28th European Conference on Operational Research

Algorithmic Future of Operations Research

Yurii Nesterov (CORE/INMA, UCL)

July 3, 2016

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Observations:

- Traditional methods are not efficient in Huge Dimension
- We need to learn the way Optimization is incorporated in Nature.

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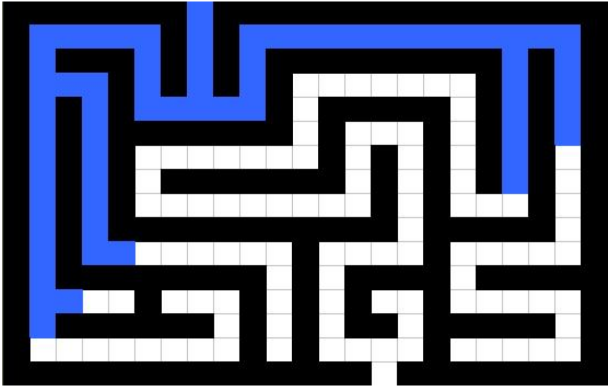

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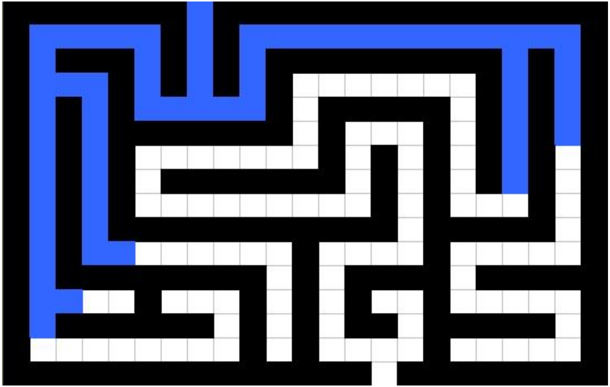

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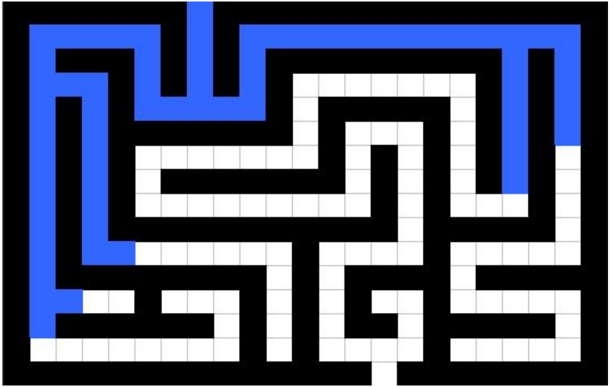

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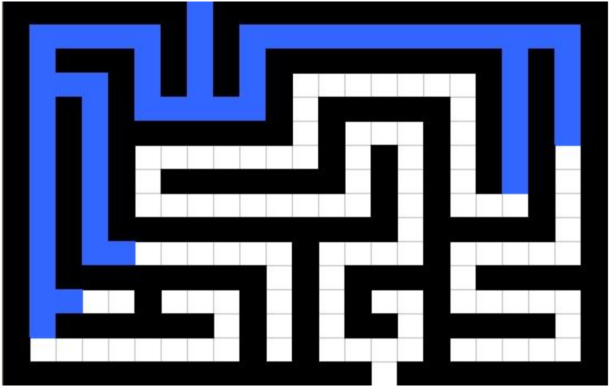

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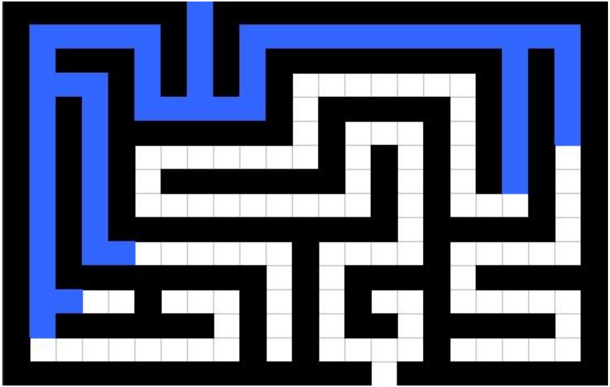

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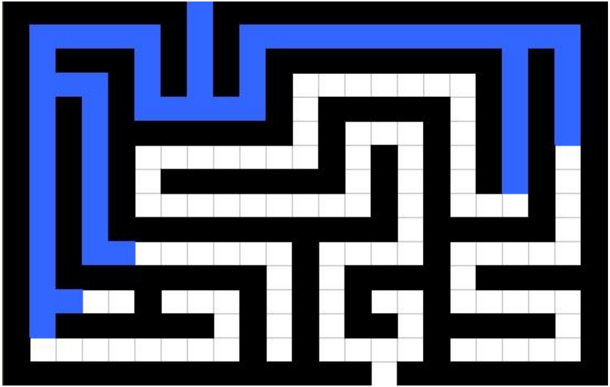

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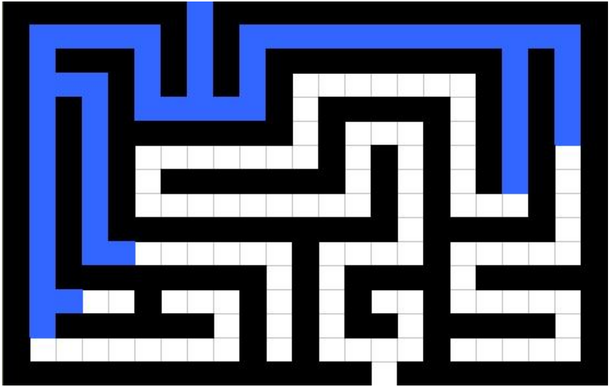

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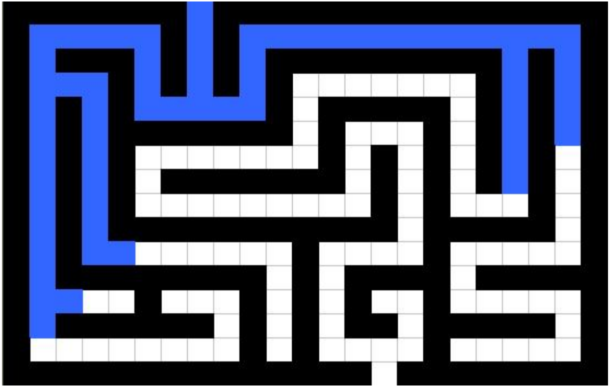

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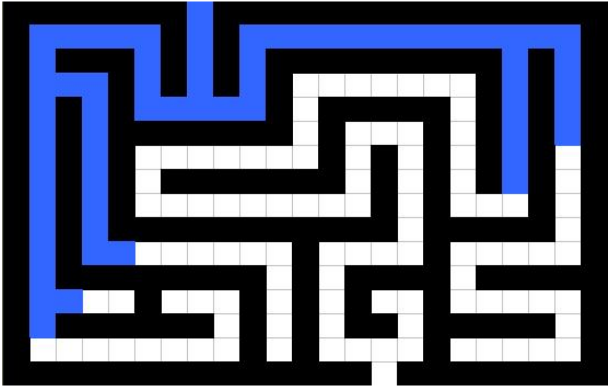

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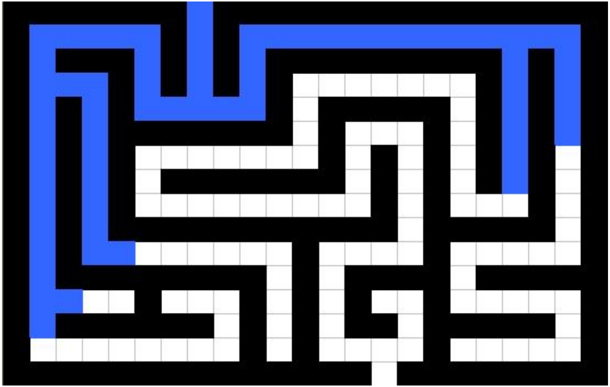

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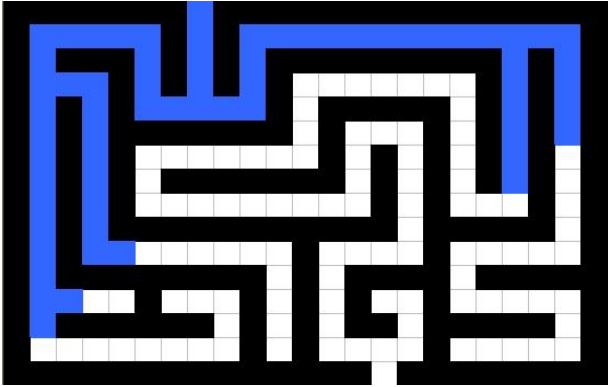

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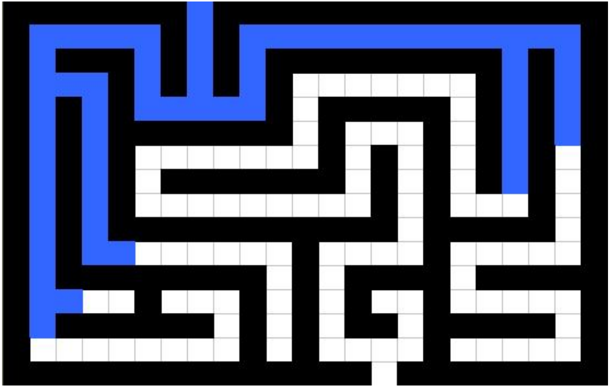

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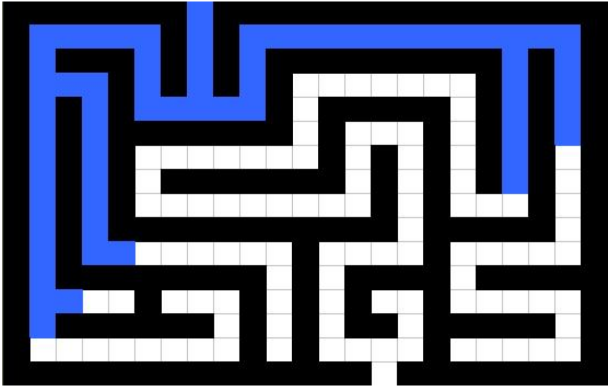

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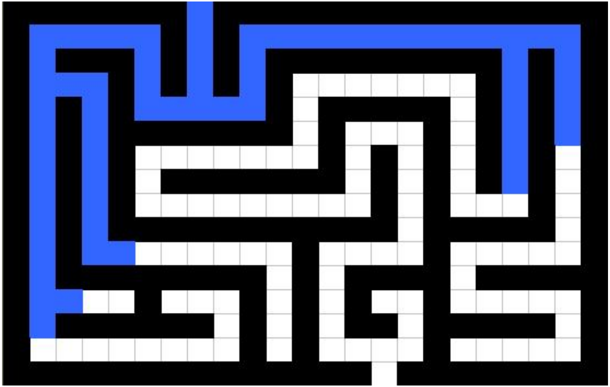

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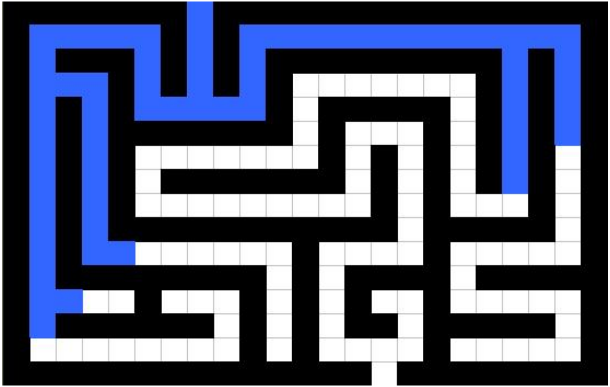

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NB: Continuous time $\Rightarrow$ Discrete time $\Rightarrow$ Polynomial complexity.

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NB: 1) Many naturally growing systems have hidden optimality.

2) They are created by *unintentional* optimization.

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Examples: Nature, Social life, etc.

Conclusion

Conclusion

... Many things to do ...

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Thank you for your attention!