A Branch-and-Price Algorithm for Train Stop Scheduling Problem

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1 Introduction and Problem Formulation

Train stoppage planning is a key strategic decision in passenger rail operations and is commonly referred to as the Train Stop Scheduling Problem (TSSP) [1]. The TSSP considered in this study involves multiple trains operating on a single, non-branching railway corridor, where passenger origin–destination (OD) demands must be assigned to trains subject to capacity constraints, while minimizing the total number of train stops.

The railway corridor is modeled as an ordered set of stations $N = \{1, \ldots, n\}$ connected sequentially, with stations 1 and n denoting the terminals. Passenger demand is specified for all OD pairs $O = \{(i, j) : i, j \in N,\, i < j\}$ and service classes $L = \{1, \ldots, l\}$. A set of trains $T = \{1, \ldots, m\}$ operates on the corridor, where each train $t \in T$ has class-dependent capacity $C^{r,t}$ for $r \in L$. Following [1], the TSSP is formulated as the following mixed-integer linear program:

$$\text{Minimize} \sum_{t \in T} \sum_{i \in N} y_i^t \tag{1}$$

$$\text{Subject to:} \sum_{\substack{i' \in N: \\ i' \leq i}} \sum_{\substack{j \in N: \\ j > i}} x_{i',j}^{r,t} \;\leq\; C^{r,t} \qquad \forall i \in N \setminus \{n\}, t \in T, r \in L \tag{2}$$

$$\sum_{t \in T} x_{i,j}^{r,t} \;\geq\; d_{i,j}^r \qquad \forall (i, j) \in O, r \in L \tag{3}$$

$$x_{i,j}^{r,t} \;\leq\; M_{i,j}^{r,t}\, y_i^t \qquad \forall (i, j) \in O, t \in T, r \in L \tag{4}$$

$$x_{i,j}^{r,t} \;\leq\; M_{i,j}^{r,t}\, y_j^t \qquad \forall (i, j) \in O, t \in T, r \in L \tag{5}$$

$$x_{i,j}^{r,t} \;\geq\; 0 \qquad \forall (i, j) \in O, t \in T, r \in L \tag{6}$$

$$y_i^t \;\in\; \{0, 1\} \qquad \forall i \in N, t \in T. \tag{7}$$

Binary variables y_i^t indicate whether train t stops at station i, while continuous variables $x_{i,j}^{r,t}$ represent the number of passengers of class r traveling between stations i and j on train t. The objective (1) minimizes the total number of train stops. Constraints (2) enforce segment-wise capacity limits, (3) ensure full demand satisfaction, and constraints (4)–(5) link passenger assignment to train stoppages at the corresponding origin and destination stations.

The formulation was previously studied in [1], where the LP relaxation was shown to be extremely weak, motivating the introduction of several classes of valid inequalities. Nevertheless, exact solutions were reported only for instances with up to 14 stations. This motivates the development of more scalable exact algorithms capable of solving realistic TSSP instances.

2 Solution Methodology and Computational Results

We propose a branch-and-price framework based on Dantzig–Wolfe (D–W) decomposition to solve the TSSP exactly. The formulation (1)–(7) is decomposed by retaining the demand satisfaction constraints

(3) and the convexity constraint in the master problem, while constraints (2), (4), and (5) are kept in the pricing problem. The LP relaxation of the master problem is solved via delayed column generation. An initial restricted master problem is constructed using heuristic generated columns, after which additional columns with negative reduced cost are obtained by solving the pricing problem. The column generation process terminates when no improving column can be identified.

If the resulting solution is integral in the original stopping variables y, the algorithm terminates. Otherwise, a branching scheme is applied on the y variables. A primal heuristic is embedded within the branch-and-bound framework to obtain high-quality integer solutions (incumbents) early in the search.

The algorithm is implemented in Julia, with CPLEX 22.1.1 used to solve both the restricted master and pricing problems at each node of the branch-and-bound tree. Computational experiments are conducted on the benchmark instances of [1]. As reported in Table 1, the proposed D–W reformulation yields a strong root node bound, with an average lower bound (LB2) of approximately 96% of the optimal integer solution. In contrast, the default branch-and-cut algorithm of CPLEX fails to solve any instance within the 2-hour time limit due to the extremely weak LP relaxation bound (LB0 = 29.4%).

Moreover, the proposed branch-and-price approach consistently outperforms the cutting-plane based method of [1], achieving stronger root bounds (LB2 vs. LB1) on all test instances and lower solution times on several instances. These results demonstrate the effectiveness of the proposed decomposition-based exact method for solving the TSSP. Due to space limitations of the extended abstract, detailed computational results are omitted.

Table 1: Lower bound and CPU time

Problem	Z^*_{IP}	CPLEX			From [1]		Branch-and-Price		
		LB0	Gap	Time (s)	LB1	Time (s)	LB2	Gap	Time (s)
A1	34	29.4	18.5	7200	92.6	201	97.3	5.9	600
A2	34	29.4	14.8	7200	90.2	1258	95.8	0.0	212
A3	34	29.4	14.2	7200	90.2	1062	96.0	2.9	600
A4	34	29.4	18.2	7200	88.6	1484	95.0	5.9	600
A5	34	29.4	14.4	7200	91.0	928	97.5	2.9	600
A6	34	29.4	13.6	7200	91.8	757	97.1	2.9	600
A7	34	29.4	13.7	7200	90.2	1074	96.3	5.9	600
A8	34	29.4	13.8	7200	89.4	1982	95.2	0.0	140
A9	34	29.4	14.6	7200	88.6	1917	94.7	2.9	600
A10	34	29.4	16.2	7200	88.6	2276	94.9	0.0	194
Average	34.0	29.4	15.2	7200	90.1	1293.9	96.0	2.9	474.6

3 Conclusions and Future Work

This study presents a branch-and-price algorithm for the Train Stop Scheduling Problem and shows that the proposed approach can solve larger problem instances exactly with significant reductions in computational time compared to existing exact methods. The strong root node relaxation and the use of problem-specific branching play a key role in the observed performance improvements.

Future work will focus on further enhancing scalability. In particular, heuristic pricing strategies will be developed to quickly generate promising columns, invoking exact pricing only when no improving column is found heuristically. In addition, the incorporation of cutting planes into the restricted master problem to develop a branch-price-and-cut framework is expected to further strengthen the lower bound and reduce the size of the search tree. Finally, tailored primal heuristics within the branching process may help obtain high-quality incumbent solutions at early stages, thereby accelerating convergence.

References

[1] Faiz Hamid and Yogesh K. Agarwal. Train stop scheduling problem: An exact approach using valid inequalities and polar duality. *European Journal of Operational Research*, 313(1):207–224, 2024.