routing costs. Furthermore, Figures 6.5-6.7 show the changes as in Table 6.5, on each cost component, except the smallest fixed vehicle costs, as λ increases. Here, we use the average value of each cost component for the three levels of hub capacity to reveal their changes. The three figures verify the decrease of the contribution of the allocation costs and the increase of the routing costs in the total cost. For example, Figure 6.6 shows the average cost structure for instances with 10 nodes and 6 potential hubs as λ increases. It can be seen that the values for each cost component keep the same except the routing costs when $\lambda = 1$ and $\lambda = 100$, as well as for the solutions when $\lambda = 500$ and $\lambda = 1000$. Although the above conclusions are obtained only on the basis of the optimal solutions for the instances with 10 nodes, similar effects of different hub capacity levels and values of parameter λ can be observed for other instances as shown in Tables 6.2-6.4. We can conclude that a significant increase of λ improve the routing part of the solution.

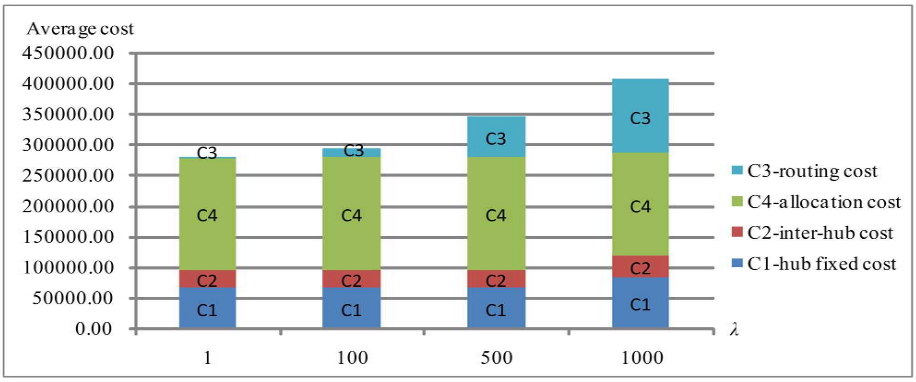


Figure 6.5: Cost structure of instance 3-10 with different λ values

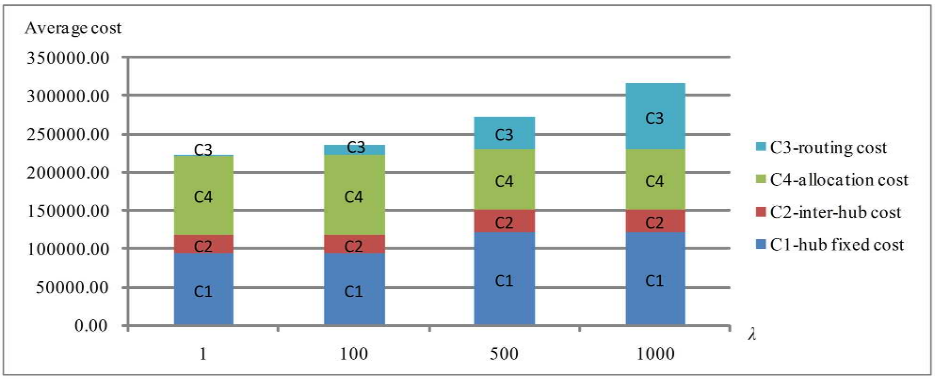


Figure 6.6: Cost structure of instance 6-10 with different λ values

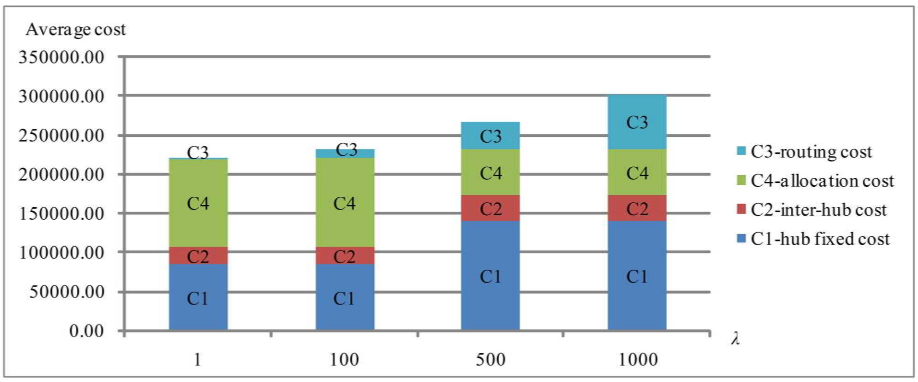


Figure 6.7: Cost structure of instance 10-10 with different λ values

6.4.3 Computational results of the memetic algorithm

Computational experiments were implemented to evaluate the performance of the memetic algorithm in solving the HLRP of postal systems. The MA was implemented in C++ language and all tests have been run on the same computer as the one aforementioned. After tuning the parameters of the MA, on postal instances, the following stopping criteria are retained: the algorithm stops when the maximum number of iteration reaches 200 or when the best individual remains unchanged after 100 consecutive iterations. The other parameters are set to the same values as the ones used in Chapter 4, except the penalty parameter for the fitness function changed to 10000.

Tables 6.6-6.8 compare the results obtained by CPLEX and the MA based on different instances for $\lambda = 100$ or $\lambda = 500$. In the three tables, the instance name is denoted by the number of potential hubs $|H|$ and the number of non-hub nodes $|N|$. The second column gives the hub capacity level $\Gamma'_k \in \{L, M, S\}$. In the last columns, the following notations are used to compare the results:

- LB: lower bound or best lower bound found by CPLEX in three hours.
- UB: upper bound (best objective value) found by CPLEX in three hours, marked "*" if the solution is optimal.
- $\%LB$: deviation in % of the upper bound from the lower bound found by CPLEX. Here, $\%LB = (UB - LB)/UB \times 100\%$. The bold one represents the sbest value compared to the other method.
- Time (s): CPU time in seconds used by CPLEX.
- Open hub: the index of open hubs in the best solution obtained by each corresponding method.
- Z_{best}: best objective value found by the MA in 10 runs, marked "*" if the solution is optimal.
- $\%LB'$: deviation in % between the best objective value and the lower bound of CPLEX, Here, $\%LB' = (Z_{best} - LB)/Z_{best} \times 100\%$.
- $\%UB$: deviation in % between the best objective value and upper bound of CPLEX, Here, $\%UB = (Z_{best} - UB)/UB \times 100\%$.
- T_{best} (s): CPU time in seconds used for the best solution obtained by the MA.

Table 6.6 reports the results obtained by CPLEX and the proposed MA based on instances with 3, 6 potential hubs when $\lambda = 100$. For all of the 42 test instances, the MA can reach the same or better solutions than CPLEX including 6 optimal solutions and 26 new best solutions. When CPLEX can't find feasible solutions in 3 hours (8 instances), especially for the large instances with small hub capacity, the MA can provide a solution of good quality in reasonable computing time. For the 36 instances unproved to optimality, the gaps between the best objective values from the MA and the lower bounds $\%LB'$ change between 0.65% and 11.19% with an average value around 3.60%. Compared to the results obtained by CPLEX, the gaps between upper bounds and lower bounds $\%LB$ for the 36 instances vary between 0.65% and 26.43% with an average value 5.75%. Especially for the instance 6-40 with small hub capacity level, the best solution obtained by the MA greatly improves the one found by CPLEX. With respect to the CPU time used by each method, the MA is able to find the competitive solutions in a shorter computational time (396.23s verse 9561.77s, on average). From the column "Open hub", it can be observed that the locations of open hubs in the best solution obtained by each method have no obvious changes except 5 instances (bold ones) Thus, it may be concluded that generally, the improvement of the best solutions from the MA arise in the decisions of allocation strategies or routing design.

Table 6.6: Result comparison between CPLEX and the MA on the postal instances (3, 6 hubs) $\lambda = 100$

Instance name		CPLEX					MA				
H-N	Γ'_k	LB	UB	$\%LB$	Time(s)	Open hub	Z_{best}	$\%LB'$	$\%UB$	T_{best}	Open hub
3-10	L	259821.39	259821.39*	**0.00**	163.94	1, 2	259821.39*	**0.00**	0.00	0.49	1, 2
	M	259821.39	259821.39*	**0.00**	117.14	1, 2	259821.39*	**0.00**	0.00	0.49	1, 2
	S	364904.19	364904.19*	**0.00**	7978.17	1, 2, 3	364904.19*	**0.00**	0.00	0.51	1, 2, 3
3-20	L	268459.98	270518.13	**0.76**	10800.00	1, 3	270518.13	**0.76**	0.00	2.99	1, 3
	M	268761.38	270518.13	**0.65**	10800.00	1, 3	270518.13	**0.65**	0.00	2.98	1, 3
	S	328708.88	334800.12	**1.82**	10800.00	1, 2, 3	334800.11	**1.82**	0.00	2.98	1, 2, 3
3-25	L	293308.79	297496.91	**1.41**	10800.00	1, 2, 3	297496.91	**1.41**	0.00	5.83	1, 2, 3
	M	293336.05	302342.11	2.98	10800.00	1, 2, 3	297496.91	**1.40**	-1.60	5.95	1, 2, 3
	S	304249.45	310030.89	1.86	10800.00	1, 2, 3	309937.50	**1.84**	-0.03	5.76	1, 2, 3
6-10	L	234910.84	234910.84*	**0.00**	317.38	2, 5, 6	234910.84*	**0.00**	0.00	1.05	2, 5, 6
	M	234910.84	234910.84*	**0.00**	403.39	2, 5, 6	234910.84*	**0.00**	0.00	2.11	2, 5, 6
	S	245500.25	245500.25*	**0.00**	3814.15	1, 2, 5, 6	245500.25*	**0.00**	0.00	1.08	1, 2, 5, 6
6-20	L	268435.13	270518.13	**0.77**	10800.00	1, 3	270518.13	**0.77**	0.00	8.00	1, 3
	M	268757.87	270518.13	**0.65**	10800.00	1, 3	270518.13	**0.65**	0.00	8.86	1, 3
	S	292370.32	297178.45	**1.62**	10800.00	1, 3, 4	297178.45	**1.62**	0.00	8.22	1, 3, 4
6-25	L	265872.05	267789.09	**0.72**	10800.00	2, 5	267789.09	**0.72**	0.00	18.92	2, 5
	M	265255.76	267789.09	**0.95**	10800.00	2, 5	267789.09	**0.95**	0.00	18.69	2, 5
	S	286625.48	295568.71	**3.03**	10800.00	1, 2, 5	295568.71	**3.03**	0.00	19.01	1, 2, 5
6-30	L	292289.05	297287.49	1.68	10800.00	1,4	296932.26	**1.56**	-0.12	16.01	1, 4
	M	292316.43	298564.16	2.09	10800.00	1, 4	297409.95	**1.71**	-0.39	17.28	1, 4
	S	316487.21	NFS	-	10800.00	-	324324.83	**2.42**	-	16.63	1,3,4
6-40	L	308797.37	321978.94	4.09	10800.00	5	315897.38	**2.25**	-1.89	65.47	5
	M	308873.39	332968.79	7.24	10800.00	1,6	317330.38	**2.67**	-4.70	50.03	1,6
	S	313732.68	426430.67	26.43	10800.00	1,2,5,6	323458.77	**3.01**	-24.15	49.63	**1,3,6**
6-50	L	287194.66	302718.58	5.13	10800.00	4,5	299150.70	**4.00**	-1.18	112.03	4,5
	M	287175.43	306157.99	6.20	10800.00	4,5	300697.36	**4.50**	-1.78	141.78	4,5
	S	303451.00	NFS	-	10800.00	-	324076.13	**6.36**	-	115.18	1,4,5
6-60	L	316437.56	346053.23	8.56	10800.00	2, 3	330906.30	**4.37**	-4.38	268.50	2,3
	M	316414.18	358524.56	11.75	10800.00	2, 3	332065.06	**4.71**	-7.38	236.12	2,3
	S	335417.71	NFS	-	10800.00	-	353049.84	**4.99**	-	275.19	2, 3, 6
6-70	L	314003.43	368781.84	14.85	10800.00	1, 4	327382.33	**4.09**	-11.23	588.32	**4**
	M	319838.22	415776.74	23.07	10800.00	1, 4, 5	351378.56	**8.98**	-15.49	490.54	**1,4**
	S	356989.88	NFS	-	10800.00	-	391698.83	**8.86**	-	474.12	1,4,5
6-80	L	277448.49	329740.26	15.86	10800.00	2, 4	298102.93	**6.93**	-9.59	881.70	2,4
	M	277588.47	326834.02	15.07	10800.00	2, 4	299668.53	**7.37**	-8.31	850.53	2,4
	S	316711.70	NFS	-	10800.00	-	339025.06	**6.58**	-	788.27	2,3,4
6-90	L	322579.88	359922.33	10.38	10800.00	1, 3	340518.86	**5.27**	-5.39	1428.78	**2**
	M	326323.08	366901.02	11.06	10800.00	1, 3	348169.04	**6.27**	-5.11	1348.68	1,3
	S	334598.62	NFS	-	10800.00	-	360177.21	**7.10**	-	1495.32	1,2,3
6-100	L	282779.22	331894.29	14.80	10800.00	2, 4	314710.74	**10.15**	-5.18	2301.63	**1,2**
	M	282990.41	NFS	-	10800.00	-	314710.74	**10.08**	-	2260.16	1,2
	S	333468.26	NFS	-	10800.00	-	375464.72	**11.19**	-	2255.91	1,2,5
Average				5.75	9561.77			**3.60**	-3.17	**396.23**	
3-hub instances				1.05	8117.69			**0.87**	-0.18	**3.11**	
6-hub instances				7.44	9955.60			**4.34**	-4.25	**503.45**	

Table 6.7: Result comparison between CPLEX and the MA on the postal instances (10 hubs) $\lambda = 100$

Instance name		CPLEX					MA				
H-N	Γ_k'	LB	UB	$\%LB$	Time(s)	Open hub	Z_{best}	$\%LB'$	$\%UB$	T_{best}	Open hub
	L	234910.84	234910.84*	**0.00**	1438.69	3,4,7	234910.84*	**0.00**	0.00	2.13	3,4,7
10-10	M	234910.84	234910.84*	**0.00**	1040.76	3,4,7	234910.84*	**0.00**	0.00	2.75	3,4,7
	S	236779.79	236779.79*	**0.00**	1227.46	3,7,8	236779.79*	**0.00**	0.00	2.09	3,7,8
	L	250505.55	252809.85	**0.91**	10800.00	1,7	252809.85	**0.91**	0.00	19.11	1,7
10-20	M	250179.95	252809.85	**1.04**	10800.00	1,7	252809.85	**1.04**	0.00	18.03	1,7
	S	266422.96	270183.11	1.39	10800.00	1,7,8	269764.92	**1.24**	-0.15	19.60	1,7,8
	L	253069.18	256352.73	**1.28**	10800.00	6,9	256352.73	**1.28**	0.00	39.23	6,9
10-25	M	252591.86	257822.19	2.03	10800.00	6,9	256481.99	**1.52**	-0.52	38.80	6,9
	S	273942.00	367590.64	25.48	10800.00	4,5,6,9	280431.85	**2.31**	-23.71	41.86	**1,6,9**
	L	270892.80	276821.75	2.14	10800.00	3,9	275360.63	**1.62**	-0.53	37.33	3,9
10-30	M	270878.77	278896.76	2.87	10800.00	3,9	276687.78	**2.10**	-0.79	36.32	3,9
	S	286681.02	NFS	-	10800.00	-	295004.49	**2.82**	-	38.31	1,3,9
	L	268602.31	282736.75	5.00	10800.00	8,10	274513.80	**2.15**	-2.91	104.88	8,10
10-40	M	268548.78	294264.14	8.74	10800.00	8,10	276789.76	**2.98**	-5.94	115.58	**1,8**
	S	285860.38	NFS	-	10800.00	-	299100.56	**4.43**	-	100.22	8,9,10
	L	265684.56	324494.12	18.12	10800.00	2,10	275352.94	**3.51**	-15.14	287.95	**5,10**
10-50	M	265384.60	338123.28	21.51	10800.00	5,10	276048.12	**3.86**	-18.36	259.67	5,10
	S	284937.55	NFS	-	10800.00	-	298627.47	**4.58**	-	251.04	2,5,10
	L	283854.05	342960.42	17.23	10800.00	9,10	295522.05	**3.95**	-13.83	520.65	9,10
10-60	M	283838.34	375197.35	24.35	10800.00	9,10	298002.54	**4.75**	-20.57	499.11	9,10
	S	313801.78	389177.1	19.37	10800.00	3,9,10	337975.98	**7.15**	-13.16	522.06	3,8,10
	L	281518.39	327083.16	13.93	10800.00	1,10	301476.60	**6.62**	-7.83	950.91	1,10
10-70	M	281380.38	356117.14	20.99	10800.00	1,10	304576.96	**7.62**	-14.47	973.72	1,10
	S	295347.22	NFS	-	10800.00	-	325013.12	**9.13**	-	1032.50	1,5,9,10
	L	277709.32	346621.14	19.88	10800.00	2,4	295241.88	**5.94**	-14.82	1932.92	2,4
10-80	M	277767.54	312321.32	11.06	10800.00	2,4	296620.03	**6.36**	-5.03	1930.12	2,4
	S	313183.276	NFS	-	10800.00	-	341556.63	**8.31**	-	1751.92	2,3,4
	L	288353.69	389418.76	25.95	10800.00	1,5,8	316394.76	**8.86**	-18.75	3284.85	**5,8**
10-90	M	288094.52	347803.93	17.17	10800.00	1,8	317534.98	**9.27**	-8.70	2925.90	1,8
	S	304595.304	NFS	-	10800.00	-	337997.58	**9.88**	-	3286.32	1,7,8
	L	282789.124	NFS	-	10800.00	-	314553.35	**10.10**	-	4401.96	2,4
10-100	M	282865.292	NFS	-	10800.00	-	321427.99	**12.00**	-	4953.71	1,7
	S	304771.005	NFS	-	10800.00	-	345387.55	**11.76**	-	4703.42	2,4,7
Average				10.85	9930.51			**4.79**	-7.72	**1063.18**	

Table 6.7 provides the results based on the instances with 10 potential hubs and $\lambda = 100$ to compare the performance of CPLEX and the proposed MA. For all 33 test instances, the MA finds all known best solutions in reasonable computing time including 27 new best solutions in which CPLEX didn't find feasible solutions for 9 instances. For 30 instances unsolved optimally, the gaps between the best objective values from the MA and the lower bounds $\%LB'$ vary between 0.91% and 12.00%, and around 60% of these gaps are lower than 5%. Compared to the gaps of CPLEX ($\%LB$), the MA outperforms CPLEX for most instances (4.79% verse 10.85%, on average).

The results reported in Table 6.8 also show the good performance of the MA compared to CPLEX on 27 small and medium instances when $\lambda = 500$. The MA can reach all optimal solutions (9 instances) obtained by CPLEX in a shorter computational time, which is less than 3 seconds for the best solution. When CPLEX cannot find the optimal solutions within the time limit, the MA finds 10 new best solutions from the 18 instances unsolved optimally in a shorter time (see the value of $\%UB$). The 10 new best solutions obtained by the MA have lower gaps compared to CPLEX (4.47% verse 7.52%), especially for the instances 10-25 associated to the best improvement (13%). With respect to the CPU time used by each method to reach the best solution with $\lambda = 500$, the MA can also find solutions of good quality in a shorter time (11.69s verser 7356.10s, on average). In addition, the locations of open hubs in the best solutions obtained by two methods have no differences except one instances (bold ones).

Table 6.8: Result comparison between CPLEX and the MA on the postal instances $\lambda = 500$

Instance name		CPLEX					MA				
H-N	Γ'_k	LB	UB	$\%LB$	Time(s)	Open hub	Z_{best}	$\%LB'$	$\%UB$	T_{best}	Open hub
	L	309106.20	309106.20*	**0.00**	302.58	1,2	309106.20*	**0.00**	0.00	0.50	1,2
3-10	M	309106.20	309106.20*	**0.00**	177.47	1,2	309106.20*	**0.00**	0.00	0.50	1,2
	S	426895.31	426895.31*	**0.00**	3000.91	1,2,3	426895.31*	**0.00**	0.00	0.50	1,2,3
	L	339070.22	345205.48	**1.78**	10800.00	1,3	345205.48	**1.78**	0.00	2.96	1,3
3-20	M	340119.04	345205.48	**1.47**	10800.00	1,3	345205.48	**1.47**	0.00	2.91	1,3
	S	395598.01	420514.51	5.93	10800.00	1,2,3	419214.53	**5.63**	-0.31	3.00	1,2,3
	L	374702.47	386654.92	3.09	10800.00	1,2,3	386023.32	**2.93**	-0.16	6.44	1,2,3
3-25	M	373711.17	386654.92	3.35	10800.00	1,2,3	386023.32	**3.19**	-0.16	6.65	1,2,3
	S	383784.64	403284.36	**4.84**	10800.00	1,2,3	403284.36	**4.84**	0.00	6.39	1,2,3
	L	274511.42	274511.42*	**0.00**	44.46	2,3,5,6	274511.42*	**0.00**	0.00	1.07	2,3,5,6
6-10	M	274511.42	274511.42*	**0.00**	41.26	2,3,5,6	274511.42*	**0.00**	0.00	1.48	2,3,5,6
	S	283408.62	283408.62*	**0.00**	39.23	1,2,3,5,6	283408.62*	**0.00**	0.00	1.09	1,2,3,5,6
	L	338429.11	345205.48	**1.96**	10800.00	1,3	345205.48	**1.96**	0.00	8.06	1,3
6-20	M	338038.62	345205.48	**2.08**	10800.00	1,3	345205.48	**2.08**	0.00	8.10	1,3
	S	361440.91	380983.87	5.13	10800.00	1,3,4	377469.96	**4.25**	-0.92	8.86	1,3,4
	L	345075.71	372723.32	7.42	10800.00	2,5	355144.88	**2.84**	-4.72	17.72	2,5
6-25	M	348170.71	355144.88	**1.96**	10800.00	2,5	355144.88	**1.96**	0.00	19.52	2,5
	S	365660.68	410486.60	10.92	10800.00	1,2,5	393545.52	**7.09**	-4.13	18.56	1,2,5
	L	271741.11	271741.11	**0.00**	302.38	1,2,3,7,8	271741.11	**0.00**	0.00	2.24	1,2,3,7,8
10-10	M	271741.11	271741.11	**0.00**	221.72	1,2,3,7,8	271741.11	**0.00**	0.00	2.06	1,2,3,7,8
	S	273209.22	273209.22	**0.00**	84.58	1,2,3,7,8	273209.22	**0.00**	0.00	2.22	1,2,3,7,8
	L	319195.00	326030.83	**2.10**	10800.00	1,7	326030.83	**2.10**	0.00	17.94	1,7
10-20	M	319240.09	326030.83	**2.08**	10800.00	1,7	326030.83	**2.08**	0.00	17.00	1,7
	S	335522.62	369024.72	9.08	10800.00	1,7,8	345521.73	**2.89**	-6.37	19.22	1,7,8
	L	332852.76	348412.80	4.47	10800.00	6,9	343708.53	**3.16**	-1.35	56.85	6,9
10-25	M	332659.94	351358.39	5.32	10800.00	6,9	347267.96	**4.21**	-1.16	41.72	6,9
	S	347434.78	437234.34	20.54	10800.00	1,5,6,9	379848.59	**8.53**	-13.12	42.14	**1,6,9**
Average				3.46	7356.10			**2.33**	-1.20	11.69	
3-hub instances				2.27	7586.77			**2.20**	-0.07	3.32	
6-hub instances				3.27	7213.88			**2.24**	-1.09	9.38	
10-hub instances				4.84	7267.63			**2.55**	-2.45	22.38	

Tables 6.9-6.11 report the detailed results obtained by the MA for all generated instances (small, medium and large ones) with different hub capacity levels when $\lambda = 100$. The "Best run" represents the best solution found in 10 runs by the MA including the best objective value Z_{best}, the corresponding running time (T_{best} (s)) and the details of the best solution found by the MA. "Open hub" gives the index of the located hubs and "Num routes" represents the total number of local routes in the best solution. The "Average on 10 runs" shows the statistical result of 10 runs of the MA. The average objective value $\overline{Z}$ of the solutions found in 10 runs is shown in the column of "Average value" and then the following statistical indicators are used to evaluate the performance of the MA:

- Aver gap (%): the average deviation in % between each value obtained by the MA and the best value. Here, $Avergap = \frac{\overline{Z}-Z_{best}}{Z_{best}} \times 100\%$;
- $CV(\%)$: the coefficient of variance for the objective values of the 10 runs with the average value. Here, $CV = SD/\overline{Z} \times 100\%$, where SD is the standard deviation of all the objective values of the 10 runs;
- $CV^{'}(\%)$: the coefficient of variance for all objective values of the 10 runs with the best objective value. Here, $CV^{'} = SD/Z_{best} \times 100\%$;
- T_{aver} (s): the average running time for the 10 runs of the MA.

Table 6.9: Results from the MA for the postal instances with 3 potential hubs $\lambda = 100$

Instance name		Best run				Average on 10 runs				
H-N	$\Gamma_k^{'}$	Z_{best}	T_{best}(s)	Open hub	Num routes	Average value	Aver gap	CV	$CV^{'}$	T_{aver}(s)
3-10	L	259821.39	0.49	1, 2	2	259821.39	0.00	0.00	0.00	0.57
	M	259821.39	0.49	1, 2	2	259821.39	0.00	0.00	0.00	0.60
	S	364904.19	0.51	1, 2, 3	3	364904.19	0.00	0.00	0.00	0.52
3-20	L	270518.13	2.99	1, 3	2	272877.61	0.87	0.88	0.89	2.93
	M	270518.13	2.98	1, 3	2	273949.32	1.27	1.16	1.17	2.97
	S	334800.11	2.98	1, 2, 3	3	339106.83	1.29	1.03	1.04	3.13
3-25	L	297496.91	5.83	1, 2, 3	3	302142.81	1.56	0.97	0.99	6.58
	M	297496.91	5.95	1, 2, 3	3	302654.82	1.73	1.13	1.15	6.23
	S	309937.50	5.76	1, 2, 3	3	314017.49	1.32	0.98	0.99	5.86
Average			3.11				0.89	0.68	0.69	3.26

From Tables 6.9-6.11, we can observe that the MA can solve effectively all the postal instances in a reasonable time, even the largest ones including 100 nodes with 10 potential hubs. For all the test instances, the MA can find feasible solutions for the HLRP in postal systems in less than 4 seconds on average for 9 instances with 3 potential hubs and 10 minutes on average for 33 instances with 6 potential hubs. And with respect to the 33 instances with 10 potential hubs, the MA can solve all the problems in less than 1 hour with up to 100 nodes. Moreover, the small average resulting gaps (0.89% for the instances with 3 hubs, 2.99% for the instances with 6 hubs and 3.06% for the ones with 10 potential hubs) prove also the robustness and usefulness of the memetic algorithm for the postal instances. Moreover, the coefficient of variance with the average objective value CV (0.68%, 2.09% and 0.25% on average, respectively) and with the best objective value CV' (0.69%, 2.16% and 2.31% on average, respectively), demonstrate also the good stability of the MA. For the best solution, it can be seen that the assignment solution is changed or the number of open hubs is increased as the capacity of the hubs decreases, and the total cost for most instances increases because more hubs may be operated to satisfy the total demand of suppliers and clients, and better route composition is formed. In addition, the

Table 6.10: Results from the MA for the instances with 6 potential hubs $\lambda = 100$

Instance name		Best run				Average on 10 runs				
H-N	$\Gamma_k^{'}$	Z_{best}	T_{best}(s)	Open hub	Max routes	Average value	Aver gap	CV	$CV^{'}$	T_{aver}(s)
6-10	L	234910.84	1.05	2,5,6	3	237066.64	0.92	0.80	0.81	1.25
	M	234910.84	2.11	2,5,6	3	237130.03	0.94	0.54	0.54	1.26
	S	245500.25	1.08	1,2,5,6	4	250310.99	1.96	1.30	1.33	1.23
6-20	L	270518.13	8.00	1,3	2	273899.37	1.25	0.87	0.88	9.11
	M	270518.13	8.86	1,3	2	274133.29	1.34	0.71	0.71	8.61
	S	297178.45	8.22	1,3,4	3	304138.54	2.34	1.99	2.03	8.26
6-25	L	267789.09	18.92	2,5	2	274891.28	2.65	2.43	2.49	21.22
	M	267789.09	18.69	2,5	2	273589.82	2.17	1.93	1.97	19.40
	S	295568.71	19.01	1,2,5	3	302770.88	2.44	1.50	1.54	19.24
6-30	L	296932.26	16.01	1, 4	2	308712.29	3.97	1.60	1.66	16.94
	M	297409.95	17.28	1, 4	2	305956.07	2.87	2.20	2.26	17.08
	S	324324.83	16.63	1,3,4	3	335059.28	3.31	2.79	2.88	17.20
6-40	L	315897.38	65.47	5	2	320797.23	1.55	1.65	1.68	59.30
	M	317330.38	50.03	1,6	3	326162.81	2.78	2.61	2.69	51.13
	S	323458.77	49.63	1,3,6	3	336070.42	3.90	3.04	3.16	50.26
6-50	L	299150.70	112.03	4,5	4	308519.58	3.13	1.26	1.30	109.16
	M	300697.36	141.78	4,5	4	309596.37	2.96	1.45	1.49	117.97
	S	324076.13	115.18	1,4,5	4	338677.95	4.51	2.38	2.49	121.06
6-60	L	330906.30	268.50	2,3	4	337927.73	2.12	1.04	1.06	247.53
	M	332065.06	236.12	2,3	4	342363.63	3.10	2.86	2.95	233.17
	S	353049.84	275.19	2, 3, 6	4	363056.55	2.83	2.44	2.51	243.13
6-70	L	327382.33	588.32	4	4	333163.99	1.77	1.17	1.19	549.15
	M	351378.56	490.54	1,4	5	366154.11	4.21	2.33	2.43	506.34
	S	391698.83	474.12	1,4,5	5	404561.09	3.28	1.88	1.94	482.35
6-80	L	298102.93	881.70	2,4	5	308974.37	3.65	3.68	3.81	927.79
	M	299668.53	850.53	2,4	5	308710.22	3.02	3.59	3.70	834.91
	S	339025.06	788.27	2,3,4	5	357704.39	5.51	3.62	3.82	841.78
6-90	L	340518.86	1428.78	2	5	349230.66	2.56	1.11	1.13	1407.98
	M	348169.04	1348.68	1,3	5	362925.50	4.24	2.33	2.43	1408.56
	S	360177.21	1495.32	1,2,3	5	396200.77	5.22	3.29	3.46	1453.11
6-100	L	314710.74	2301.63	1,2	6	328007.33	4.23	1.62	1.69	2292.74
	M	314710.74	2260.16	1,2	6	326652.84	3.79	3.57	3.70	2206.95
	S	375464.72	2255.91	1,2,5	6	390575.16	4.02	3.50	3.64	2224.90
Average			503.45				2.99	2.09	2.16	500.30

Table 6.11: Results from the MA for the postal instances with 10 potential hubs $\lambda = 100$

Instance name		Best run				Average on 10 runs				
H-N	Γ_k'	Z_{best}	T_{best}(s)	Open hub	Max routes	Average value	Aver gap	CV	CV'	T_{aver}(s)
	L	234910.84	2.13	3, 4, 7	3	237399.91	1.06	0.84	0.85	2.16
10-10	M	234910.84	2.75	3, 4, 7	3	238205.57	1.40	0.95	0.96	2.50
	S	236779.79	2.09	3, 7, 8	3	237969.34	0.50	0.76	0.77	2.36
	L	252809.85	19.11	1, 7	2	257235.85	1.75	1.72	1.75	21.31
10-20	M	252809.85	18.03	1, 7	2	257666.68	1.92	1.66	1.70	17.90
	S	269764.92	19.60	1, 7, 8	3	276242.78	2.40	2.19	2.24	18.56
	L	256352.73	39.23	6, 9	2	261379.25	1.96	1.80	1.83	47.75
10-25	M	256481.99	38.80	6, 9	2	260421.42	1.54	1.44	1.47	41.43
	S	280431.85	41.86	1, 6, 9	3	289482.40	3.23	2.18	2.25	41.02
	L	275360.63	37.33	3,9	2	280739.90	1.95	1.94	1.98	39.40
10-30	M	276687.78	36.32	3,9	2	282411.08	2.07	1.88	1.91	45.81
	S	295004.49	38.31	1,3,9	3	304235.00	3.13	1.93	1.99	37.77
	L	274513.80	104.88	8,10	3	283227.76	3.17	2.75	2.83	105.03
10-40	M	276789.76	115.58	1,8	2	282630.79	2.11	2.34	2.39	123.03
	S	299100.56	100.22	8,9,10	3	309055.35	3.07	3.07	3.17	108.22
	L	275352.94	287.95	5,10	3	282571.43	2.57	2.20	2.25	249.75
10-50	M	276048.12	259.67	5,10	4	286464.35	3.77	2.64	2.65	258.79
	S	298627.47	251.04	2,5,10	3	304926.42	2.11	2.34	2.39	255.93
	L	295522.05	520.65	9,10	4	309777.82	4.82	2.79	2.70	609.34
10-60	M	298002.54	499.11	9,10	4	303522.67	1.85	1.28	1.29	507.29
	S	337975.98	522.06	3,8,10	5	348729.24	3.18	2.58	2.66	512.65
	L	301476.60	950.91	1,10	5	315131.96	4.53	2.43	2.33	966.34
10-70	M	304576.96	973.72	1,10	5	317809.70	4.34	2.55	2.66	1070.62
	S	325013.12	1032.50	1,5,9,10	6	343711.45	5.75	2.90	3.07	1086.71
	L	295241.88	1932.92	2,4	5	307107.40	4.02	2.34	2.44	1959.67
10-80	M	296620.03	1930.12	2,4	5	304154.33	2.54	2.20	2.25	1873.90
	S	341556.63	1751.92	2,3,4	5	356773.12	4.46	3.31	3.46	1815.27
	L	316394.76	3284.85	5,8	5	331238.79	4.39	2.83	2.96	3240.97
10-90	M	317534.98	2925.90	1,8	5	328874.29	3.36	1.99	2.06	3081.00
	S	337997.58	3286.32	1,7,8	6	357486.38	5.77	3.18	3.36	3142.32
	L	314553.35	4401.96	2,4	6	330312.92	5.01	1.75	1.84	4805.89
10-100	M	321427.99	4953.71	1,7	6	335344.12	4.33	4.87	5.01	4862.02
	S	345387.55	4703.42	2,4,7	7	355412.73	2.90	2.67	2.75	4908.06
Average			1063.18				3.06	2.25	2.31	1086.69

difficulty of solving problems can be detected through the computational time that increases as the number of potential hubs and non-hub nodes increases and the hub capacity become more tighter.

Figure 6.8 and 6.9 show the tendency of the average gap and the coefficient of variance $CV'(\%)$ depending on the number of nodes, respectively, for 6 and 10 potential hubs. They demonstrate the stability of the MA. It can be seen from Figure 6.8 that the average gap between the best single run and the average run of the MA is less than 5.00%. In addition, for all instances, when the number of nodes exceeds 50, the average gap is relatively stable without significant fluctuation except one instance with 70 nodes and 10 potential hubs. And Figure 6.9 shows that the coefficient of variance of all the objective values from the best one is less than 4.0%. Even when the number of nodes exceeds 50, it is close to 3.00%. This shows that the solutions in all the 10 runs are close to the best one. Hence, it can offer the decision maker several reliable solutions near the optimum.

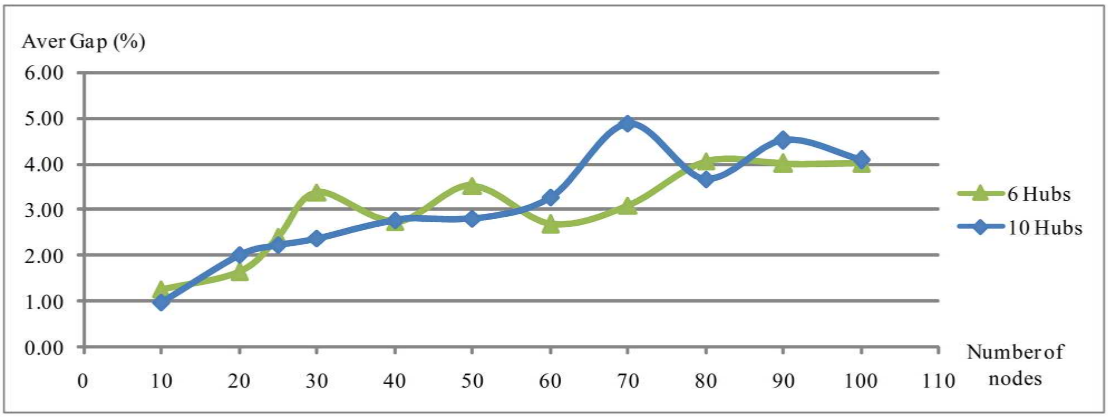


Figure 6.8: The tendency of the average gap depending on the problem scale of postal instances

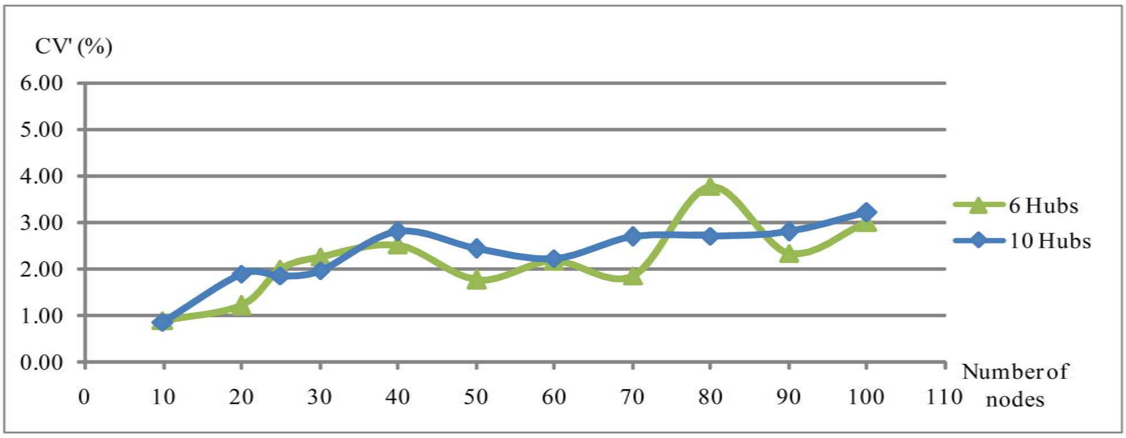


Figure 6.9: The tendency of CV' depending on the problem scale of postal instances

Furthermore, in order to validate the consolidation role of the hubs in the hub-and-spoke organization for postal instances [32], we compare the hub arc flows (inter-hub) with the spoke (non-hub node) flows in Table 6.12 based on instance 6-20, 6-25, 10-20 and 10-25 with $\lambda = 100$. In this table, the average flow transfered between hubs $Flow_{hub}$ and the average spoke flow allocated to hubs $Flow_{spoke}$ in the best solutions are reported in column 5 and 6, respectively. The first two columns present the instance name and hub capacity level. Column 3 and 4 provide respectively the number of hub arcs and the spokes in the best solution for each instance. In addition, it also reports the minimum flow between hubs $Minflow_{hub}$ and the maximum spoke flow $Maxflow_{spoke}$ in column 7 and 8, respectively. The last column provides an average percentage (%) of the number of spokes with flow larger than the hub arc flow. It can be seen from this table that the average hub arc flow is always larger than the average spoke flow for these instances. There are small percentage of spokes with a flow larger than the minimum hub arc flow (3.37%, on average). Indeed, there are at most 2 spoke flows larger than the hub arc flow for the instance 6-20 with the small hub capacity. All these observations demonstrate the interest of hub terminals to aggregate flows in the postal system network and the efficiency of the inter-hub transportation. They also justify the underlying hypothesis of the problem.

Table 6.12: The comparison of hub arc flows and spoke flows in postal instances

Instance	Γ_k	Hub arcs	Spokes	$Flow_{hub}$	$Flow_{spoke}$	$Minflow_{hub}$	$Maxflow_{spoke}$	Average %
6-20	L	2	18	773.29	201.57	600.34	909.95	5.56%
	M	2	18	773.29	201.57	600.34	909.95	5.56%
	S	6	17	398.82	201.26	299.72	909.95	11.76%
6-25	L	2	23	677.54	150.18	586.50	782.08	4.35%
	M	2	23	677.54	150.18	586.50	782.08	4.35%
	S	6	22	384.33	150.39	259.05	782.08	4.35%
10-20	L	2	18	763.57	162.90	606.15	515.98	0.00%
	M	2	18	763.57	162.90	606.15	515.98	0.00%
	S	6	17	477.39	142.13	280.70	259.18	0.00%
10-25	L	2	23	697.98	132.76	627.36	381.26	0.00%
	M	2	23	685.30	132.76	593.43	381.26	0.00%
	S	6	22	373.34	121.46	277.57	339.87	4.55%
Average				620.50	159.17	493.65	622.47	3.37

6.5 Conclusion

In this chapter, the application of the hub location-routing problem in postal service systems is studied. We analyzed the individual characteristics of this application and we presented a mathematical formulation of the HLRP in postal systems where the pickup and delivery at one node are operated simultaneously. Meantime, the memetic algorithm proposed in Chapter 4 is adapted to solve large postal instance problems. In the computational experiments based on the instances inspired from the AP data set, we analyzed the effects of different parameters on the solutions, including the hub capacity level, the number of potential hubs and the weight of the routing cost in the total cost. In particular, the cost structures of each solution with different parameter values are reported. It is observed that the allocation cost (handling cost) has a larger proportion in the total cost even when the weight of the routing cost increases. In addition, the solutions have no obvious changes when the weight of the routing cost λ equals 1 and 100 on the one hand, as well as $\lambda = 500$ and $\lambda = 1000$ on the other hand, for most of the test instances. Other experiments, implemented on all instances for $\lambda = 100$ and small to medium for $\lambda = 500$ prove that the MA outperforms the solver CPLEX in terms of solution quality and computational time. Finally, the effectiveness and stability of the memetic algorithm is verified again through many computational experiments on all instances generated and $\lambda = 100$. It demonstrated that the MA can provide promising solutions for the HLRP in postal service systems with up to 100 non-hub nodes, although we have not proven their optimality. This is due to the difficulty to obtain good lower bounds. It is worth to notice that there are nearly 60 thousands integer variables and about 260 thousands inequalities in the model for the problem involving 100 non-hub nodes and 10 potential hubs.

7

Conclusion and prospect

This thesis was devoted to proposing and testing new models and solution techniques for the hub location-routing problem (HLRP). Two particular applications have been studied : for the less-than-truckload (LTL) shipments of freight transport providers and for the postal systems. This is a very relevant real problem, especially for transportation network design and the logistics system optimization in an LTL context. A particular attention has been paid to the capacitated single allocation HLRP, which considered a restriction of capacity for the hubs and the vehicles. The aim was to determine the location of hubs, the single allocation of non-hub nodes to hubs, the routing of flows between each origin and destination, as well as the optimal vehicle routing for local collection and delivery tours. In order to solve this problem aiming to minimize the total operating cost of LTL networks, mathematical models and solution methods were proposed, including heuristic and exact algorithms. Numerous computational experiments based on generated instances inspired from the well known AP data set have been conducted to evaluate the performance of the methods in terms of finding feasible solutions, improving lower bounds or computing time. Results obtained by each method showed their individual strengths in particular aspects. In more details, the main research work and theoretical contributions of this thesis are summarized as follows:

(1) A state of the art about the hub location-routing problem has been given based on the current published works, as well as a literature review of closely related problems ,ie the Hub Location Problem (HLP), the Location and Routing Problem (LRP) and the Vehicle Routing Problem (VRP). They summarized individual features, classical mathematical models and main solution methods of each problem. The main goal of this literature survey was to analyze the relations and differences among related problems and to suggest directions for the analysis of the HLRP with respect to the problem constraints, solving methods and application areas.

(2) New models have been proposed for the capacitated single allocation HLRP with separate collection and delivery processes or not, respectively. Two general linear programming formulations are devoted to a resolution by CPLEX for small size instances and to give some insights into the CSAHLRP of general freight transport based on different parameter values. A three-index vehicle flow based formulation has been proposed to implement a branch-and-cut algorithm to solve the HLRP. Another mathematical model focuses on

the HLRP in postal service systems where collection and delivery are operated simultaneously in one vehicle route. It is mainly devoted to the area of solution analysis and evaluations based on many different parameter values.

(3) A memetic algorithm (MA) has been proposed to solve the CSAHLRP for general freight transports as well as postal services systems problems. It combines an iterative local search (ILS) procedure into the framework of a genetic algorithm (GA) to find feasible solutions of good quality in a competitive time. Computational results on small and medium size instances, compared to the CPLEX results prove that the proposed MA outperformed CPLEX in terms of finding best solutions and computational time. In addition, such approach allows us to find good and reliable solutions for large size instances with up to 100 non-hub nodes efficiently.

(4) A branch-and-cut (B&C) algorithm has been developed to solve the CSAHLRP for general freight transport based on some families of valid inequalities, which strengthen the linear programming (LP) relaxation into a cutting plane procedure. In addition, the best solution from the memetic algorithm is used as the initial solution at the root node of the proposed B&C algorithm. Computational results demonstrate a good performance of our B&C algorithm for solving the small and medium instances optimally with up to 30 non-hub nodes and 10 potential hubs. In addition, it gives large improvements on the lower bounds obtained by CPLEX. Furthermore, results based on large instances show that the proposed complete B&C algorithm can find some new best solutions compared to those obtained by the MA.

(5) New sets of instances for the CSAHLRP have been generated based on the Australian Post (AP) data set and a real cost data base of the French Comité National Routier (CNR). Then for each of the proposed method, a large number of computational experiments have been conducted including parameter tuning, results comparisons and performance evaluations.

Following the researches conducted for this thesis, there is still much research to be done on this subject, to extend the models that have been proposed and enforce the solution methods. The following research directions and issues for future work may be proposed:

1. Other variants of the HLRP exist in the literature, such as the p-hub location-routing problem, the multiple allocation hub location-routing problem or the two-layer hub location-routing problem. Our models and proposed methods could be adapted to study these problems.
2. Our branch-and-cut algorithm could be improved to solve exactly larger size instances of the HLRP. In particular new valid inequalities could be developed or the ones derived from other related problems could be adapted, and other branching strategies and separation algorithm could be adapted. It would also be interesting to adapt, our exact solution method for solving the HLRP in postal service systems.
3. A new efficient lower bound could be developed in order to better evaluate the performance of our heuristic methods than it has been done through linear relaxation.
4. It would be challenging to apply our methods to solve real cases from logistics providers, and larger instance sets should be generated including more potential hubs or more non-hub nodes to further test the capability of our methods to solve large, realistic problems.

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COMMUNAUTÉ D'UNIVERSITÉS ET ÉTABLISSEMENTS

Centrale Nantes

Thèse de Doctorat

Mi ZHANG

Le problème de localisation de hubs et tournées combinées pour le transport des marchandises par camions incomplets

Hub location routing problem for less than truckload shipments of freight transport providers

Résumé
Cette thèse porte sur le problème de localisation de hubs et tournées combinées (HLRP : hub location-routing problem), qui permet de détermine le nombre et la localisation des hubs (ou plateformes de consolidation), l'affectation des clients et fournisseurs aux hubs et le schéma type des tournées de collecte et livraison associées à chaque hub. Des modèles mathématiques et des algorithmes d'optimisation ont été développés pour le CSAHLRP (capacitated single alllocation HLRP), qui considère des contraintes de capacité sur les hubs et les véhicules, et d'affectation unique des clients et fournisseurs aux hubs. L'objectif des ces modèles est de minimiser le coût total d'exploitation du réseau de transport. Ces modèles sont appliqués au cas de la messagerie (transport de colis par camions incomplets) dans lequel les collectes et les livraisons sont séparées. Un modèle est également adapté au système postal dans lequel les collectes et livraisons peuvent être regroupées dans une même tournée. Deux méthodes de résolution ont été proposées pour le CSAHLRP : un algorithme mémétique et un algorithme de branchement et coupes. De nombreuses expérimentations numériques montrent la performance des méthodes développées et permettent d'évaluer les effets des différents paramètres sur les solutions. La performance des algorithmes développés a été également comparée avec celle d'un solveur commercial.

Mots clés
Problème de localisation de hubs et tournées, transport de marchandises, algorithme mémétique, algorithme de branchement et coupes, les systèmes postaux.

Abstract
This thesis studies the hub location-routing problem (HLRP), which determines the number and location of hub facilities concentrating flows and through which flows are to be routed from origins to destinations, together with the design of both collection and delivery routes associated to each hub. The state of the art shows that only very few works directly address the HLRP. Mathematical models and optimization algorithms are developed for the capacitated single allocation hub location-routing problem (CSAHLRP), which considers capacitated hubs, capacitated vehicle and single allocation of non-hub nodes to hubs to minimize the total operating cost of LTL logistics network. These models are applied to the case of less-than-truckload shipments of freight transport providers with separated collections and deliveries. One model is also adapted for the postal systems with simultaneous collections and deliveries. A metaheuristic and an exact method (Branch and cut algorithm) are proposed to solve the CSAHLRP. Extensive computational experiments validate our models, evaluate the effects of different parameters on solutions and show the performance of each proposed method. The performances of the algorithms are also compared to that of a commercial solver.

Key Words
Hub location-routing problem, less-than-truckload shipments, memetic algorithm, branch-and-cut algorithm, postal service systems.