Balancing resource distribution for the healthcare districting problem

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1 Introduction

One of the main challenges in healthcare is achieving an efficient use of available resources, which is essential to ensure equitable and timely access to medical care. One way to address this challenge is to properly allocate the population to different healthcare centers, with the aim of optimizing coverage, since, according to the World Health Organization, coverage rates have been declining since 2015. However, it is recognized that the necessary resources exist to reverse this trend through better planning and distribution[1]. In this context, the unequal allocation of resources can lead to various difficulties for patients, such as longer waiting times and delays in care or referrals across different levels of the healthcare system[2]. Therefore, a more equitable and well-planned management of resources is essential not only to improve healthcare coverage but also to facilitate effective care across all levels of the system [3, 1].

One way to achieve an efficient allocation of resources is through the redesign of districts, defined as groups of territorial units organized for administrative purposes and service planning. This process involves regrouping these units into districts with the goal of optimizing resource utilization. This problem is referred to in the literature as territorial design, or districting, which aims to divide a region into balanced parts. In the context of healthcare planning, this approach is adapted to incorporate additional features, such as territorial contiguity, district compactness, and other criteria that enhance the efficiency of resource management [2].

2 Problem statement

The healthcare districting problem can be represented using a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes $\mathcal{V}$ correspond to territorial units and the edges $\mathcal{E}$ represent neighborhood relationships between them. Each territorial unit must be assigned to a district in such a way that every district contains at least one unit and that all units assigned to the same district form a geographically connected set to satisfy the contiguity requirement, which constitutes the main challenge of the problem. Additionally, solutions must ensure population homogeneity, so that the population in each district does not deviate significantly from the average, and geographic compactness, so that the units within each district are sufficiently close to minimize travel times to healthcare centers. The objective is to minimize the maximum surplus population over the available resources in each district. This surplus is evaluated at both the primary healthcare level, corresponding to family health centers, and the secondary healthcare level, which refers to high-complexity hospitals, with their relative importance weighted by a parameter β.

To address this problem, three mixed-integer linear programming (MILP) formulations are proposed, which mainly differ in the strategy used to enforce district contiguity, the first employs MTZ constraints, the second is based on a single-flow within each district (DSF), and the third uses a multi-commodity approach (MCF). The structure of these formulations is analyzed to identify strategies that improve computational performance. In particular, symmetry-breaking constraints are incorporated to eliminate equivalent solutions arising from different ways of connecting units within districts, and a variable-fixing

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[1] https://www.who.int/data/gho/data/themes/topics/service-coverage

[2] https://www.paho.org/es/temas/salud-universal

technique is implemented through the solution of a maximum weighted clique problem, with the aim of identifying sets of territorial units that cannot be assigned to the same district due to compactness requirements.

3 Numerical experiments

We assess the performance of the three MILP formulations, MTZ, DSF, and MCF, using randomly generated instances with 36, 54, and 72 territorial units, which need to be assigned to 4, 6, or 8 districts. For each instance, we evaluate the impact of varying the number of districts on model performance, as well as the effect of applying the proposed strategies, including symmetry-breaking constraints and center fixing through the maximum weighted clique, which reduce the search space and improve computational efficiency.

The results show that MCF performs poorly on larger instances, likely due to the high number of variables required to represent complete paths between territorial units and the root node. DSF generally solves a larger number of instances when the number of districts is small, benefiting from the flow-based structure that scales well with fewer districts. As the number of districts increases, MTZ demonstrates better scalability, as the growth in the number of variables and constraints is slower compared to DSF, allowing it to solve more instances to optimality and maintain lower computation times. When the proposed strategies are applied, all formulations exhibit significant improvements, DSF solves the largest number of instances overall, and MTZ maintains its efficiency across different problem sizes. In particular, symmetry-breaking constraints provide the most consistent gains by reducing the search space and eliminating equivalent solutions, while the impact of center fixing through the maximum weighted clique depends on the size of the clique that can be identified for each instance.

4 Future Prospects

Future research could focus on several directions to further enhance the efficiency and applicability of the proposed healthcare districting models. One possibility is to explore additional symmetry-breaking constraints, which have already shown significant improvements in computational performance. Another direction is to evaluate the scalability of the presented methods with larger instances, to better understand their limits and potential for real-world applications. Moreover, alternative objective functions could be considered for each level to better capture different healthcare priorities. Finally, extending the models to a multi-level framework could provide a more realistic representation of healthcare delivery, where the population is first assigned to primary care centers, which in turn refer patients to secondary hospitals as needed, allowing the optimization to account for the interaction between different structural levels.

References

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