

Shortest Path Interdiction and Solution Diversity

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Diversity of solution alternatives and network interdiction are closely related concepts. However, there is no theoretical investigation on this relationship in general, nor on the specific links between interdiction problems and solution diversity. In this talk, I will be establishing a formal link between solution diversity and shortest path interdiction by extending ideas underlying Menger's theorem to weighted shortest path problems. Specifically, this talk will cover the *Maximising the Shortest Path (MXSP)* problem, a bilevel network interdiction model in which an interdictor chooses a set of edge costs to increase to maximize the length of a shortest $s - t$ path [4]. The MXSP is then linked to diverse shortest paths as defined in Hanaka et al. [3], using the sum of pairwise Hamming distances as a diversity measure.

Based on the aforementioned definitions MXSP and diverse shortest paths, sets of maximally diverse shortest paths correspond to edge-disjoint path collections. This correspondence can be used to formulate a polynomial-time greedy algorithm that for each set of edge-disjoint paths computes the optimal value of MXSP under the restriction that only this set of paths must be interdicted and that interdiction costs are uniform. This can be done at once for every possible interdiction budget. The values of the restricted MXSP problems are upper (dual) bounds for the unrestricted MXSP, where all paths need to be interdicted. Thus, any set of edge-disjoint paths implies a set of upper (dual) bounds for the MXSP.

The above synergises with the concept of *complementary column generation*, as introduced by Ghoniem and Sherali: When adding a new pattern in column generation, at the same time, add other patterns that *complement* the chosen pattern by completing it to a feasible solution [1]. Thus, when MXSP is solved through column generation of paths, completing an $s - t$ path to a set of edge-disjoint $s - t$ paths also induces a set of upper (dual) bounds at the same time. In addition, this implies termination criteria for when additional interdiction budget becomes ineffective, providing stopping rules for iterative solution methods. I evaluate the strength of the bounds and the effect on performance in a computational study on randomly generated grid graphs and a real-world road network of Rome from the 9th DIMACS Implementation Challenge. The results show that bounds obtained from edge-disjoint path sets are often tight or near-tight, with average gaps around 10% and little degradation as instance size increases. In a nontrivial fraction of instances, the bounds coincide with the optimal interdiction value.

Finally, I discuss links to work by Goffin and Vial on cutting planes [2], and by Viens et al., who explore the effect of diversity in Bender's decomposition [5]. Both cover diversity as a concept and both observe computational advantages to considering diversity of solutions. We conjecture that this be a promising paradigm for algorithms that iteratively add columns or rows to mixed-integer programming problems, such as column generation or cutting plane methods.

References

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