

# Scientific Program Schedule

## Tuesday

08:30 - 08:45

### Welcome Session

*José Fernando Oliveira, Adriana Cristina Cherri*

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08:45 - 09:45

### Keynote 1

*Chair: Juan J. Lastra-Díaz*

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- Mathematical models for 2D irregular cutting problems  
*Franklina M. B. Toledo*

10:15 - 11:55

### Session T2

*Chair: Antonio Martinez Sykora*

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- T2.1* – A Measure-Theoretic View on Feasibility in Continuous Packing  
*Sergiy Yakovlev, Lyudmyla Kirichenko, Olha Matsyi*
- T2.2* – Modeling and linearization approaches for the quadratic bin packing problem  
*Fatih Burak Akcay, Maxence Delorme, Renata Sotirov*
- T2.3* – Models and Decomposition methods for the Balanced Vector Bin Packing Problems with configurable items  
*Antonio Martinez Sykora, Carlos Lamas-Fernandez*
- T2.4* – Solving Hard Instances from Knapsack and Bounded Knapsack Problems: A new state-of-the-art solver  
*Renan Fernando Franco da Silva, Thiago Alves de Queiroz, Rafael Crivellari Saliba Schouery*

13:30 - 15:10

### Session T3

*Chair: Khadija Alice HADJ SALEM*

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- T3.1* – A new linear integer model for two-dimensional bin packing problem and customer order spread  
*Daniel José Schulmeister, Kelly Cristina Poldi*
- T3.2* – Improving Stone Slab Cutting Patterns through Post-Positioning Compaction with Vacuum Lifting Systems Usage  
*José Fernando Oliveira, Tiago Silveira, Elsa Silva, Maria Antónia Carravilla, Rui Guerreiro, Fernando Sousa*
- T3.3* – Ornamental Stone Industry: Rectangular Cutting Problems with Overcuts and a Lifter-Based Approach  
*Walison Oliveira, Tiago Silveira, Elsa Silva, Marina Andretta, Rui Guerreiro, Fernando Sousa, Maria Antónia Carravilla, José Fernando Oliveira*
- T3.4* – Preliminary results from instance space analysis for 2D guillotine cutting stock mathematical models  
*Khadija Alice HADJ SALEM, Tony Wauters, Alysson Costa, José Fernando Oliveira*

15:40 - 16:55

### Session T4

*Chair: Carlos Diego Rodrigues*

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- T4.1* – Heuristics for an integrated bin-packing and scheduling problems  
*Gustavo Alencar Rolim, Silvio Alexandre de Araujo*
  - T4.2* – The one-dimensional cutting stock problem with sequence-dependent setups  
*Eduardo silva, Antonio Chaves, Silvio Alexandre de Araujo*
  - T4.3* – The two-dimensional packing and scheduling problem with total tardiness minimization
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*Carlos Diego Rodrigues, Bruno Prata*

## Wednesday

08:30 - 09:30

**Keynote 2**

*Chair: José Fernando Oliveira*

– Irregular Packing Heuristics: past, present and future.  
*Tony Wauters*

10:00 - 11:40

**Session W2**

*Chair: Adriana Cherri*

*W2.1* – 3D Irregular Cutting and Packing Problem from Tokamak Reactor Decommissioning

*Yifu Wei, Julia Bennell, Hamish Carr, Ipek Caliskanelli*

*W2.2* – Advancing Sparrow 3D for High Resolution Irregular Packing: Bottleneck Analysis and Continuous Orientation Optimization

*Jan Martens, Jonas Tollenaere, Tony Wauters*

*W2.3* – Exact method for the nesting problem with guillotine cuts

*Adriana Cherri, Luiz Henrique Cherri, José Fernando Oliveira*

*W2.4* – Using Discrete Fast Fourier Transform for Efficient Voxel-Based Collision Detection

*Alessio La Greca, Tony Wauters*

13:15 - 14:55

**Session W3**

*Chair: J. M. Valerio de Carvalho*

*W3.1* – An Integrated Two-Echelon Location-Routing Model for Urban Solid Waste Collection with Ecopoints  
*Danylo Viana, Adriana Cherri*

*W3.2* – Formulation for the Bin Sizing Problem in E-commerce Distribution Centers

*Gabriel González, Mariá Nascimento*

*W3.3* – Prisoner-to-compartment assignment feasibility in the Prisoner Transportation Problem

*J. M. Valerio de Carvalho, Éverton Santi, Luciano Ferreira*

*W3.4* – Three-dimensional packing feasibility in an integrated vehicle routing and loading problem: formulations and logic-based Benders decomposition algorithms

*Douglas Nogueira Nascimento, Franklina Toledo, Leandro Coelho*

15:25 - 16:15

**Session W4**

*Chair: Thiago Alves de Queiroz*

*W4.1* – An Effective Tree Search Heuristic for the Container Loading Problem

*Thiago Alves de Queiroz, Lucas Guesser, Flávio Keidi Miyazawa*

*W4.2* – Approaches to applying Interval Graphs in Local Search to the Three-Dimensional Bin Packing Problem with rotation.

*Daniel Maia, Denis Borenstein*

16:15 - 16:30

**Closing session**

*Tony Wauters*

## Thursday

Morning

**ESICUP General Matters Meeting**

# Abstracts

Keynote 1

## Mathematical models for 2D irregular cutting problems

Franklina Toledo

*Universidade de São Paulo – Campus São Carlos, Brazil*

The irregular cutting problem consists of packing a set of pieces into an object. The object may be regular or irregular, and at least one piece must have irregular shape. We have two main constraints associated with the problem: i) the pieces must be contained entirely within the object, and ii) they cannot overlap. The most frequent objective is to minimize the material waste. Such problems appear in many industrial sectors, such as the garment, footwear, metallurgy and glass industries. In this talk, we present the problem, some of its applications, and some mathematical models developed to address 2D irregular cutting problems.

**Keywords:** Irregular packing, Mathematical Models

Keynote 2

## Irregular Packing Heuristics: past, present and future.

Tony Wauters

*KU Leuven, Belgium*

Irregular cutting and packing problems, often referred to as nesting problems, constitute a fundamental class of combinatorial optimization challenges with wide-ranging industrial applications, including manufacturing, logistics, and material processing. Characterized by the need to efficiently place complex, non-standard shapes within constrained spaces, these problems are inherently NP-hard and have historically resisted exact solution approaches at scale. This presentation provides a structured overview of the evolution of heuristic methods for irregular packing, tracing their development from early constructive and local search techniques to more sophisticated metaheuristic and hybrid frameworks. The discussion begins with a historical perspective, highlighting how classical heuristics such as bottom-left placement, no-fit polygon (NFP) representations, and early improvement strategies laid the groundwork for practical nesting solutions. It then examines the present state of the field, focusing on recent advancements including metaheuristics. Particular attention is given to emerging trends such as improved geometric representations, and scalable algorithms capable of handling real-world complexity. Finally, the presentation outlines promising directions for future research.

**Keywords:** Irregular packing, Heuristics

Session T2.1

## A Measure-Theoretic View on Feasibility in Continuous Packing

Sergiy Yakovlev, Lyudmyla Kirichenko, Olha Matsyi

*Poland*

A global measure-based approach to feasibility modeling in continuous two-dimensional packing problems is proposed. Instead of classical systems of local non-overlap and containment constraints, feasibility is characterized by a single scalar quantity called the configuration area, defined as the area of intersection between the container and the union of all placed objects. It is shown that this global condition is equivalent to classical geometric feasibility up to zero-measure boundary contacts, independently of the number and shapes of objects. Systems of pairwise geometric constraints can be replaced by a single global feasibility condition that does not depend on the combinatorial structure of object interactions. The proposed formulation provides a compact feasibility representation and significantly simplifies the structural complexity of packing models. From an optimization perspective, feasibility is encoded through one scalar quantity that can be naturally incorporated into penalty-based formulations and hybrid optimization schemes. The approach is geometry-agnostic and can be applied to heterogeneous objects, irregular shapes, and containers with internal forbidden regions without introducing additional shape-specific constraints. This study was supported by the National Science Center of Poland (project no.2023/05/Y/ST6/00263) and the Ministry of Education and Science of Ukraine (project no.3-41-262).

**Keywords:** Cutting and packing

Session T2.2

**Modeling and linearization approaches for the quadratic bin packing problem**Fatih Burak Akcay, Maxence Delorme, Renata Sotirov  
*Tilburg University, Netherlands*

The Quadratic Bin Packing Problem (QBPP) is a generalization of the classical Bin Packing Problem in which pairwise interaction values (which may be negative, positive or zero) between items are considered. The objective is to minimize the sum of bin usage costs and pairwise interaction values among items placed in the same bin.

We present MILP formulations for the QBPP obtained through different linearization strategies and compare the formulations with respect to model size, LP relaxation strength, and computational performance. We report preliminary computational results on a set of instances.

**Keywords:** Bin packing, Combinatorial optimization, Cutting and packing

Session T2.3

**Models and Decomposition methods for the Balanced Vector Bin Packing Problems with configurable items**Antonio Martinez Sykora, Carlos Lamas-Fernandez  
*University of Southampton, United Kingdom*

The one-dimensional Bin Packing Problem (BPP) is one of the most studied combinatorial optimisation problems, with deep connections to scheduling, resource allocation, and cutting-stock applications. An additional layer of complexity arises in applications requiring balanced or fair load distribution. In contrast to classical objectives that focus solely on minimising the number of bins, balanced bin packing introduces an equity-based objective: bins of the same or different types should be filled such that the load distribution satisfies fairness, regularity, or deviation-minimisation criteria. Balancing is particularly relevant in applications like helicopter loading problems, where the weight of the load has a direct impact on the flying range, requiring a good balance to maximise the flying range of a given mission; or the workload scheduling for parallel computing, where high utilisation imbalance leads to performance bottlenecks. In some of these applications, it is important to consider also the extension where items are not characterised by a single, fixed size for each dimension (or resource requirement), but instead admit a set of configurations (also called incarnations). We introduce a unified and flexible mathematical formulation that integrates heterogeneity and balancing requirements, exploiting structural properties of the problem that influence solvability and study its decomposition potential.

**Keywords:** Combinatorial optimization, Bin packing

Session T2.4

**Solving Hard Instances from Knapsack and Bounded Knapsack Problems: A new state-of-the-art solver**Renan Fernando Franco da Silva, Thiago Alves de Queiroz, Rafael Crivellari Saliba Schouery  
*University of Campinas, Brazil*

The Knapsack and Bounded Knapsack problems are classical NP-hard problems with many practical applications. Although COMBO and BOUKNAP were introduced over 25 years ago, they remain state-of-the-art due to strong implementations and bounding techniques. We present a new solver for both problems that builds on core-based dynamic programming and surrogate relaxation while introducing multiplicity reduction, refined dominance rules, and a new divisibility bound. Our solver preserves COMBO's near-linear behavior on several instances and achieves significant speedups on harder benchmarks.

**Keywords:** Cutting and packing, Combinatorial optimization

Session T3.1

**A new linear integer model for two-dimensional bin packing problem and customer order spread**Daniel José Schulmeister, Kelly Cristina Poldi  
*Unicamp/IMECC, Brazil*

This research proposes a novel model for the two-dimensional bin packing problem with minimization of customer order spread. The problem focuses on optimizing order processing costs and the amount bins used, typical objectives of contexts operating under a make-to-order policy. To address this problem, an integer linear programming

model based on cutting patterns with up to three guillotine stages is developed. Computational experiments demonstrate that the proposed approach outperforms previously reported methods in both solution quality and computational efficiency on benchmark instances.

**Keywords:** Bin packing, Cutting and packing, Combinatorial optimization

Session T3.2

### **Improving Stone Slab Cutting Patterns through Post-Positioning Compaction with Vacuum Lifting Systems Usage**

José Fernando Oliveira, Tiago Silveira, Elsa Silva, Maria Antónia Carravilla, Rui Guerreiro, Fernando Sousa

*Universidade do Porto, Portugal ; Federal University of Alfenas, Brazil*

This work focuses on improving cutting patterns in stone slab processing by applying a compaction heuristic after the positioning phase. Initial layouts are generated by a placement heuristic which explicitly considers overcut constraints caused by the advance of the diamond blade, thereby ensuring geometric feasibility during cutting. After positioning, a compaction phase is performed with the specific purpose of bringing pieces closer together in order to accommodate additional items within the same slab. Rather than redefining the entire layout, the method reorganizes already positioned pieces, reducing unused regions and creating feasible space for inserting new pieces. In industrial settings, intermediate repositioning of partially cut pieces may be required and is executed using vacuum lifting systems. Since lifting operations increase processing time, compaction seeks to improve material utilization while keeping the number of required lifter operations controlled. The result of compaction is instance-dependent, since geometric feasibility is governed by overcut constraints, admissible cutting structures, and the spatial configuration of the layout. Computational results indicate that, when feasible, post-positioning compaction can significantly reduce waste by enabling the insertion of additional pieces, while maintaining operational viability.

**Keywords:** Cutting and packing

Session T3.3

### **Ornamental Stone Industry: Rectangular Cutting Problems with Overcuts and a Lifter-Based Approach**

Walison Oliveira, Tiago Silveira, Elsa Silva, Marina Andretta, Rui Guerreiro, Fernando Sousa, Maria Antónia Carravilla, José Fernando Oliveira

*University of São Paulo, Brazil ; Universidade do Porto, Portugal*

This work addresses the Two-Dimensional Single Knapsack Problem (2D-SKP) in the ornamental stone industry, considering overcuts and the use of vacuum lifters via floating-cuts. We propose a mixed-integer optimization model to allocate items on a rectangular plate, maximizing occupied area, penalizing the use of vacuum lifters, and respecting geometric, non-overlapping, and guillotine-cut overcut constraints. Experiments based on real world instances indicate the model is promising for industrial applications, achieving effective utilization of allocated area.

**Keywords:** Cutting and packing

Session T3.4

### **Preliminary results from instance space analysis for 2D guillotine cutting stock mathematical models**

Khadija Alice HADJ SALEM, Tony Wauters, Alysson Costa, José Fernando Oliveira  
*KU Leuven, Belgium ; University of Melbourne, Australia ; Universidade do Porto, Portugal*

The two-dimensional cutting stock problem (2D-CSP) is a well-studied NP-hard optimization problem belonging to the cutting and packing (C&P) family of problems. It involves cutting a set of smaller rectangular items from a set of larger available rectangular plates. The objective is to fulfill a given demand while minimizing the total amount of raw material used to reduce waste. In the 2D-CSP considered here, the cutting patterns are restricted to non-exact two-stage guillotine cuts. In this work, we apply, for the first time, instance space analysis (ISA) to study mathematical models for the 2D-CSP. We focus on four model categories: pattern-based, strip-based, item-based, and arc-flow models. To evaluate the performance of these models, including existing and newly

adapted formulations, we consider both classical and newly generated instances. We use the time required by the black-box MIP solver Gurobi to find a proven optimal solution for each model as a performance metric. We analyze model performance with respect to instance features, and present and discuss preliminary results.

**Keywords:** Combinatorial optimization, Cutting and packing, Cutting stock

Session T4.1

### Heuristics for an integrated bin-packing and scheduling problems

Gustavo Alencar Rolim, Silvio Alexandre de Araujo  
*UNESP, Brazil*

This study investigates the one-dimensional Bin Packing Problem with Variable Pattern Processing Time (1BP-VPT), an integrated bin packing and scheduling problem in which packing decisions influence item completion times. We develop three heuristics: a polynomial-time constructive procedure, a Variable Neighborhood Descent (VND) local search, and a Large Neighborhood Search (LNS) inspired by methods for the Batch Scheduling Problem (BSP). In addition, we revisit the state-of-the-art Sequential Value Correction (SVC) heuristic, using iterated racing for automatic algorithm configuration to obtain a more effective parameter setting, and propose a hybrid that combines SVC with VND. Extensive computational experiments reveal several new best known solutions on challenging benchmarks. Finally, we examine the advantages, limitations, and trade-offs of the heuristics approaches.

**Keywords:** Bin packing

Session T4.2

### The one-dimensional cutting stock problem with sequence-dependent setups

Eduardo silva, Antonio Chaves, Silvio Alexandre de Araujo  
*Universidade Federal de Uberlândia, Brazil*

This study addresses the Cutting Stock Problem (CSP) with sequence-dependent setups on cutting patterns. We propose a novel assignment-based formulation to minimize distinct patterns and sequence setups. A Random-Key Optimizer (RKO) metaheuristic is proposed to solve large instances, utilizing a decoder for pattern construction and sequencing. The RKO is compared against a column-generation heuristic on 360 benchmark instances using Gurobi, the RKO approach proved superior on most instances. It delivers high-quality solutions for large instances within reasonable computational times.

**Keywords:** Combinatorial optimization, Cutting stock

Session T4.3

### The two-dimensional packing and scheduling problem with total tardiness minimization

Carlos Diego Rodrigues, Bruno Prata  
*DEMA / UFC, Brazil*

Two-dimensional packing problems often seek to minimize either the number of bins used or the waste resulting from cutting patterns. However, real-world applications introduce new criteria and constraints that play a key role in obtaining high-quality solutions for practical scenarios. This work presents a new variant of the Two-Dimensional Guillotine Cutting Problem that incorporates due dates for each item. The objective is therefore to minimize the total tardiness. To address this problem, we propose a mixed-integer linear programming model along with several constructive heuristics.

**Keywords:** Combinatorial optimization, Cutting and packing, Cutting stock

Session W2.1

### 3D Irregular Cutting and Packing Problem from Tokamak Reactor Decommissioning

Yifu Wei, Julia Bennell, Hamish Carr, Ipek Caliskanelli  
*University of Leeds, United Kingdom*

The waste from Tokamak fusion reactor needs to be cut into legal pieces and packed into containers remotely by robotic arms, then, stored in disposal facilities. In this process, an optimised solution of cut and bin packing, considering practical constraints, has potential to reduce the overall cost by millions. In this research, we propose two patterns of cutting and bin packing algorithm, combining with GPU-empowered algorithm, advanced machine learning, heuristics and metaheuristics. These algorithms efficiently solve the problem and support the decision-making in real world applications.

**Keywords:** Cutting and packing, Combinatorial optimization, Bin packing

Session W2.2

### **Advancing Sparrow 3D for High Resolution Irregular Packing: Bottleneck Analysis and Continuous Orientation Optimization**

Jan Martens, Jonas Tollenaere, Tony Wauters

*KU Leuven, Belgium*

The 3D irregular strip packing problem involves positioning objects within a container of fixed width and depth to minimize total height. Despite practical applications in additive manufacturing and logistics, 3D irregular packing is underrepresented in literature compared to 2D nesting, leaving a gap in scalable methodologies. Recent efforts include the Sparrow3D algorithm, adapted from the state of the art 2D Sparrow heuristic. While Sparrow3D performs well on benchmarks, scalability is hindered by geometric complexity and large item quantities. We identify two primary bottlenecks. First, increasing item counts significantly slows collision detection. Second, high geometric complexity reduces overlap quantification speed and makes it difficult for local search to receive effective guidance. To address these limitations, we propose advancements to the Sparrow3D framework. We introduce a restructured collision detection engine to handle intricate 3D meshes more efficiently, alongside refined heuristic guidance for complex geometry. Furthermore, we depart from traditional discrete orientation sets by integrating a mechanism for continuous 3D rotations. While this expands the search space, it allows the heuristic to exploit geometric features that discrete methods overlook, helping the search escape local optima. Our efforts aim to retain accuracy while reducing the computational load for large scale industrial instances.

**Keywords:** Cutting and packing, Nesting problems, Combinatorial optimization

Session W2.3

### **Exact method for the nesting problem with guillotine cuts**

Adriana Cherri, Luiz Henrique Cherri, José Fernando Oliveira

*Sao Paulo State University - UNESP, Brazil ; Universidade do Porto, Portugal*

This study proposes the first exact method for the strip packing problem with irregular pieces using guillotine cut constraints. Based on the Dotted Board Model, we have developed a branch-and-cut algorithm that eliminates infeasible patterns using valid cuts derived from D-functions. The pieces are convex, with discrete rotations, and the final cut is obtained by a valid sequence of guillotine cuts. We used unitary discretisation and a mathematical model that contain non-overlap, demand and length limit constraints. The combinatorial explosion was treated with clique coverage in the conflict

**Keywords:** Nesting problems, Combinatorial optimization, Cutting and packing

Session W2.4

### **Using Discrete Fast Fourier Transform for Efficient Voxel-Based Collision Detection**

Alessio La Greca, Tony Wauters

*KU Leuven, Belgium*

3D Irregular Cutting & Packing problems are notoriously difficult to confront because of two interrelated challenges they pose: an optimization one, where a good placement of the items must be found, and a geometric one, dealing with detecting collisions between items placed in a container. In fact, being able to efficiently determine whether two items overlap, and by how much, can be crucial for the development of a good optimization heuristic. In recent years, voxels have been proposed as a powerful tool to approximate the item's irregularities and allow for simpler collision detection mechanisms. Still, performing the correlation between two voxel grids – required to check the amount of overlap between them – has a quadratic complexity in the size of the input if carried out

naively. Recent studies, however, proved how the Discrete Fast Fourier Transform can achieve the same result with significant speedup. In this talk, we will give an intuitive explanation of how it works in the context of voxel-based 3D packing, providing some computational results that demonstrate the performance boosts it is capable of achieving.

**Keywords:** Cutting and packing, Combinatorial optimization, Nesting problems

Session W3.1

## An Integrated Two-Echelon Location-Routing Model for Urban Solid Waste Collection with Ecopoints

Danylo Viana, Adriana Cherri  
*São Paulo State University (UNESP), Brazil*

This study proposes a mixed-integer linear programming model formulated as a Two-Echelon Location-Routing Problem (2E-LRP), integrating the decisions of ecopoint location, allocation of illegal dumping points, and routing of specialized fleets for urban solid waste management in Natal, Brazil. Ecopoints are voluntary drop-off units that serve as intermediate facilities for the reception, sorting, and temporary storage of waste generated by the population, enabling proper disposal in place of illegal dumping points. Ecopoints structure the operation in two levels: at the first, generators forward waste to intermediate units; at the second, specialized fleets transport it to licensed final destinations via closed routes by waste type. The model considers five waste types (gypsum, CDW, rubble, green waste, and recyclables). The solution obtained with CPLEX 22.1 (gap of 5.0% in 1,048 minutes) installs 6 of 23 technically suitable candidate sites, fully covers all 600 illegal dumping points at distances between 0.06 km and 17.84 km, processes 426.1 m<sup>3</sup>/week over 1,358.60 km traveled, and yields a weekly cost of R\$ 31,975.04 (57.5% operational and 42.5% logistics).

**Keywords:** Combinatorial optimization

Session W3.2

## Formulation for the Bin Sizing Problem in E-commerce Distribution Centers

Gabriel González, Mariá Nascimento  
*Instituto Tecnológico de Aeronáutica (ITA), Spain ; Brazil*

We address the bin sizing problem in e-commerce distribution centers: selecting bin types from a catalog and assigning products to minimize total volume. Operational constraints include product type limits per bin, quantity restrictions, and stacking rules ensuring item accessibility. We exploit these constraints to reduce the 3D packing problem to 1D block positioning, where the configurations are optimally determined. The resulting MILP formulation significantly reduces complexity. Experiments with real-world data demonstrate practical applicability.

**Keywords:** Bin packing, Combinatorial optimization

Session W3.3

## Prisoner-to-compartment assignment feasibility in the Prisoner Transportation Problem

J. M. Valerio de Carvalho, Éverton Santi, Luciano Ferreira  
*Universidade do Minho, Portugal ; Brazil*

The Prisoner Transportation Problem, originally formulated by Christiaens et al. (2020), is an NP-hard combinatorial problem. The objective is to service a set of transport requests in which prisoners must be transferred from their respective detention facilities to designated destinations and subsequently returned. Given a homogeneous fleet, it consists of assigning requests to vehicles to serve all requests, respecting the problem constraints such as route duration, capacity, ride time, time windows, and multi-compartment assignment of conflicting prisoners, in order to optimize a given minimum-cost or minimum-duration objective function. The PTP extends the classical Dial-a-Ride Problem by introducing the critical requirement of compartmentalized prisoner allocation within vehicles.

A route is only feasible if the prisoner-to-compartment assignment obeys all the compartment conflicts, which indicate that two or more prisoners cannot simultaneously occupy the same compartment, and vehicle conflicts, which denote that they may not be transported in the same vehicle. We review the PAH feasibility check heuristic, by Christiaens et al. (2020), show that there may be global inconsistency in solutions that are independently consistent at each pick-up vertex of the route, and present integer programming models.

**Keywords:** Bin packing

Session W3.4

### **Three-dimensional packing feasibility in an integrated vehicle routing and loading problem: formulations and logic-based Benders decomposition algorithms**

Douglas Nogueira Nascimento, Franklina Toledo, Leandro Coelho  
*Universidade de São Paulo (USP), Brazil; Canada*

This work addresses the Three-Dimensional Loading Capacitated Vehicle Routing Problem (3L-CVRP) with time windows. We propose two integrated formulations based on continuous box positioning: a three-index model that can handle both homogeneous and heterogeneous fleets, and a two-index model for homogeneous fleets that avoids vehicle-indexed routing variables. Preprocessing rules are applied to tighten time windows and discard unreachable arcs and reduce non-overlap variables based on container and box dimensions. To solve the problem, we develop logic-based Benders decomposition (LBBD) algorithms in which the routing master is iteratively strengthened with feasibility cuts from route-level packing verification. Packing feasibility is checked using a two-step procedure: a multi-trial greedy constructive heuristic that combines different box orderings with distance- and compactness-based placement criteria, and, if needed, an exact packing feasibility model. The heuristic dynamically maintains a set of candidate placement points and stops as soon as a feasible loading is found. We consider both fixed-vertical and full six-orientation loading settings. Computational experiments on benchmark instances yielded optimal solutions in 46 of 124 experiments (including 13 first-proven optimals) and 33 new best-known solutions. This study was partly funded by the São Paulo Research Foundation (FAPESP), Brazil, grants #2023/16405-1 and #2024/14591-5.

**Keywords:** Cutting and packing, Combinatorial optimization

Session W4.1

### **An Effective Tree Search Heuristic for the Container Loading Problem**

Thiago Alves de Queiroz, Lucas Guessier, Flávio Keidi Miyazawa  
*Federal University of Goiás, Brazil*

This study addresses the 3D single-container loading problem. We propose a tree search-based heuristic built on a beam search strategy that features cache-friendly structures, a memoization table, and an adaptive two-stage fitness function. Parameters were tuned via irace. Testing on 1,600 instances showed 95.91% volume usage in 30s, which is a +0.59% gain over the state-of-the-art. In just 5s, it reached 95.40%, still outperforming prior methods. Code and data are public to ensure reproducibility.

**Keywords:** Cutting and packing, Combinatorial optimization

Session W4.2

### **Approaches to applying Interval Graphs in Local Search to the Three-Dimensional Bin Packing Problem with rotation.**

Daniel Maia, Denis Borenstein  
*UFRGS, Brazil*

Performing Local Search for the Three-Dimensional Bin Packing Problem (3D-BPP) with rotations is challenging due to its large solution space. Interval Graphs (IG) offer an effective way to explore this space providing an analytical view of solution and representing multiple solutions simultaneously through packing classes.

The proposed approach removes one bin from a good solution and attempts to reinsert its items into the remaining bins. The insertion process is guided by the property that the total weight of the vertices in any independent set must not exceed the bin dimension along the axis represented by the graph. This constraint significantly tends to induce the secondary requirement that at least one graph with no edges between any pair of vertices. Finally, IGs are verified using incremental PQ-tree reductions, so that each inserted item introduces only additional constraints without recomputing the structure from scratch.

Unlike previous methods that start from the second constraint, which allows many combinations, this approach first exploits the weight-based constraint to better guide the Local Search. This strategy has the potential to enable the use of Interval Graphs in Local Search in 3D-BPP with rotation.

**Keywords:** Bin packing, Combinatorial optimization