Fair and Efficient Bandwidth Allocation with the Reference Point Methodology

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1. Introduction

A fair way of distribution of the bandwidth among competing demands becomes a key issue in computer networks [3] and telecommunications network design, in general [11, 12]. The so-called Max-Min Fairness (MMF) [1, 5, 6] is widely considered as such ideal fairness criteria. Indeed, the lexicographic max-min optimization used in the MMF approach generalizes equal sharing at a single link bandwidth to any network while maintaining the Pareto optimality. Certainly, allocating the bandwidth to optimize the worst performances may cause a large worsening of the overall throughput of the network. Therefore, network management must consider two goals: increasing throughput and providing fairness. The purpose of this work is to show that there exists a multiple criteria model that allows to represent consistently the overall efficiency and fairness goals. Moreover, the criteria measure actual network throughput for various levels (targets) of flows. Thereby, the criteria can easily be introduced into the model and they allow to apply effectively the reference point methodology where the decision maker specifies preferences in terms of aspiration levels (reference point), i.e., by introducing desired (acceptable) levels for several criteria.

The problem of network dimensioning with elastic traffic can be formulated as follows [11]. Given a network topology $G =< V, E >$, consider a set of pairs of nodes as the set $I = \{1, 2, \ldots, m\}$ of services representing the elastic flow from source v_i^s to destination v_i^d. For each service, we have given the set P_i of possible routing paths in the network from the source to the destination. This information can be summarized in the form of binary matrices $\mathbf{\Delta_e} = (\delta_{eip})_{i\in I;p\in P_i}$ assigned to each link $e \in E$, where $\delta_{eip} = 1$ if link e belongs to the routing path $p \in P_i$ (connecting v_i^s with v_i^d) and $\delta_{eip} = 0$ otherwise. For each service $i \in I$, the elastic flow from source v_i^s to destination v_i^d is a variable representing the model outcome and it will be denoted by x_i. This flow may be realized along various paths $p \in P_i$ and it is modeled as $x_i = \sum_{p\in P_i} x_{ip}$ where x_{ip} (for $p \in P_i$) are nonnegative variables representing the elastic flow from source v_i^s to destination v_i^d along the routing path p. Although, the single-path model requires additional multiple choice constraints to enforce nonbifurcated flows.

The network dimensioning problem depends on allocating the bandwidth to several links in order to maximize flows of all the services (demands). Typically, the network is already operated and some bandwidth is already allocated (installed) and decisions are rather related to the network expansion. Therefore, we assume that each link $e \in E$ has already capacity a_e while decision variables ξ_e represent the bandwidth newly allocated to link $e \in E$ thus expanding the link capacity to $a_e + \xi_e$. Certainly, all the decision variables must be nonnegative: $\xi_e \geq 0$ for all $e \in E$ and there are usually some bounds (upper limits) on possible expansion of the links capacities: $\xi_e \leq \bar{a}_e$ for all $e \in E$. Finally, the following constraints must be fulfilled:

$$0 \leq x_{ip} \leq M u_{ip},\ u_{ip} \in \{0, 1\} \qquad \forall\, i \in I;\ p \in P_i \tag{1}$$

$$\sum_{p\in P_i} u_{ip} = 1, \quad \sum_{p\in P_i} x_{ip} = x_i \qquad \forall\, i \in I \tag{2}$$

$$\sum_{i\in I} \sum_{p\in P_i} \delta_{eip} x_{ip} \leq a_e + \xi_e, \quad 0 \leq \xi_e \leq \bar{a}_e \qquad \forall e \in E \tag{3}$$

$$\sum_{e\in E} c_e \xi_e \leq B \tag{4}$$

where (1)–(2) represent single-path flow requirements using additional binary (flow assignment) variables u_{ip} and define the total service flows. Next, (3) establish the relation between service flows and links bandwidth. The quantity $y_e = \sum_{i \in I} \sum_{p \in P_i} \delta_{eip} x_{ip}$ is the load of link e and it cannot exceed the available link capacity. Further, while allocating the bandwidth to several links the decisions must keep the cost within available budget B (4) where for each link $e \in E$ the cost of allocated bandwidth is c_e.

The network dimensioning model can be considered with various objective functions, depending on the chosen goal. One may consider two extreme approaches. The first extreme is the maximization of the total throughput (the sum of flows) $\sum_{i \in I} x_i$. On the other extreme, the network flows between different nodes should be treated as fairly as possible which leads to the maximization of the smallest flow or rather to the lexicographically expanded max-min optimization (the so-called max-min ordering) allowing also to maximize the second smallest flows provided that the smallest remain optimal, the third smallest, etc. This approach is widely recognized in networking as the so-called Max-Min Fairness (MMF) [1, 5]. The throughput maximization can always result in extremely unfair solutions allowing even for starvation of certain flows while the MMF solution may cause a large worsening of the throughput of the network. In an example built on the backbone network of a Polish ISP, it turned out that the throughput in a perfectly fair solution could be less than 50% of the maximal throughput [10]. Network management may be interested in seeking a compromise between the two extreme approaches [2, 4]. In the following, we shall describe an approach that allows to search for various compromise solutions with multiple linear criteria. All these criteria represent partial throughput for several target levels of flows.

2. Fair Allocations and Partial Throughputs

The bandwidth allocation problem we consider may be viewed as a special case of general resource allocation problem where a set I of m services is considered and for each service $i \in I$, its measure of realization x_i is a function $x_i = f_i(\xi)$ of the allocation pattern $\xi \in A$. This function, called the individual objective function, in our applications expresses the service flow and a larger value of the outcome means a better effect (higher service quality or client satisfaction). This leads us to a vector maximization problem:

$$\max \{(x_1, x_2, \ldots, x_m) : \; \mathbf{x} \in Q\} \tag{5}$$

where $Q = \{(x_1, \ldots, x_m) : x_i = f_i(\xi), \quad i \in I, \; \xi \in A\}$ denotes the attainable set for outcome vectors $\mathbf{x}$. For the network dimensioning problems, we consider, the set Q is defined by constraints (1)–(4). Model (5) only specifies that we are interested in maximization of all outcomes x_i for $i \in I$. In order to make it operational, one needs to assume some solution concept specifying what it means to maximize multiple outcomes. The commonly used concept of the Pareto-optimal solutions, as feasible solutions for which one cannot improve any criterion without worsening another, depends on the rational dominance $\succeq_r$ which may be expressed in terms of the vector inequality: $\mathbf{x}' \succeq_r \mathbf{x}''$ iff $x'_i \geq x''_i$ for all $i \in I$.

In order to ensure fairness in a system, all system entities have to be equally well provided with the system's services. This leads to concepts of fairness expressed by the equitable rational preferences [9, 7]. First of all, the fairness requires impartiality of evaluation, thus focusing on the distribution of outcome values while ignoring their ordering. Hence, we assume that the preference model is impartial (anonymous, symmetric).

$$(x_{\tau(1)}, x_{\tau(2)}, \ldots, x_{\tau(m)}) \cong (x_1, x_2, \ldots, x_m) \tag{6}$$

for any permutation τ of I. Further, fairness requires equitability of outcomes which causes that the preference model should satisfy the (Pigou–Dalton) principle of transfers:

$$\mathbf{x} - \varepsilon \mathbf{e}_{i'} + \varepsilon \mathbf{e}_{i''} \succ \mathbf{x}, \quad 0 < \varepsilon < x_{i'} - x_{i''} \tag{7}$$

where $\mathbf{e}_i$ denotes the ith unit vector. The principle of transfers states that a transfer of any small amount from an outcome to any other relatively worse–off outcome results in a more preferred outcome vector. The rational preference relations satisfying additionally axioms (6) and (7) are called hereafter *fair (equitable)* *rational preference relations*. We say that outcome vector $\mathbf{x}'$ *fairly dominates* $\mathbf{x}''$ ($\mathbf{x}' \succ_e \mathbf{x}''$), iff $\mathbf{x}' \succ \mathbf{x}''$ for all fair rational preference relations $\succeq$. In other words, $\mathbf{x}'$ fairly dominates $\mathbf{x}''$, if there exists a finite sequence of vectors $\mathbf{x}^j$ ($j = 1, 2, \ldots, s$) such that $\mathbf{x}^1 = \mathbf{x}''$, $\mathbf{x}^s = \mathbf{x}'$ and $\mathbf{x}^j$ is constructed from $\mathbf{x}^{j-1}$ by application of either permutation of coordinates, equitable transfer, or increase of a coordinate. An allocation pattern $\xi \in A$ is called *fairly (equitably) efficient* if $\mathbf{x} = \mathbf{f}(\xi)$ is fairly nondominated. Note that each fairly efficient solution is also Pareto-optimal, but not vice verse. The theory of majorization [8] includes the results which allow us to express the relation of fair (equitable) dominance as a vector inequality on the cumulative ordered outcomes [7]. This can be mathematically formalized as follows. First, introduce the ordering map $\Theta : R^m \to R^m$ such that $\Theta(\mathbf{x}) = (\theta_1(\mathbf{x}), \theta_2(\mathbf{x}), \ldots, \theta_m(\mathbf{x}))$, where $\theta_1(\mathbf{x}) \leq \theta_2(\mathbf{x}) \leq \cdots \leq \theta_m(\mathbf{x})$ and there exists a permutation τ of set I such that $\theta_i(\mathbf{x}) = x_{\tau(i)}$ for $i = 1, \ldots, m$. Next, apply to ordered outcomes $\Theta(\mathbf{x})$, a linear cumulative map thus resulting in the *cumulative ordering map* $\bar{\Theta}(\mathbf{x}) = (\bar{\theta}_1(\mathbf{x}), \bar{\theta}_2(\mathbf{x}), \ldots, \bar{\theta}_m(\mathbf{x}))$ defined as $\bar{\theta}_i(\mathbf{x}) = \sum_{j=1}^{i} \theta_j(\mathbf{x})$. The coefficients of vector $\bar{\Theta}(\mathbf{x})$ express, respectively: the smallest outcome, the total of the two smallest outcomes, the total of the three smallest outcomes, etc. The theory of majorization allow us to derive the following statement [7]. Outcome vector $\mathbf{x}'$ fairly dominates $\mathbf{x}''$, if and only if $\bar{\theta}_i(\mathbf{x}') \geq \bar{\theta}_i(\mathbf{x}'')$ for all $i \in I$ where at least one strict inequality holds.

The ordered achievement vectors describe a distribution of outcomes generated by a given decision $\mathbf{x}$. In the case when there exists a finite set of all possible outcomes of the individual objective functions, we can directly deal with the distribution of outcomes described by frequencies of several outcomes. Let $V = \{v_1, v_2, \ldots, v_r\}$ (where $v_1 < v_2 < \cdots < v_r$) denote the set of all attainable outcomes (all possible values of the individual flows $x_i = f_i(\xi)$ for $\xi \in A$). We introduce integer functions $h_k(\mathbf{x})$ ($k = 1, \ldots, r$) expressing the number of values v_k taken in the outcome vector $\mathbf{x}$. Having defined the functions h_k we can introduce cumulative distribution functions $\bar{h}_k(\mathbf{x}) = \sum_{l=1}^{k} h_l(\mathbf{x})$ for $k = 1, \ldots, r$. The function $\bar{h}_k$ expresses the number of outcomes smaller or equal to v_k. Since we want to maximize all the outcomes, we are interested in the minimization of all the functions $\bar{h}_k$.

In order to take into account the principle of transfers we need to distinguish values of outcomes smaller or equal to v_k. For this purpose we weight vector $\bar{\mathbf{h}}(\mathbf{x})$ to get:

$$\hat{h}_k(\mathbf{x}) = \sum_{l=1}^{k-1} (v_{l+1} - v_l)\bar{h}_l(\mathbf{x}) = \sum_{l=1}^{k-1}(v_k - v_l)h_l(\mathbf{x}) = \sum_{i=1}^{m} \max\{v_k - x_i, 0\} \tag{8}$$

for $k = 2, \ldots, r$ and $\hat{h}_1(\mathbf{x}) = 0$. In other words, $\hat{h}_k(\mathbf{x})$ expresses the total of differences between v_k and all the outcomes x_i smaller than v_k. Since $(v_k - v_l) > 0$ for $1 \leq l < k$, it follows from (8) that vector function $\hat{\mathbf{h}}(\mathbf{x})$ provides a unique description of the distribution of coefficients of vector $\mathbf{x}$. Moreover, for achievement vectors $\mathbf{x}', \mathbf{x}'' \in V^m$, $\hat{\mathbf{h}}(\mathbf{x}') \leq \hat{\mathbf{h}}(\mathbf{x}'') \quad \Leftrightarrow \quad \bar{\Theta}(\mathbf{x}') \geq \bar{\Theta}(\mathbf{x}'')$ [9]. This permits one to express fair efficiency for problem (5) in terms of the standard efficiency for the multiple criteria problem with objectives $\hat{\mathbf{h}}(\mathbf{x})$:

$$\min \{(\hat{h}_1(\mathbf{x}), \hat{h}_2(\mathbf{x}), \ldots, \hat{h}_r(\mathbf{x})) : \ \mathbf{x} \in Q\}. \tag{9}$$

Namely, a feasible solution $\mathbf{x} \in Q$ is an fairly efficient solution of the multiple criteria problem (5), if and only if it is an efficient solution of the multiple criteria problem (9). Note that $\hat{h}_1(\mathbf{x}) = 0$ for any $\mathbf{x}$ which means that the first criterion is constant and redundant in problem (9). Moreover, $mv_r - \hat{h}_r(\mathbf{x}) = \sum_{i=1}^{m} x_i$. thus representing the total throughput. Similarly, one may define for all k the complementary quantities $\eta_k(\mathbf{x}) = mv_k - \hat{h}_k(\mathbf{x}) = \sum_{i=1}^{m} \min\{x_i, v_k\}$ expressing the corresponding partial throughputs generated by flows ranged to v_k. Therefore, the entire multiple criteria model (9) can be reformulated into η_k criteria to be maximized. In order to reduce the problem size one may attempt the restrict the number of distinguished outcome values.

Let us consider a sequence of indices $K = \{k_1, k_2, \ldots, k_q\}$, where $v_{k_1} < v_{k_2} < \ldots < v_{k_{q-1}} < v_{k_q}$, and the corresponding restricted form of the multiple criteria model (9):

$$\max \{(\eta_{k_1}, \ldots, \eta_{k_q}) : \ \mathbf{x} \in Q\} \tag{10}$$

with only $q < r$ criteria. Multiple criteria model (9) allows us to generate any fairly efficient solution of problem (5). While restricting the number of criteria in the multiple criteria model (10) we can essentially still expect reasonably fair efficient solution and only *unfairness* may be related to the distribution of flows within classes of skipped criteria. In other words, we have guaranteed some rough fairness while it can be possibly improved by redistribution of flows within the intervals $(v_{k_j}, v_{k_{j+1}}]$ for $j = 1, 2, \ldots, q-1$. Finally, we may generate various fairly efficient bandwidth allocation patterns as efficient solutions of the multiple criteria problem:

$$\begin{array}{rl} \max & (\eta_k)_{k \in K} \\ \text{s.t.} & \mathbf{x} \in Q \\ & \eta_k = \sum_{i \in I} t_{ki}, \quad k \in K \\ & t_{ki} \leq x_i, \quad t_{ki} \leq v_k, \quad i \in I,\ k \in K \end{array} \tag{11}$$

where auxiliary variable t_{ki} represent $\min\{x_i, v_k\}$. Note that formulation (11) adds only linear constraints to the attainable set Q defined by constraints (1)–(4).

3. Reference Point Analysis

The simplest way to model a large gamut of fairly efficient allocations with the multiple criteria problem (11) may depend on the use some combinations of criteria $(\eta_k)_{k \in K}$. Better controllability and the complete parameterization of nondominated solutions for discrete problems can be achieved with the direct use of the reference point methodology introduced by Wierzbicki [13] and later extended leading to efficient implementations of the so-called aspiration/reservation based decision support (ARBDS) approach The ARBDS approach allows the decision maker (DM) to specify the requirements in terms of aspiration and reservation levels, i.e., by introducing acceptable and required values for several criteria. Depending on the specified aspiration and reservation levels, a special scalarizing achievement function is built and maximized thus generating an efficient solution. The solution is accepted by the DM or some modifications of the aspiration and reservation levels are introduced to continue the search for a better solution. When applying the ARBDS methodology to the multiple target model (11), one may generate various fairly efficient solutions of the original problem (5). The generic scalarizing achievement function takes the following form [13]:

$$\sigma(\eta) = \min_{k \in K}\{\sigma_k(\eta_k)\} + \varepsilon \sum_{k \in K} \sigma_k(\eta_k) \tag{12}$$

where ε is an arbitrary small positive number and σ_k, for $k \in K$, are the partial achievement functions measuring actual achievement of the individual outcome η_k with respect to the corresponding aspiration and reservation levels (η_k^a and η_k^r, respectively). Thus the scalarizing achievement function is, essentially, defined by the worst partial (individual) achievement but additionally regularized with the sum of all partial achievements. The regularization term is introduced only to guarantee the solution efficiency in the case when the maximization of the main term (the worst partial achievement) results in a non-unique optimal solution.

The partial achievement function σ_k can be interpreted as a measure of the DM's satisfaction with the current value (outcome) of the k-th criterion. It is a strictly increasing function of outcome η_k with value $\sigma_k = 1$ if $\eta_k = \eta_k^a$, and $\sigma_k = 0$ for $\eta_k = \eta_k^r$. We use the piecewise linear partial achievement function [9]:

$$\sigma_k(\eta_k) = \begin{cases} \gamma(\eta_k - \eta_k^r)/(\eta_k^a - \eta_k^r), & \eta_k \leq \eta_k^r \\ (\eta_k - \eta_k^r)/(\eta_k^a - \eta_k^r), & \eta_k^r < \eta_k < \eta_k^a \\ \beta(\eta_k - \eta_k^a)/(\eta_k^a - \eta_k^r) + 1, & \eta_k \geq \eta_k^a \end{cases}$$

where β and γ are arbitrarily defined parameters satisfying $0 < \beta < 1 < \gamma$. In our implementation the values $\beta = 0.01$ and $\gamma = 100$ have been used. This partial achievement function is strictly increasing and concave which guarantees its LP computability with respect to outcomes η_k. Recall that in our model outcomes η_k represent partial throughputs for ranged flows x_i. Hence, the reference vectors (aspiration and reservation) represent, in fact, some reference distributions of outcomes (flows). If considering perfectly equal flows ϕ as the reference (aspiration or reservation) distribution, one needs to set the corresponding levels as $\eta_k = m\phi$ for $\phi \leq v_k$ and $\eta_k = mv_k$ otherwise.

The reference distribution approach has been tested on a sample network dimensioning problem with elastic traffic. The network topology is patterned after the backbone network of a Polish ISP [10]. The network consists of 12 nodes and 18 links. Flows between any pair of different nodes have been considered (132 flows). For each flow, two alternative paths have been specified that could be used for transport. All links have unit costs equal to one, and the total budget is $B = 1000$. We have analyzed the network dimensioning problem defined by constraints (1)–(4). Thus the model under consideration allows flows to choose one of two paths for transport (1)–(2) while providing also some free link capacity for certain links (3). Actually, free link capacity was set to 10, and the upper limit on the expansion capacity was set to 30.

The final input to the model consisted of the reservation and aspiration levels for the total throughput within ranges of the specified target flow values. We have preselected 11 target flow values: ten values $v_k = k$ ($k = 1, 2, \ldots, 10$), and $v_{11} = 20$. For simplicity of the analysis, all aspiration levels were set larger than the maximum possible value $mv_k + 1$, and only reservation levels were used to control the outcome flows. One of the most significant parameters was the reservation level for the largest target value representing actually the required network throughput. This value denoted by η_r^r was selected separately from the other reservation levels. All the other reservation levels were formed as a linearly increasing sequence of the ordered values with slope (step) s. Exactly, a value ϕ is selected as required minimal flow and further the (ordered) required flows are defined as $\phi + (i-1)s$. Hence, one gets the reservation levels $\eta_k^r = mv_k$ for $v_k \leq \phi$ and $mv_k - \bar{k}(\bar{k}-1)s/2$ where $\bar{k} = (v_k - \phi)/s$. For the sake of simplicity, we select value ϕ as one of the lower target values v_k. Thus we have 3 control parameters: reservation level for total throughput, minimum required flow, and the slope of ordered required flows.

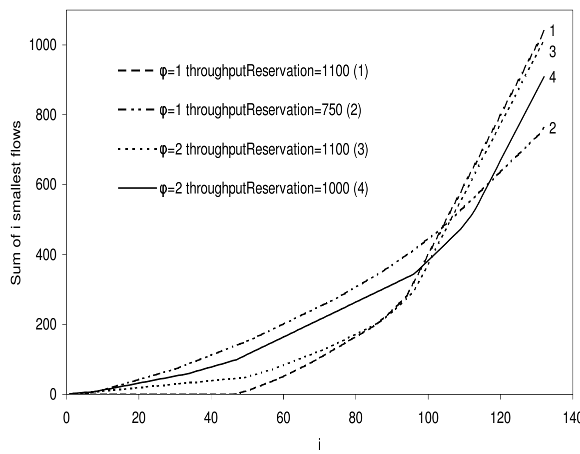

Figure 1: Flows distribution for varying parameters in the interactive analysis.

Consider a network designer who wishes to extend link capacities of a network under analysis. First, she may search for a solution that has a high overall throughput by setting 1100 as the corresponding reservation, and some reasonable slope value $s = 0.04$ and the required minimal flow value $\phi = 1$. Solution 1, shown in Fig. 1, indeed provides a high throughput (about 1050) but is unfair with more than 40 flows completely starved. When the network designer relaxes the throughput reservation to 750, she gets Solution 2 that is quite close to perfectly fair, but has a low throughput (750). To identify a relatively fair solution with a larger throughput, the network designer returns to $\eta_m^r = 1100$, but increases the required smallest flow to $\phi = 2$. This leads to Solution 3 with all positive flows, (but about 40 of them below 2) and a total throughput of about 1000. The solution plot is very similar to that of Solution 1, thus depicting strong unfairness. To reach a more fair solution, the network designer accepts a decrease of the total throughput. When setting $\eta_m^r = 1000$, she gets Solution 4 with a total throughput about 900, but similarly fair as Solution 2. In Solution 4, about 80% of (smallest) flows remain on the level close to those of Solution 2, while about 20% receive much larger values, thus increasing the total throughput. The network designer finds Solution 4 an acceptable compromise. The presented analysis is simplified, but it demonstrates that it is possible to easily find a satisfactory fair and efficient allocation pattern in a few interactive steps. The computation time to generate a single solution does not exceed 200 seconds when using CPLEX 9.1 on an 1.4GHz PC. Moreover, the plots of cumulated ordered flows turn out to be a convenient graphical interface to support the search process.

4. Concluding Remarks

A central issue in networking is how to allocate bandwidth to flows efficiently and fairly. We have shown that there exists a multiple criteria model that allows to represent consistently the overall efficiency and fairness goals. The criteria measure actual network throughputs for various levels (targets) of flows. Thereby, the criteria can easily be introduced into the model. While looking for fairly efficient bandwidth allocation the reference point methodology can be applied to the multiple target partial throughputs. Our initial experiments on the sample network topology demonstrated the versatility of the described methodology. The use of reservation levels, controlled by a small number of simple parameters, allowed us to search for solutions best fitted to various possible preferences of a network designer.

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References

[1] Bertsekas, D., and R. Gallager, *Data Networks*, Prentice-Hall, Englewood Cliffs, NJ, 1987.

[2] Bonald, T., and L. Massoulie, "Impact of Fairness on Internet Performance", *Proc. ACM Sigmetrics*, pp. 82–91, 2001.

[3] Denda, R., A. Banchs, and W. Effelsberg, "The Fairness Challenge in Computer Networks", *Lect. Notes in Comp. Sci.*, vol. 1922, pp. 208–220, 2000.

[4] Goel, A., A. Meyerson, and S. Plotkin. "Combining Fairness with Throughput: Online Routing wih multiple Objectives", *J. Comput. Syst. Sci.*, vol. 63, pp. 62–79, 2001.

[5] Jaffe, J., "Bottleneck Flow Control", *IEEE Trans. on Communications*, vol. 7, pp. 207–237, 1980.

[6] Kleinberg, J., Y. Rabani, and E. Tardos. "Fairness in Routing and Load Balancing", *J. Comput. Syst. Sci.*, vol. 63, pp. 2–20, 2001.

[7] Kostreva, M.M., and W. Ogryczak, "Linear Optimization with Multiple Equitable Criteria", *RAIRO Oper. Res.*, vol. 33, pp. 275–297, 1999.

[8] Marshall, A.W., and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*, Academic Press, New York, 1979.

[9] Ogryczak, W., *Linear and Discrete Optimization with Multiple Criteria: Preference Models and Applications to Decision Support* (in Polish). Warsaw Univ. Press, Warsaw, 1997.

[10] Ogryczak, W., T. Śliwiński, and A. Wierzbicki, "Fair Resource Allocation Schemes and Network Dimensioning Problems", *J. Telecommunications and Info. Tech.*, no. 3, pp. 34–42, 2003.

[11] Pióro, M., and D. Medhi, *Routing, Flow and Capacity Design in Communication and Computer Networks*. Morgan Kaufmann, San Francisco, 2004.

[12] Tang, A., J. Wang, and S.H. Low, "Is Fair Allocation Always Inefficient", *IEEE INFOCOM*, vol. 1, pp. 35-45, 2004.

[13] Wierzbicki, A.P., "A Mathematical Basis for Satisficing Decision Making", *Math. Modelling*, vol. 3, pp. 391–405, 1982.