Dynamic Programming and Economics

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Households' problem:

$$Max\ E\sum_{t=0}^{\infty}\beta^t\ \frac{(C_t^{\alpha}L_t^{1-\alpha})^{1-\sigma}-1}{1-\sigma}$$

subject to:

$$C_t + I_t = z_t\, K_t^{\theta}\, N_t^{1-\theta} = r_t\, K_t + w_t\, N_t$$

$$L_t + N_t = 1$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

$$z_{t+1} = \rho\, z_t + \varepsilon_t$$

ε's ~ Normal Probability Distribution

Dynamic contracts (mechanism design) with private information (not directly observable by the designer):

1. **Where is it useful?**

   a) Optimal taxation (private information about productivity or preferences)

   b) Health insurance (private information about one's health)

   c) Repeated auctions (private information about values)

   d) Managerial compensation (private information about manager's cost of effort)

   e) Firm financing (firms have private information about productivity or costs)

2. Why is it hard to compute?

a) *Revelation principle*: without loss of generality design contracts where individuals prefer to report about their types truthfully.

b) The constraints (*incentive constraints*) thus require that the expected lifetime utility from telling the truth must be greater than the expected lifetime utility from any other reporting strategy.

c) To find the optimal contract, one in general needs to introduce additional *co-state* variables (Kydland and Prescott (1980)).

d) In case of private information, one would need to introduce a *very large* number of co-state variables in general (Fernandes and Phelan (2000)).

i. Let $\theta \epsilon \Theta$ be the private information variable (agent's type). Agent that is of type $\theta \epsilon \Theta$ can report any $\hat{\theta} \epsilon \Theta$.

ii. To make sure that the incentive constraints hold, one needs to make sure that such a reporting strategy is no better than reporting truthfully.

iii. One needs to include one state variable for the truthteller (*promised utility*), and one co-state variable for each alternative type (*threat utility*).

iv. If both θ_1–type agent and θ_2–type agent report , then one needs to include the lifetime utility of the θ_1–type agent as a state, and a lifetime utility of a θ_2–type agent that reports θ_1 as a co-state.

v. Thus, if there are *N* types of agents, one includes *N – 1* co-state variables. If there is a continuum of types, one includes a function as a state variable.

vi. Curse of dimensionality!

3. **Are there any special cases when the problem has a simpler solution?**

    a) Yes, if the shocks are i.i.d. Then all the co-state variables turn out to be *identical* to the promised utility.

    b) The assumption of i.i.d. shocks is a very limiting assumption (are your earnings i.i.d.?).

4. **How to solve the problem if the shocks are Markov?**

   a) Do we really need to include all the co-state variables?

   b) E.g. if a θ_2–type agent is never thinking of reporting θ_1 then the lifetime utility of a θ_2–type agent that reports θ_1 is not needed as a co-state variable.

   c) Things get simplified with a continuum of shocks: in many cases it is possible to conjecture that the only reports that the agent will consider are the ones that are marginally different from the true type (*First – order approach*).

d) With the first-order approach, one only needs to include one co-state variable corresponding to the marginal change in agent’s type (Kapička (2011)).

e) One potential drawback: In most cases the conjecture needs to be verified only numerically ex-post. One needs to verify the incentive constraints for all $(\hat{\theta}, \theta) \epsilon \Theta^2$ combinations, for all points in the state space.

f) However, in many applications this is the only way to go forward.

References

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