Model and Algorithms for the Density, Coverage and Connectivity Control Problem in Flat WSNs

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1. Introduction

In the last decade, several technological advances in embedded systems and wireless communication allowed the arising of a new kind of ad-hoc network, the Wireless Sensor Network (WSN). A WSN is a special kind of mobile network composed by hundreds or even thousands of autonomous and compact devices, called sensor nodes, which can perform sensing, processing and wireless communication tasks. In a WSN application, the source nodes collect data from a phenomenon and disseminate them towards the sink node, using multi-hop communication.

Some of the WSNs' main limitations are energy restriction, small bandwidth, and low processing and communication capacities. The energy restriction is due to the limited battery capacity and the impossibility of recharge or replacement, specially for networks established in regions of difficult access or composed of hundred or thousand nodes. The nodes' low cost and reduced size are the main reason for the small capacity of their components.

In this scenario the popularization of WSNs in a near future depends on making them more reliable, specially in aspects such as area coverage, node connectivity, and efficient resource usage. Recent works show that an efficient density control in high density sensor networks saves significant amounts of energy [6] and can improve the data dissemination, since it decreases packets collisions and radio interference. In a density control scheme some nodes are scheduled to sleep or change to energy-saving state, while others continue to collect, process and disseminate data to sink nodes.

Our work proposes an ILP formulation and algorithms to solve the Density, Coverage and Connectivity Control Problem (DCCCP) in flat WSNs subject to node failures due to mechanical problems or when battery runs out of energy. We present a global algorithm which has a complete vision of the network and so can build up good topologies using the available nodes. However, spreading the solutions generated by this algorithm can be expensive in terms of energy, time and network load. Thus, we also propose a local algorithm, which is called every time a node failure occurs and solves the problem considering only the failure neighborhood. By combining both algorithms we obtain a hybrid approach which benefits from the best features of each one of them.

2. Related Work

Megerian and Potkonjak [2] use Integer Linear Programming (ILP) to model the Coverage Problem in Wireless Sensor Networks and solve the problem by using a greedy algorithm. Vieira et al [7] use computational geometry and graph theory to solve the same problem. Nakamura et al [3] propose a mathematical ILP model to the multi-period coverage and connectivity problem in flat WSNs and solve it with the commercial optimization package CPLEX [1]. The model adds to the coverage and connectivity constraints, a set of constraints regarding the node energy limits. Quintão et al [5] compare the CPLEX solutions of a multi-period ILP model, that deals only with density and coverage control, to the ones obtained with an evolutionary algorithm, achieving good results regarding objective function values and specially computational time.

Ye et al [8] present the algorithm PEAS (*Probing Environment and Adaptive Sleeping*). PEAS consists of two algorithms, which determine (1) which sensor nodes should work and how a wake-up sensor node makes the decision of whether going back to sleep, and (2) how the average sleep time of the sensor nodes is dynamically adjusted to keep a relatively constant wake-up rate.

Zhang and Hou [9] present the protocol OGDC (*Optimal Geographical Density Control*). According to the authors, this is a full distributed and local search algorithm based on the following: if the communication (or radio) range is at least twice of the sensing range, then a complete coverage of a convex area implies the connectivity among the working set of nodes. Based on this assumption, authors present a set of optimality conditions under which a subset of working sensor nodes can be chosen for full coverage and also propose an algorithm that assures these conditions when the network has a high density and each node knows its own position in the network area.

3. Problem Definition

For modeling purposes, we discretize the monitoring area in a set of points that require sensing, called demand points, and consider that the node coverage area is a circle of range R, where R is the sensing range. If the distance between a demand point and a sensor node is less than R, the node is able to cover this point. Thus, the problem can be defined as: *Given a set of sensor nodes S, a sink node m, a monitoring area A and a set of demand points D, the Density, Coverage and Connectivity Control Problem (DCCCP) in flat Wireless Sensor Networks consists of assuring that at least one sensor node $l \in S$ covers each demand point $j \in D$, and that there is a path between each active sensor node $l \in S$ and the sink node m. The WSNs are subject to node failures.* The DCCCP solutions minimize the energy consumed in the network in an attempt to maximize its lifetime.

4. Mathematical Model

The DCCCP can be modeled it as an ILP problem. The following parameters are used in our formulation:

S set of sensor nodes	O^s set of arcs (i,j) outgoing the sensor node $i \in S$
D set of demand points	AE_i activation energy for node $i \in S$
A^d set of arcs connecting sensor nodes to demand points	ME_i maintenance energy for node $i \in S$,
A^s set of arcs connecting sensor nodes	TE_{ij} transmission energy between nodes i and j, $\{i,j\} \in \{A^s \cup A^m\}$
A^m set of arcs connecting sensor nodes to the sink node	RE_i reception energy for node $i \in S$
I^d set of arcs (i,j) incoming on the demand point $j \in D$	
I^s set of arcs (i,j) incoming on the sensor node $j \in S$	

Table 1: Parameters of the model

The model variables are presented in Table 2.

x_{ij} variable that has value 1 if node i covers demand point j, and 0 otherwise
z_{lij} decision variable that has value 1 if arc (i,j) is in the path between sensor node l and the sink node m, and 0 otherwise
y_i decision variable that has value 1 if node i is active, and 0 otherwise
e_i variable to indicate the energy consumed by node

Table 2: Variables of the model

The ILP formulation proposed is presented below.

The objective function (1) minimizes the network energy consumption given in terms of current consumed for each task.

$$min \sum_{i \in S} e_i \tag{1}$$

The set of constraints (2), (3), and (4) deals with the coverage problem. Constraints (2) specifically assure that, if possible, at least one sensor node will cover each demand point. Constraints (3) indicate that a node can only cover a point if it is active. Constraints (4) set limits for variables x.

$$\sum_{ij \in I^d} x_{ij} \geq 1, \forall j \in D \tag{2}$$

$$x_{ij} \leq y_i, \forall i \in S, \forall ij \in A^d \tag{3}$$

$$0 \leq x_{ij} \leq 1, \forall ij \in A^d \tag{4}$$

The set of constraints (5), (6), (7) and (8) are related to the connectivity problem. Constraints (5) and (6) assure a path between each active sensor node $l \in S$ and the sink node m and constraints (7) and (8) only allow active nodes to be part of these paths.

$$\sum_{ij \in I^s} z_{lij} - \sum_{jk \in O^s} z_{ljk} = 0, \forall j \in (S \cup m - l), \forall l \in S \tag{5}$$

$$-\sum_{jk \in O^s_j} z_{ljk} = -y_l, j = l, \forall l \in S \tag{6}$$

$$z_{lij} \leq y_i, \forall i \in S, \forall l \in (S - j), \forall ij \in (A^s \cup A^m) \tag{7}$$

$$z_{lij} \leq y_j, \forall j \in S, \forall l \in (S - j), \forall ij \in (A^s \cup A^m) \tag{8}$$

The energy constraints (9) define that a node spends its energy with activation, maintenance and packets transmission and reception. The energy constraints (10) define the energy lower bound as zero.

$$(AE_i + ME_i) \times y_i + \sum_{l \in (S-i)} \sum_{ki \in I^s_i} RE_i \times z_{lki} +$$
$$\sum_{l \in S} \sum_{ij \in O^s_i} TE_{ij} \times z_{lij} \leq e_i, \forall i \in S \tag{9}$$

$$e_i \geq 0, \forall i \in S \tag{10}$$

Constraints (11) define the decision variables as boolean.

$$h, y, z \in \{0, 1\} \tag{11}$$

The model solution consists of a subset of active nodes, represented by the variables y_i with value 1. The variables x_{ij} with value 1 indicate which active sensor node i covers which demand point j. The solution also provides a path between the active nodes and the sink node assuring the network connectivity, and given by the variables z_{lij} with value 1. The solution also estimates the network current consumption in the variables e_i. The DCCCP problem is a complex combinatorial problem, therefore we propose a Hybrid algorithm as an alternative to solve it.

5. Hybrid Algorithm for the DCCCP

In this section, we highlight the main ideas of the two algorithms that composed our solution and describe the Hybrid Algorithm for the DCCCP.

Global On-Demand Algorithm The Global On-Demand Algorithm (GOD) is a hybrid approach that combines genetic and graph algorithms. See [4] for more details on this algorithm. The genetic algorithm solves the density and coverage problems, determining the set of nodes that assures the area coverage with the lowest energy consumption cost. These nodes are then connected by Minimum Spanning Tree and Shortest Path algorithms. The algorithm is on-demand because it is triggered by the hybrid approach.

Local Online Algorithm The Local Online Algorithm (LOA) is used every time a failure occurs. It is called to locally restore the area coverage and the nodes connectivity choosing a node to replace the one that failed. Let $d(i,j)$ be the Euclidean distance between nodes i and j and C the set of children of the node that failed, the local algorithm selects a node based on the value of the Equation 12. We use the quadratic distance to choose the node that is, at the same time, closer to the parent and to the children of the node that failed.

$$Value(i) = d^2(i, parent) + \sum_{j \in C} d^2(i,j) \tag{12}$$

where *parent* is the parent of node that failed in the connectivity tree.

The local algorithm tries to connect the chosen node and all the nodes in C to an active route in the network. If that is not possible, due to the limited communication ranges, the algorithm calculates the shortest path between each disconnected nodes in C and the sink considering all the network nodes, besides the nodes that failed, and it turns on the inactive nodes in the shortest path calculated.

Hybrid Algorithm The Hybrid algorithm to solve DCCCP in flat WSNs is an attempt of combining the advantages of solving the problem in global and local ways. When the algorithm has the global vision of the network, it can find better nodes to compose the network, leading to better solutions regarding energy consumption. However this approach is not scalable and it is computationally expensive. In simulations or in real applications, the cost to disseminate the solution generated, normally using control messages, is high. The local approach can be more adequate to WSNs because it is computationally cheaper, requires less control messages and is more scalable, especially when computed by the nodes. But the local vision can lead to worse solutions because the candidates to compose the network are restricted to a specific area, leading sometimes to unnecessary node activation.

The Hybrid algorithm firstly executes GOD to create an initial solution. Then, at each node failure, LOA tries to restore the coverage and connectivity. In each simulation time unit, we compare the energy consumed with the one consumed in previous time unit and if it is 5% higher it triggers the GOD algorithm to improve the configuration, given that GOD has the global vision of the network.

6. Results

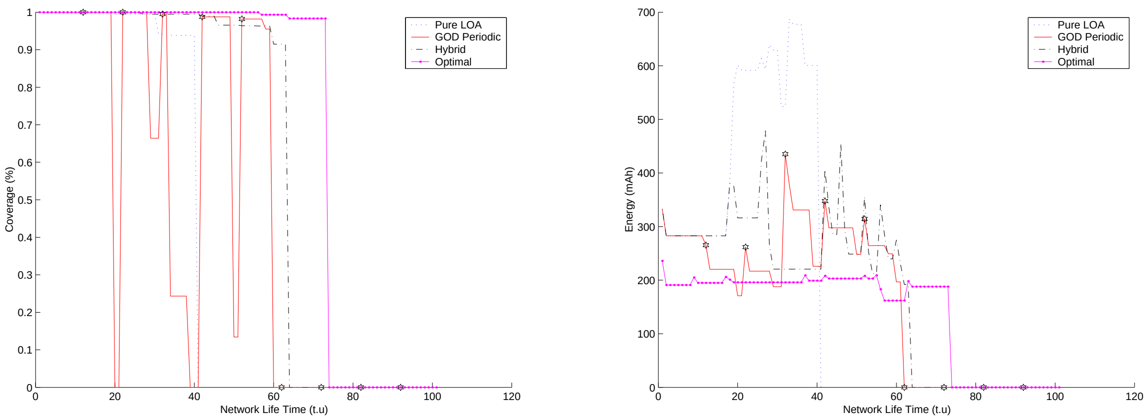
We compare the Hybrid algorithm to three approaches, the Optimal Solution, the GOD Periodic and the Pure LOA. In the Optimal Solution approach we use the optimization package CPLEX to solve the ILP model. First, we find the initial optimal solution and after that, each time a failure occurs, we exclude the nodes that failed from the input parameters and execute CPLEX again to establish the optimal solution for the new set of nodes. The solution remains the same until a failure occurs. When the node is already active, its activation energy is set to zero to calculated the new solution.

In the GOD Periodic approach, we call the GOD algorithm every 10 time units. We find a initial solution and before each algorithm call the input set S is updated and the nodes that failed between the current period and the last algorithm execution are excluded from the it. To compare the hybrid solution to the Pure LOA approach, we execute the GOD algorithm on time unit 0 to create a initial solution for the network. Then, at each node failure the LOA is called to try to restore the area coverage and the nodes connectivity.

We simulate a WSN subject to node failures and measure the coverage achieved for the algorithms. We work with two types of failures: mechanical failures, simulated by a failure generator and energy failures, that occur when then battery finishes. In the test with ILP model, we use a list with the mechanical failures that occurred in simulations and the period they occurred so we can reproduce the same comparison scenario. To identify energy failures, we calculate, for each solution, the energy consumed for the nodes until we identify one or more nodes with no energy left. We also measure the energy consumed in the network for each one of the approaches.

The Figures 1(a), and 1(b) show the results obtained. The Coverage Graphic 1(a) shows that the optimal solution keeps a high coverage until it fails completely around the simulation time 73. The coverage fail is due to connectivity problems, because none sensor left in the network is connected to the sink node. The optimal solution maintains the best coverage possible with the minimum number of sensor nodes, which leads to solutions where the nodes near the sink have a very large subtree connected to them.

When the coverage is the same for all curves the energy consumed by the optimal solution is the lowest as showed on Graphic 1(b). When the energy consumed for the algorithms is lower than the optimal solution, the best possible coverage is low or the coverage failure is high.



(a) 32-nodes network Coverage

(b) 32-nodes network Energy Consumed

Figure 1: Optimal Solution x Algorithms

The Coverage Graphic 1(a) also shows that for the algorithms the best results are from the Hybrid algorithm because its local part tries to recover from failures immediately and avoids big coverage fails as the ones that occur with the GOD Periodic. Moreover, from time to time, it uses GOD to choose a better set of active nodes. The worst behavior is from the Pure LOA because, once it works locally, it spends more energy for it actives unnecessary nodes. This unnecessary activations leads to a decrease of the number of nodes candidates as times goes by and eventually there is no node left to recover from failures. In the GOD Periodic approach, the high drops on the coverage mean that the subtree connected to node that failed is big.

The Energy Graphic 1(c) shows that the GOD Periodic approach presents the better results regarding energy consumption, followed by the Hybrid algorithm and finally from the Pure LOA approach. The GOD Periodic approach spends less energy because the global vision actives less nodes, but as it does not recover immediately from fails the combined results Coverage x Energy are not good. Besides, once the subtree previously connected to the node that failed continues to work until a new execution, this algorithm spends unnecessary energy. The local vision leads the Pure LOA approach to active more nodes and consequentially consuming more energy. The Hybrid approach balances both the energy and coverage, achieving the better combined results Coverage x Energy. We are studying ways of adding residual energy constraints to the ILP model and the algorithms to try to improve their results.

7. Final Considerations

This work proposes an ILP mathematical formulation to model the Density, Coverage and Connectivity Control Problem in flat Wireless Sensor Networks subject to nodes failures and a Hybrid algorithm as an alternative to solve it. The Hybrid algorithm uses an Global On-Demand algorithm (GOD) that rebuilds all the network when required and a Local Online Algorithm (LOA) that tries to restore locally the coverage and connectivity when failures occur. We compare the hybrid approach to the optimal solution obtained by solving the ILP model, to a GOD Periodic approach, that rebuilds all the network in pre-defined time periods, and to a Pure LOA approach that works locally every time a failure occurs. Results show that the combination of the global and local approaches leads to better solutions once it benefits from the advantages of both approaches, but it can be improved to reach better results when compared to the optimal solution.

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