Arc-flow formulations for a multiple bin size dual bin packing problem for wood reuse

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The increasing demand for raw materials such as wood undoubtedly contributes to the depletion of natural resources and global warming. To curb this phenomenon, a more sustainable and circular management of wood could be developed through a more efficient management of wood waste. This wood waste can be in the form of beams or pallets and could be considered as wooden *slats*. These slats could be combined, assembled, and glued to build Cross-Laminated Timber (CLT) panels for the construction industry.

We aim to develop optimization techniques to recycle raw wood waste to create CLT panels of variable dimensions. The input waste is in the form of slats. As a first step, we assume the lengths to be variable and heterogeneous, but that all the slats have the same width, even if this means they must be pre-cut or sorted. Their thickness is also constant. The objective is to provide the assembly schemes that maximize the amount of wood reused through the production of CLT panels.

The decision process to build a panel in this context is:

1. Assembling several slats to form a long rectangle at least as long as the panel, called a *strip*;
2. Gluing these strips together to reach the panel width, as shown in Figure 1;
3. Cut off the parts of the slats that exceed the panel, called the *trim loss* or leftovers, hatched in red in Figure 1.

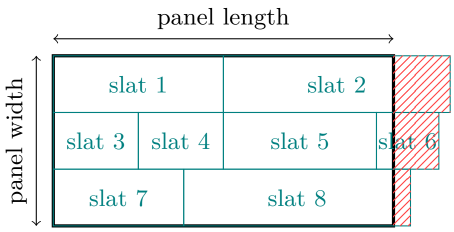

Figure 1: Toy example of assembly scheme to build one CLT panel in 2D

After a review of the Cutting & Packing literature, we identify our problem as an *Exact case of the Two-Stage Two-Dimensional Multiple Bin Size Dual Bin Packing Problem* (E-2S-2D-MBSDBPP) [1, 2] in the context of wood reuse optimization.

To reduce the trim loss, we could approach the problem from a different perspective and imagine that we build a very long strip by joining a subset of slats, which is then cut into pieces, each with the length of the type of panel to be built, which are finally glued widthwise as shown in Figure 2. The length of the long strip should be at least equal to the length of the panel type multiplied by the number of strips. The last piece cut could be longer than the length of the panel, resulting in the trim loss of this assembly scheme. This approach reduces the problem to a one-dimensional problem and allows the reuse of the cut parts of the slats, except for the last strip. Based on the literature review, this problem is identified as a *Variable-Sized Bin Covering Problem* (VS-BCP) [6].

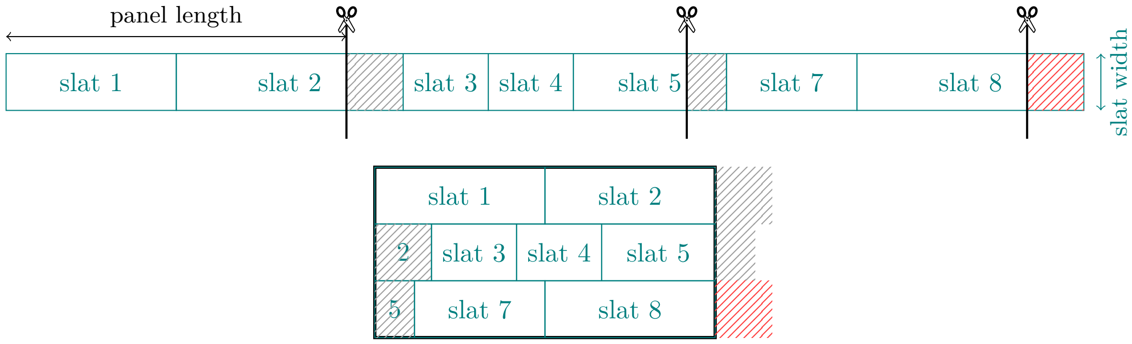

Figure 2: Toy example of an assembly scheme with the 1D approach

We propose a description of both versions of the problem, followed by an explanation of how the arc-flow graph is generated. We present the two similar arc-flow formulations with cuts, adapted from [3], [4], and [5]. Then, the different formulations are tested using the realistic instances we created, calibrated on real data and coherent with the observations of our industrial contacts. Finally, we compare the results obtained with Gurobi's B&B procedure and observe that the arc-flow formulation for the E-2S-2D-MBSDBPP, which needs smaller graphs than the one for the VS-BCP, shows very good performance. This is explained by the pseudo-polynomial character of arc-flow formulations.

The scientific contribution of this work is multiple. First, we study a two-dimensional MBSDBPP, while the DBPP has only been studied in one dimension. In addition, although the arc-flow formulation has been applied to several cutting and packing problems, to the best of our knowledge, it has never been applied to a 2S-2D-MBSDBPP.

Keywords: Dual bin packing, Cutting and Packing, Arc-flow formulation, Combinatorial Optimization

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