A Max-Min approach to delay load shedding in power distribution networks despite source-capacity uncertainties

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1. Introduction

In the context of energy, the market deregulation is deeply modifying the conditions of control of the operational safety of the networks. These evolutions result in exploiting the networks closer to their physical limits [1]. If the present operating system remains flexible insofar as the sources capacities are much higher than the customers load, this margin however cannot last in a context of quick increase of the loads. The challenge of the next years will be to exploit the networks with an available power which tends to balance with the loads. In this context, taking into account uncertainties on the sources capacity will induce challenging problems. These uncertainties are mainly due to new technologies such as renewable energies whose production remains very fluctuating. The aerogenerators can be disconnected from the network for safety reasons and can induce voltage drops. Moreover their production is very related to the weather conditions. In the same way the production of the photovoltaic cells is dependent on the sunning. When voltage drops occur, the only present solution to face these problems is to resort to load shedding.

That is the reason why, in this paper, the configuration of a power distribution network is considered in uncertain context. This network is composed of several power sources, electrical lines with their switch and customers with their load (figure 1). The set of power sources must serve the set of customers with feeding their load and satisfying some electrotechnical constraints (like no connection between two sources during functioning). Configuring the network means to choose what power source will serve what customer with opening or closing the appropriate switches. So the set of the switches positions represents the configuration of the network.

In certain context (the problem's data are considered as known and constant values), an optimization criterion usually used is the minimization of power losses. To consider this optimization problem means to be already confronted with a difficult problem. To solve it, metaheuristics have been developed [2, 3, 4, 5]. However solutions are valid only for a given instance. Uncertainties on power sources capacity lead to a performance degradation which involves a modification a posteriori (after the perturbation is occurred) of the network configuration. This reconfiguration must satisfy constraints which were violated and serve loads which were cut off. To resort to load shedding is intolerable in a service production system where the service is precisely to distribute energy. That is the reason why, the uncertainties on sources capacity must be considered when solution are built. Like this, performances (non load shedding) can be guaranteed despite the uncertainties. Thus the goal of this paper is to present this new problem in the domain of optimization under uncertainty and to propose a Max-Min approach to delay load shedding despite uncertainties on sources capacity.

After having modelled the network by a directed graph and having presented the inherent electrotechnical constraints, a criterion is proposed to value the robustness of the network against uncertainties on sources capacity. Then a mathematical formulation is presented with Mixed Integer Linear Programming. Finally some experiments have been led to value the effectiveness and the limits of the method.

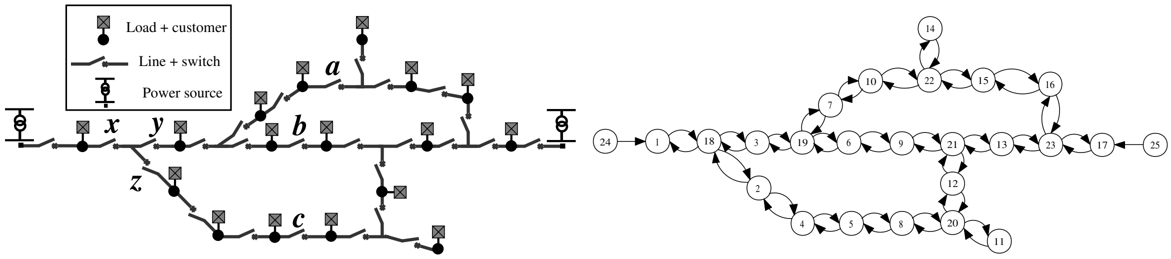

Figure 1: A power distribution network and its model

2. Problem modelling

The power distribution network is composed of m power sources which have to feed n loads (residential, commercial or industrial customers).
Using the notations and definitions given in [6], the power distribution network is modelled by a directed graph $G = (N, A)$ with $N = N_c \cup N_t \cup N_s$. On figure 1, the network on left is modelled by the graph on right.
$N_c = \{1 \dots n\}$, $N_t = \{n+1 \dots n+t\}$ and $N_s = \{n+t+1 \dots n+t+m\}$. Each node i of N_c represents a customer with its load whereas N_s represents the set of sources with their capacity. N_t represents the set of junction nodes. On figure 1, the network is composed of $n = 17$ customers $(1 \dots 17)$, of $t = 6$ junction nodes $(18 \dots 23)$ and of $m = 2$ sources (24 and 25). The junction node 18 represents the junction point between switches which are denoted x, y and z on the network on left. The junction nodes behave as customers without load, which are not to be necessarily served.
The network structure is represented by arcs between nodes of N. Each arc $A_k \in A$, with $A_k = (i, j)$, where $(i, j) \in N^2$ represents an electrical line with a switch. As flow direction is not pre-defined, each electrical line is represented by two arcs modelling the two possible directions. However, a power flow cannot arrive to a source, thus there is no incoming arc for source nodes. The existence or the non-existence of an arc is defined by the adjacency matrix H whose size is $(n+t+m)^2$. If $H_{ij} = 1$ the arc (i, j) exists and if $H_{ij} = 0$ then it does not exist.
The load of each customer and of each junction node is represented by the $n + t$ integer vector W ($\forall i \in [n+1, n+t], W_i = 0$).
Each source has a limited power capacity which is represented by the m real vector K.
The distribution network configuration is modelled by a $(n+t+m)^2$ binary matrix denoted Q. $Q_{ij} = 1$ if the switch represented by the arc (i, j) is closed and $Q_{ij} = 0$ if the same switch is opened or if it does not exist (*i.e.* $H_{ij} = 0$). The matrix Q is the result of a choice whereas the matrix H represents the network structure. Thus $H_{ij} = 0 \Rightarrow Q_{ij} = 0$, but the reverse is not true.

3. Electrotechnical constraints

The electrotechnical constraints can be defined as follows:

1. Two sources must not be in connection during functioning: an admissible configuration does not contain any path connecting two sources. By construction of matrix H, this constraint is always satisfied since their is no entrance arc to the sources.

2. Each customer must be served by one and only one source.

3. There cannot be cycle in the resulting configuration: an admissible configuration is a spanning forest of the directed graph.

4. Each source can provide only a limited quantity of power characterized by the capacity K.

5. Each line can support a maximal power flow: the arcs are constrained by a capacity of maximum flow.

In the following, it will be assumed that each capacity of maximum flow is equal to the maximal value of K. Thus a configuration which satisfies constraint 4 also satisfies constraint 5.

4. Characterization of the problem solutions

An admissible solution is an oriented forest whose roots are the source nodes. This forest must cover all the customers nodes and satisfy the previous constraints. In figure 2, an admissible solution for the example of figure 1 is presented. This solution consists in opening the switches represented by the arcs $(5, 8)$, $(6, 9)$, and $(10, 22)$ (resp. the switches a, b, and c of figure 1) while keeping closed the other switches.

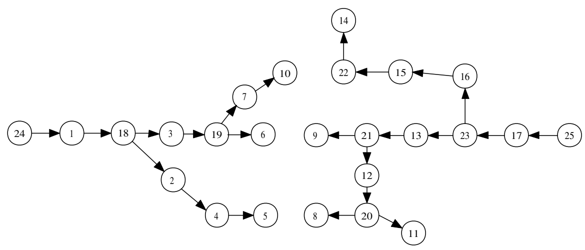

Figure 2: An admissible configuration

5. Uncertainties on sources capacity: performance and robustness

A lot of perturbations can occur in the electrotechnical domain: loss of a line, uncertainties on the customers load, uncertainties on sources capacity, break down of a switch...
However only the uncertainties on sources capacity will be taken into account in the following of the paper. Thus the customers load is regarded as known and constant. It can correspond to a reference predicted load or to the subscription of the customer. This load is denoted W_{ref}.
Moreover, the network is regarded as being free from failures: *i.e* lines and switches are always well functioning (matrix H is considered as perfectly known and constant).
In fact, as seen in introduction, taking into account the uncertainties on sources capacity induces challenging problems.
A first challenge is to define the performance to guarantee despite uncertainties. As seen previously, capacity uncertainties can induce load shedding. However, to serve all the customers is an electrotechnical constraint (see section **3.** second constraint). Thus considering capacity uncertainties means to consider the constraint of load-service as a performance to guarantee. In fact, a power distribution network can be seen as a service production system where to guarantee the service is the first aim.
A second challenge is to measure the robustness of a given configuration Q (*i.e.* its ability to ensure the load-service despite the perturbations). Considering a given configuration Q and a reference load W_{ref}, and knowing the nominal capacity K_{nom}, it is easy to value a load shedding margin denoted M_j for each source j:

$$M_j(Q, W_{ref}, K_{nom}) = \frac{K_{nom_j} - \sum\limits_{i \in B_j} W_{ref_i}}{K_{nom_j}}$$

with B_j, the set of loads i served by the source j in the configuration Q. Let consider the configuration Q of figure 2: $B_1 = \{1; 2; 3; 4; 5; 6; 7; 10; 18; 19\}$ and $B_2 = \{8; 9; 11; 12; 13; 14; 15; 16; 17; 20; 21; 22; 23\}$. So if $W_{ref_i} = 1, \forall i \in \{1..17\}$ and $K_{nom_j} = \sum\limits_{i=1}^{n+t} W_{ref_i}, \forall j \in \{24, 25\}$, then $M_{24} = \frac{17-8}{17} = 53\%$ and

$M_{25} = \frac{17-9}{17} = 47\%$.

M_j values the nominal-capacity-deviation percentage absorbable by the configuration Q without resorting to the load shedding of W_{ref}.
Then, a global robustness level for the configuration Q can be defined by the minimal load shedding margin valued by:

$$M(Q, W_{ref}, K_{nom}) = \min_{j \in [1,m]} M_j(Q, W_{ref}, K_{nom})$$

$M(Q, W_{ref}, K_{nom})$ is between 0 and 100%.
It measures a global neighborhood of K_{nom} in which the non load shedding is guaranteed.
The last challenge but not least is to build the most robust configuration. That is the subject of the following sections.

6. Definition and formulation of the robustness problem

The robustness problem consists in finding the configuration which maximizes its minimal load shedding margin. Using the previous notations, the problem can be stated as follows: knowing the reference load W_{ref}, the structure matrix H, the nominal capacity K_{nom}, find the configuration Q which maximizes the minimal load shedding margin. In the following of the paper, this problem will be referred as *Max-min Load shedding Margin Configuration Problem* (MLMCP). Note that MLMCP can be proved to be strongly NP-hard as in [8] by transformation of 3-partition problem [9].
MLMCP can be written as the following Mixed Integer Linear Programming:

$$(MILP) \begin{cases} max(M_{min}) & & \\ \sum_{i=1}^{n+t+m} Q_{ij} = 1 & \forall j \in [1, n] & (1) \\ \sum_{i=1}^{n+t+m} Q_{ij} \leq 1 & \forall j \in [n+1, n+t] & (2) \\ \sum_{i=1}^{n+t+m} x_{ij} - \sum_{k=1}^{n+t} x_{jk} = W_{ref_j} & \forall j \in [1, n+t] & (3) \\ x_{ij} - W_{ref_j} \times Q_{ij} \geq 0 & \forall (i,j) \in [1, n+t+m] \times [1, n] & (4a) \\ x_{ij} - sumW \times Q_{ij} \leq 0 & \forall (i,j) \in [1, n+t+m] \times [1, n+t+m] & (4b) \\ K_{nom_i} \times M_{min} + \sum_{j=1}^{n+t} x_{ij} \leq K_{nom_i} & \forall i \in [n+t+1, n+t+m] & (5) \\ Q_{ij} \leq H_{ij} & \forall (i,j) \in [1, n+t+m]^2 & (6) \\ Q_{ij} \in \{0, 1\} & \forall (i,j) \in [1, n+t+m]^2 & (7) \\ x_{ij} \geq 0 & \forall (i,j) \in [1, n+t+m]^2 & (8) \\ M_{min} \geq 0 & & (9) \end{cases}$$

H_{ij}, W_{ref_i}, $Knom_j$, and $sumW$ are given, whereas Q_{ij}, x_{ij}, M_j and M_{min} are decision variables. x_{ij} values the flow on arc (i, j). $sumW = \sum_{i=1}^{n+t} W_{ref_i}$.

The set of constraints (1) ensures that each customer is served by one and only one predecessor. As set of constraints (2) ensures that each junction node has at most one predecessor, the second electrotechnical constraint is satisfied (each node has at most one predecessor, so each customer node is served by one and only one source).
The set of constraints (3) expresses Kirchhoff's law. Coupled with set of constraints $(1)+(2)$, this guarantees that third electrotechnical constraint is satisfied. The set of constraints $(4a) + (4b)$ allows to link the flow x_{ij} with the switch position Q_{ij}. The set of constraints (5) allows to update the minimal load shedding margin M_{min}. The set of constraints (6) ensures that arc (i, j) belongs to the solution only if it structurally exists. The sets of constraints $(7), (8), (9)$ define decision variables type with their variation domains. Moreover, the constraint (9) ensures that fourth electrotechnical constraint is satisfied.

7. Solving and experiments

The Mixed Integer Linear Programm ($MILP$) has been implemented in C language using the free package *GNU Linear Programming Kit* [10].
It has been tested on 135 examples on a PC whose characteristics are as follows: RAM 1Gb, Intel Pentium processor IV 3GHz.
Each example consisted of an adjacency matrix H randomly generated. The reference load W_{ref} was set to 1 for each customer. The nominal capacity K_{nom} was set to $sumW$ for each source.
Examples are presented in the table 1.
Moreover an upper bound can be computed as follows. As all the sources capacity are identical ($= sumW$), at least one source must provide a power superior or equal to P_{min} given by the rounded up value of $\frac{sumW}{m}$.
So an upper bound for the highest minimal load shedding margin is valued by $M_{up} = \frac{sumW - P_{min}}{sumW}$.

Table 1: Characteristics of examples

Type	**Number of examples**	**Size** n **customers**	t **junctions**	s **sources**	**Average density** $\frac{\text{number of arcs}}{(\text{number of nodes})^2}$
type 1	45	$10 \rightarrow 50$	0	$2 \rightarrow 6$	15%
type 2	45	$10 \rightarrow 50$	0	$2 \rightarrow 6$	32%
type 3	45	$10 \rightarrow 50$	0	$2 \rightarrow 6$	49%

Table 2: Optimality and execution times

Type	**Percentage of examples optimally solved**	**Percentage of examples whose execution time is** $<$ **1 s**	$<$ **1 min**	$<$ **1 h**	$>$ **time limit**
type 1	96%	58%	91%	96%	4%
type 2	80%	38%	71%	80%	20%
type 3	69%	22%	58%	69%	31%

Table 3: Quality of results

Type	**Optimality**	**Relative distance with the upper bound** min.	av.	max.
type 1	yes (96%)	0%	4%	29%
	no (4%)	4%	6%	8%
type 2	yes (80%)	0%	2%	14%
	no (20%)	3%	11%	24%
type 3	yes (69%)	0%	2%	21%
	no (31%)	4%	12%	32%

An execution time limit arbitrarily equal to 1 hour has been fixed.
Results are presented in tables 2 and 3. The fields "*min.*", "*av.*" and "*max.*" respectively mean minimum, average and maximum.
Table 2 gives the percentage of examples optimally solved and the execution times. Table 3 gives the opportunity to measure the quality of the best obtained solutions in terms of relative distance between the obtained load shedding margin M_{obt} and the upper bound M_{up}. This distance is valued by $\frac{M_{up} - M_{obt}}{M_{up}}$.
Results presented in the table 2 allow to conclude that the proposed method guarantees that the best found solution is the optimal one for 96% of examples of type 1. But this score decreases with the size and the density of the examples as shown by the results for examples of type 2 and type 3 (resp. 80% and 69%).
In the same way, the execution times increase with the size and the density of adjacency matrices. For examples of type 1, 91% are solved before the first minute against 71% for examples of type 2 and 58% for examples of type 3.
The same conclusions can be made with the quality of results. In fact the distances between the best obtained load shedding margin and the upper bound increases with the density of examples (as shown in table 3). Moreover these distances are higher in the non-guaranteed optimal cases than in the optimal cases. However

the distances in non-optimal cases are always lower than 32% whereas even in optimal cases, the distance can reach 29%.

8. Conclusion and future works

The problem of the configuration of a power distribution network under sources capacity uncertainties has been studied in this article. A robustness level has been proposed: it evaluates the configuration ability to absorb a source capacity variation without resorting to load shedding. Then a Max-Min formulation of the robustness problem has been given help a Mixed Integer Linear Programm. The Max-Min approach has been experimented and the results make it possible to conclude that if the method is not always optimal (it does not guarantee to find the optimal solution in less than one hour), it remains a good heuristic because of its ability to find good solutions in a reasonable time ($<$ 1 hour).
However it should be interesting to develop some metaheuristics like genetic algorithms or tabu search, to compare the results and to see if metaheuristics allow to find as many optimal solutions within the same times. Moreover, a choice has been made to not consider the criterion of power losses which is the most considered in certain context. But is a robust configuration, also a less power losses consuming configuration?

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