A unified view in planning broadcasting networks

Carlo Mannino, *Dip. di Informatica e Sistemistica, Univ. di Roma "La Sapienza", Rome, Italy*
Fabrizio Rossi, *Dip. di Informatica, Univ. di L'Aquila, L'Aquila, Italy*
Antonio Sassano, Dip. di Informatica e Sistemistica, Univ. di Roma "La Sapienza", Rome, Italy
Stefano Smriglio, *Dip. di Informatica, Univ. di L'Aquila, L'Aquila, Italy*

Abstract

Wireless broadcasting network planning has been widely addressed in the literature and a large number of models and algorithms have been developed. Most such models make use of MILP formulations both to represent and to solve practical planning problems. We discuss a new systematization of the different models presented in the literature by introducing a suitable problem hierarchy. We also discuss some major drawbacks of classical solution approaches, and describe alternative decomposition strategies to overcome them.

Keywords: terrestrial broadcasting, transmitter, receiver, interference, power assignment, frequency assignment, mixed integer programs, Bender's decomposition, Dantzig-Wolfe decomposition.

1. Introduction

Planning problems in wireless networks arise in a broad variety of application settings, such as radio and video broadcasting, mobile telephony, satellite communications and military.

Radio communication occurs between a couple of devices, i.e. a radio transmitter and a receiver. A wireless network is a set of transmitters distributing services to a set of receivers. Both transmitters and receivers are characterized by a set of radio-electrical parameters. Typical transmitter parameters are activation status, emission power, transmission frequency, polarization, antenna diagram. Analogously, receiver parameters are service status, antenna orientation, serving transmitter, etc. The general network planning problem consists in establishing suitable configurations for the radio electrical parameters of the transmitters and the receivers so as to maximize the customers coverage within a specified territory area (*target area*).

In the practical planning approach, the general problem is decomposed into a sequence of single-parameter decision problems: each of the parameters (i.e. each family of decision variables) is managed independently from the others. According to this practice, also most of the scientific literature focuses on the single-parameter case. In fact, the two main streams concern with the *Frequency Assignment Problem* (FAP) in which one can establish transmitter frequency and receiver service status, and with the *Power Assignment Problem*, where one can establish emission power and activation status of each transmitter and service status of each receiver.

The introduction of new technologies and the impressive increase of the frequency spectrum occupancy ask for the investigation of new, more accurate optimization models. As a consequence, efficient algorithms to the solution of the new models need to be developed. The authors experienced this fact within a series of contextual experiences, carried out during the last seven years, in the design of the Italian audio and video broadcasting system ([7, 6, 14]). In particular, these concern with both regulation issues in charge of the Public Authority and implementation aspects relevant for private broadcasters. This paper aims to provide a systematic view of the new (and the old) problems and to develop Integer Programming models which are useful to solve relevant practical problems. The starting point is the fact that the coverage condition of a receiver can be expressed by a linear function of the emission powers of the received signals. This is the ground of the *Signal-to-Interference* (SIR) model described in Section 2., and widely addressed in the practice and in the literature. Due to the large dimension of the resulting instances, such a model is generally approximated by decomposing it into smaller, more tractable problems. This decomposition is typically carried out in a heuristic fashion. Unfortunately, this approach fails to produce satisfactory solutions for most instances of practical interest. In this abstract we illustrate two different reformulation techniques of the SIR to get over such difficulty. In particular, a Bender's like and a Dantzig-Wolfe decomposition are discussed in Section 4.

2. The power and frequency assignment problem

A *wireless network* distributes its *services* from a set T of *transmitters* over a portion of territory, referred to as *target area*. Transmitter configuration depends on a set of basic parameters: geographical location and antenna height (physical parameters), transmission frequency and emission power (radio-electrical parameters). Depending on the application, additional parameters may be involved, such as polarization (horizontal/vertical), antenna tilt and time offset. We consider emission powers and frequencies as decision variables, while all other parameters are fixed.

The frequency spectrum is subdivided into a set $\mathcal{F} = \{1, \ldots, |\mathcal{F}|\}$ of equally sized intervals called *channels* (or *frequencies*). The set of feasible frequencies for transmitter $i \in T$ is denoted by $\mathcal{F}_i \subseteq \mathcal{F}$. It may happen that $\mathcal{F}_i \subset \mathcal{F}$ because of technical or commercial constraints (international agreements at boundaries, licensing, etc.).

The *transmission frequency* assigned to transmitter $i \in T$ is denoted by f_i, whilst P_i denotes its *emission power*, ranging in the interval $[0, P_i^U]$. For convenience, we also introduce the *power fading* $p_i \in [0, 1]$, that measures power attenuation w.r.t. the maximum power P_i^U (i.e., $P_i = p_i \cdot P_i^U$).

The target area is decomposed into a set Z of "small" areas called *testpoints* (TPs). Each testpoint, identified by its coordinates, represents the behavior of all *receivers* within it. A revenue u_j is defined for each TP j, typically related to the number of customers in j (for $S \subseteq Z$, $u(S) = \sum_{j \in S} u_j$).

The signal emitted by a transmitter propagates according to transmitter directivity and orography. The power density P_{ij} ($watt/m^2$) received in TP j from transmitter i is proportional to the emission power. In particular, $P_{ij} = A_{ij} \cdot P_i$, where $A_{ij} \in [0, 1]$ is defined through a propagation model (see [16]) and is given for each pair $i \in T, j \in Z$. We refer to the matrix $[\mathbf{A}] = [A_{ij}]_{i \in T, j \in Z}$ as the *fading matrix*. Finally, given a TP j, $T(j) = \{i \in T : A_{ij} \neq 0\} \subseteq T$ denotes the set of signals received in j. Among them, exactly one is elected as the *reference transmitter*, the major (possibly the unique) candidate to distribute the service in the TP. This is represented by a (characteristic) vector $\mathbf{s} \in \{0, 1\}^{|Z| \times |T|}$, where

$$s_{jh} = \begin{cases} 1 & \text{if } h \in T(j) \text{ is reference signal of } j \in Z \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

A TP j is said *covered* if the service is received "clearly". The coverage evaluation depends on system type (i.e., analog or digital; audio or video) and receiver behavior. However, a general model applies, with some parameter customization, to most practical settings. In fact, the coverage condition for TP j when h is the reference signal is expressed by a linear inequality, referred to as *SIR inequality*, of the form:

$$\sum_{i \in W(j,h,\mathbf{f})} a_{ij}(h) p_i - \sum_{i \in I(j,h,\mathbf{f})} b_{ij}(h) p_i \geq d_j. \tag{2}$$

The coefficients $a_{ij}(h)$ and $b_{ij}(h)$ are functions of the fading coefficients A_{ij}, of the maximum power P_i^U and of the (elected) reference signal h. $W(j, h, \mathbf{f}) \subseteq T(j)$ is the set of *useful* signals, containing h and possibly other transmitters in $T(j)$. For instance, in analog broadcasting and GSM, $W(j, h, \mathbf{f}) = \{h\}$. On the contrary, in digital broadcasting (DAB-T, DVB-T, DVB-H)[14], $W(j, h, \mathbf{f})$ may contain several signals in $T(j)$, all at frequency f_h. The set $I(j,h,\mathbf{f}) \subseteq T(j)$ is the set of *interfering* signals and contains signals at frequency f interfering with f_h. Typically, this happens when the distance $|f - f_h|$ in the frequency spectrum is below a given threshold. Remark that, in some cases, such as DAB-T and DVB-T and DVB-H, a transmitter in $T(j)$ may contribute to both $W(j,h,\mathbf{f})$ and $I(j,h,\mathbf{f})$ [14].

According to (2), the coverage status of a testpoint j depends on its reference signal $h : s_{jh} = 1$, as well as on the powers and the frequencies assigned to h and to the other transmitters in T. Thus, in order to assess the overall network coverage we need to consider the power vector $\mathbf{p} \in [0,1]^{|T|}$, the frequency vector $\mathbf{f} \in \mathcal{F}^{|T|}$ and the reference signal vector $\mathbf{s} \in \{0,1\}^{|Z|\times|T|}$.

The vector triple $(\mathbf{p}, \mathbf{f}, \mathbf{s})$ is called *configuration* of the network. The *coverage area* is the subset $C(\mathbf{p}, \mathbf{f}, \mathbf{s}) \subseteq Z$ of testpoints covered under the current configuration. We denote by $u(C(\mathbf{p}, \mathbf{f}, \mathbf{s}))$ the total revenue, i.e., the sum of the revenues of the covered TP-s. Therefore, the general planning problem reads as follows:

Problem 1 (*Power and Frequency Assignment Problem* (PFAP)) *Given a network (T, Z), the fading matrix $\mathbf{A}$, the power upper bound vector $\mathbf{P}^U \in \Re_+^{|T|}$ and the feasible frequency domain $\mathcal{F}_i \in \mathcal{F}$ for all $i \in T$, the* Power and Frequency Assignment Problem *is the one of finding a network configuration $(\mathbf{p}, \mathbf{f}, \mathbf{s})$ such that the function $u(C(\mathbf{p}, \mathbf{f}, \mathbf{s}))$ is maximized.*

The PFAP is NP-hard [14].

3. Problem hierarchy

Most of the wireless planning problems addressed in the literature may be derived from the PFAP by suitable assumptions on the configuration triple. Specialized versions can be obtained by including specific variables and constraints.

PFAP → GFAP (*Generalized Frequency Assignment*)
Then Generalized Frequency Assignment Problem occurs when $\mathbf{p} = \tilde{\mathbf{p}}$ is fixed, while frequency and server assignments need to be established. A special case of the GFAP is probably the most addressed problems in OR telecom applications, namely:

GFAP → FAP (*Frequency Assignment Problem*)
Both $\mathbf{p} = \tilde{\mathbf{p}}$ and $\mathbf{s} = \tilde{\mathbf{s}}$ are fixed and we only need to establish optimal transmission frequencies ([2, 10]). Actually, in most cases a simplified model is considered, by neglecting multiple interference and by assuming that each testpoint is interfered by one transmitter at a time. By these assumptions an instance of the FAP can be represented by means of a simple, undirected graph, namely the *interference graph*, and the FAP becomes the problem of finding a (generalized) max k-cut in the interference graph. Indeed, this reduction introduces a modelling approximation which is often unacceptable in practice. Even worse, we cannot tell if the actual optimal value is approximated from above or from below.

FAP → NSIM (*Wireless Network Simulation*)
All of the radio-electrical parameters are fixed to $(\tilde{\mathbf{p}}, \tilde{\mathbf{f}}, \tilde{\mathbf{s}})$. In this setting, the PFAP reduces to the problem of evaluating network coverage $u()$.

PFAP→ PAP (*Power assignment problem*)
Frequencies are fixed to $\mathbf{f} = \tilde{\mathbf{f}}$. UMTS planning may be considered as a specialized version of the PAP ([4, 9, 15]). A special case of PAP, namely the *Antenna Siting Problem* (ASP), is obtained when $\mathbf{p}$ is restricted to assume integer values, i.e. $\mathbf{p} \in \{0,1\}^{|T|}$. In other words, (ASP) is the problem of selecting a suitable set of reference signals to switch on and assign to testpoint as to maximize coverage.

PAP$\rightarrow$ **WND** (*Wireless network design*). Both frequencies $\mathbf{f} = \tilde{\mathbf{f}}$ and servers $\mathbf{s} = \tilde{\mathbf{s}}$ are fixed. This problem typically arises in practical network planning. In fact, in most practical cases both transmission frequencies and service areas of the available transmitters are established in advance, in order to fulfil specific service requirements, and the network engineers are left with the problem of establishing suitable antenna diagrams, namely powers in the 36 horizontal directions .

WND$\rightarrow$ **NSIM** (*Wireless Network Simulation*) All of the radio-electrical parameters are fixed to $(\tilde{\mathbf{p}}, \tilde{\mathbf{f}}, \tilde{\mathbf{s}})$.

The complete problem hierarchy is depicted in Figure 1.

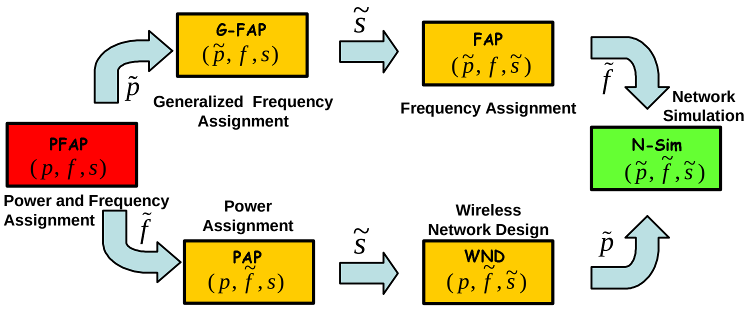

Figure 1: Problem hierarchy

4. Solution approaches to the PFAP

The PFAP has been introduced in [14], where a Mixed Integer Linear Programming formulation based on SIR inequality along with a primal heuristic are presented. The MILP model exhibits large integrality gap when applied to large scale real-world instances and its practical applicability decreases as the number of frequencies increases. A similar bad behavior has been reported in [9] and [15], where a SIR model is applied to a restricted version of PFAP in which both site selection (transmitter activation status) and frequencies have to be optimized.

In order to tackle large scale practical instances a possible solution strategy is to decompose PFAP into a sequence of single-parameter subproblems, namely FAP and PAP, for which many effective algorithms are available (see resp. [2] and [5] for references). We tested such an approach within different joint projects with the Public Authority [1] and major broadcasters [14], concerning with the design of the Italian audio and video broadcasting system. It turned out that using natural heuristic decompositions this approach does not behave satisfactorily.

In two recent papers two *exact* decomposition approaches have been investigated. In [13] a non-compact set packing formulation is obtained by Dantzig-Wolfe decomposition from the SIR model; in [12] a non-compact set covering formulation is derived through Bender's like decomposition.

A Dantzig-Wolfe decomposition approach to the PFAP The Dantzig-Wolfe decomposition principle allows to reduce the PFAP to the solution of a sequence of the PAP, somehow imitating a most common practical approach. The decomposition is based on a non compact *Set Packing* reformulation of the PFAP which turns out to be very tight in an extensive computational experience. Remarkably, the column generation amounts to solve instances of manageable size of the power assignment problem on a single frequency network. We test the resulting branch-and-price algorithm on reference instances of the Italian Authority of Communications [7]. The branch-and-price algorithm solves to optimality such very large scale instances, which are far behind the solution tractability for the standard SIR formulation.

A Bender's like decomposition approach to the PFAP This decomposition is based on a non compact *Set Covering* reformulation of the PFAP, which can be traced back to the classical Bender's reformulation for MILP (see [8]). A set of testpoint R is *coverable* if there exists a configuration triple such that all of testpoints in R are covered. Clearly, if $R' \subseteq R$ and R is coverable, then R' is also coverable. Thus, denoting by $\mathcal{I}$ the family of the coverable sets of testpoints, the pair $(R, \mathcal{I})$ is an independence system. It is well known that the problem of finding a maximum weight independent set in an independence system $(R, \mathcal{I})$ can be formulated as the following set covering problem: $\min\{\sum_{r\in R} c_r y_r : \sum_{r\in C} y_r \geq 1, C \in \mathcal{C}, y \in \{0,1\}^{|R|}\}$, where $\mathcal{C}$ is the family of circuits of $\mathcal{I}$, c is a weight vector and $\mathbf{y}$ is the incidence vector of the complement of an independent set. In our case, the circuit family $\mathcal{C}$ corresponds to the family of the uncoverable sets of testpoints, which grows exponentially with $|R|$. As a consequence, the practical applicability of the above set covering formulation strongly depends on the definition of effective routines to identify violated circuits. Since testpoints coverage is represented by a system of linear inequalities, circuit identification can be reduced to identify suitable infeasible subsystems (*IIS identification*, see Amaldi et al. [3]). Finally, as observed by Codato and Fischetti [8], violated set covering inequalities may be interpreted as special cases of (combinatorial) Bender's cuts. Computational testing shows that this reformulation is much tighter than the natural MILP formulation and allows to solve to optimality otherwise unsolvable instances.

The above reformulations can be applied alternatively or jointly, depending on the specific instances we are dealing with. In particular, the Dantzig-Wolfe approach is preferable whenever the column generation phase can be tackled efficiently: analogously, the Bender's like approach is workable if we have at hand efficient separation routines to identify violated inequalities. This happens, for example, in solving instances of the WND (see hierarchy), but also instances of other classes of optimization problems in wireless network design, such as the *Time Offset Optimization Problem* [11]. Finally, the PAP instances encountered in the column generation phase of the Dantzig-Wolfe decomposition approach may be well suited to be solved by means of combinatorial Bender's cuts.

References

[1] Autorità Nazionale per le Garanzie nelle Comunicazioni, `http://www.agcom.it.`

[2] K. I. Aardal, S.P.M. van Hoesel, A.M.C.A. Koster, C. Mannino and A. Sassano, Models and solution techniques for the frequency assignment problem,*4OR*, **1**, 4, 2003, 261-317

[3] E. Amaldi, M. E. Pfetsch and L. E. Trotter Jr.. *On the maximum feasible subsystem problem, IISs and IIS-hypergraphs* Math. Programming, Ser. A, Vol. 95, 3, pp. 533-554, (2003)

[4] E. Amaldi, A. Capone, F. Malucelli *Radio planning and coverage optimization of 3G cellular networks* ACM Wireless Networks, to appear.

[5] E. Amaldi, A. Capone, F. Malucelli, C. Mannino, *Optimization problems and models for planning cellular networks*, Handbook of Optimization in Telecommunication, Eds. M. Resende and P. Pardalos, Springer Science, (2006).

[6] Autorità per le Garanzie nelle Comunicazioni, *Piano Nazionale di Assegnazione delle Frequenze Radiotelevisive*, http://www.agcom.it/provv/pnf/target01.htm

[7] Autorità per le Garanzie nelle Comunicazioni, *Libro Bianco sulla televisione digitale terrestre*, http://www.agcom.it/provv/libro_b_00/librobianco00.

[8] G. Codato, M. Fischetti, *Combinatorial Benders' Cuts for Mixed-Integer Linear Programming*, to appear in Operations Research, 2006

[9] Eisenblätter, A., Fügenschuh, A., Geerdes, H.F., Junglas, D., Koch, T., Martin, A., Integer Programming Methods for UMTS Radio Network Planning, Proc. of WiOpt'04", 2004

[10] Leese, R., Hurley, S., Methods and Algorithms for Radio Channel Assignment, Oxford Lecture Series in Mathematics and its Applications 23, Oxford Press, New York 2002.

[11] C. Mannino, F. Rossi, A. Sassano, S. Smriglio, *The time-offset optimization problem in digital broadcasting*, to appear in Discrete Applied Mathematics.

[12] C. Mannino, S. Mattia, A. Sassano, *Wireless Network Design by Set Covering*, working paper.

[13] Mannino, C., Marinelli, F., Rossi, F., Smriglio, S., “Power and frequency assignment in wireless networks”, *Working Paper*

[14] Mannino, C., Rossi, F., Smriglio, S., “The Network Packing Problem in Terrestrial Broadcasting”, *Operations Research* **54**, 6, (2006) pp. 611-626.

[15] R. Mathar, M. Schmeinck, *Optimisation Models for GSM Radio*, *International Journal of Mobile Network Design and Innovation*, **1**, 1, 2005, 70-75.

[16] Rappaport, T.S., *Wireless Communications*, Prentice Hall, 2002.