A two-stage stochastic programming approach for the optimal sizing of a one-way station-based electric car sharing network

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1 Problem description

Electric car sharing services allow users to rent vehicles for short periods, typically a few hours, under time- or distance-based pricing schemes. In one-way station-based systems, vehicles may be collected at any station in the network and returned to a different location. The rapid expansion of such services is commonly explained by their environmental and societal benefits, as well as their potential to foster the adoption of electric vehicles. However, the economic and operational viability of an electric vehicle sharing system (EVSS) critically depends on how effectively its design and operations are planned.

This study focuses on the strategic sizing of a one-way station-based EVSS operating in a limited urban service area. Assuming that station locations are fixed, we aim to determine both the appropriate fleet size and the number of charging facilities to be deployed at each station. Addressing this problem raises three major challenges.

First, strategic investment decisions strongly constrain the set of feasible operational policies. Ensuring that the resulting system is both realistic and implementable requires explicitly accounting for key operational aspects, including vehicle availability, relocation activities across the network, and battery energy management. The integration of such tactical and operational considerations substantially increases the model complexity and may impair computational tractability. Second, car-sharing demand exhibits significant day-to-day variability and is subject to multiple sources of uncertainty, which naturally leads to a stochastic optimization framework. Third, the long-term viability of an EVSS depends not only on its profitability but also on its service reliability. Frequent trip rejections may result in user dissatisfaction and eventual attrition, making it essential to guarantee a high request acceptance rate, even when some trips are not immediately profitable. Consequently, the problem requires a joint optimization of economic performance and service quality.

To the best of our knowledge, no existing contribution addresses these three challenges within a unified framework. The closest related work is [2], which studies EVSS design under demand and electricity price uncertainty using a risk measure. However, their approach focuses exclusively on economic objectives and neglects both vehicle charging and relocation operations.

2 Problem modeling

We model the problem as a two-stage stochastic integer program. To represent demand uncertainty, we consider a discrete set of scenarios. Each scenario corresponds to a typical day of operation and is described by a set of trip requests, specified by their origin-destination stations, departure time, trip duration, battery consumption, and associated revenue.

The decision-making process is decomposed into two stages. First-stage decisions correspond to strategic sizing choices that must be made before demand uncertainty is revealed. They involve determining the number of chargers to be built at each station and the total number of vehicles to buy. After the realization of a given demand scenario, second-stage decisions define the corresponding operational plan.

These include the selection of accepted trip requests, the planning of vehicle relocations between stations, and the management of the charging of the vehicles parked at a station in-between two trips throughout the day. Similar to e.g. [1], our modelling of the related operational constraints relies among others on a discretization of the battery state of charge of each vehicle. We thus do not track each individual vehicle in the network but only monitor the number of vehicles with a battery state-of-charge at (or above) a certain level present at each station and each time period.

To account for both economic efficiency and customer service quality, we propose a bi-objective optimization framework that simultaneously addresses profitability and service reliability. In addition, a conditional value-at-risk (CVaR) measure is introduced to control the risk of poor service performance under unfavourable scenarios. For a given confidence level, this CVaR-based risk measure captures the expected service quality within the worst $100(1 - \alpha)$ of scenarios. The resulting objective function maximizes a normalized weighted combination of the expected economic profit and the CVaR of the service level at a given confidence level α.

3 Solution approach and results

The resulting formulation is a large-scale mixed-integer linear program with the classical block-separable structure of two-stage stochastic models. However, the direct application of standard Benders decomposition is impractical. Namely, each scenario subproblem is itself a high-dimensional MILP, potentially involving up to 100,000 binary and integer variables. Furthermore, the poor quality of the first-stage solution provided by the master problem at the beginning of the solution process means that a large number of iterations will be needed before a Benders-like decomposition algorithm may converge.

To overcome this computational bottleneck, we propose an approximate solution approach. The method exploits the fact that the linear relaxation of each scenario subproblem is remarkably tight, due to its close connection to network flow formulations. Building on this observation, we first solve a relaxed version of the model in which all second-stage variables are continuous. This is achieved through an original approximate Benders framework, in which suitably aggregated second-stage variables and constraints derived from the scenario subproblems are embedded directly into the master problem. The resulting expanded master problem provides first-stage solutions of significantly improved quality as compared to a standard master problem, which translates into a drastic decrease in the number of iterations carried out by the Benders decomposition algorithm. Once the relaxed solution is obtained, the first-stage decisions are fixed, and the scenario subproblems are then solved independently as MILPs to recover an integer-feasible solution to the original problem.

Computational results on synthetically generated instances, involving up to 10 stations, 30 scenarios, and 350 trip requests per scenario, highlight the efficiency of the proposed approach when compared to a state-of-the-art MILP solver. The approximate Benders strategy consistently produces near-optimal solutions within a two-hour time limit, whereas the monolithic formulation often fails to identify any feasible solution.

Finally, a case study relying on real traffic data from the Eastern Massachusetts region is conducted to investigate the trade-off between economic performance and service quality. The results highlight the ability of the proposed framework to generate a broad set of efficient solutions, from which decision-makers may select according to their risk aversion and managerial preferences. In particular, the analysis shows that substantial gains in service quality can be achieved with only limited impact on the long-term profitability of the system.

References

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