OSPF Routing with Optimal Oblivious Performance Ratio Under Polyhedral Demand Uncertainty*†

Ayşegül Altın, *Department of Industrial Engineering, Bilkent University, Ankara, Turkey*
Pietro Belotti, *Tepper School of Business, Carnegie Mellon University, Pittsburgh PA*
Mustafa Ç. Pınar, *Department of Industrial Engineering, Bilkent University, Ankara, Turkey*

Keywords: OSPF, oblivious routing, traffic engineering, ECMP, branch-and-price, traffic uncertainty.

1. Introduction

Open Shortest Path First (OSPF) is a *link-state* routing protocol developed for Internet Protocol (IP) networks where routers send the traffic between all nodes in an internetwork along the corresponding shortest paths composed of *available* links of the underlying network. There are different approaches for determining the link metric to define these shortest paths. The traditional one is to fix link weights using some measure like physical distances or the inverse of link capacities (De Giovanni et al. [5]). However, the management of link metric to optimize some design and routing criteria is the focus of the recent references (Tomaszewski et al. [1], Fortz and Thorup [2], Holmberg and Yuan [4], Broström and Holmberg [8]).

The "fairness" of a routing can be measured by the utilization of the most congested link. If some flow is distributed among the links in proportion to their capacities such that none of them becomes the bottleneck link, the routing is relatively fair. Moreover, when there is a *set* of feasible demand matrices rather than a single one, the routing needs to be assessed irrespective of the realization of the traffic matrix. Such a routing is called *oblivious* since the path between every node pair is chosen independently of the current demand matrix.

Since it is not likely for traffic engineers to estimate the traffic demands with certainty in advance, considering some level of uncertainty in the definition of demand matrices would strengthen the traffic engineering efforts. Applegate and Cohen [3] study oblivious routing in the case of very limited information of traffic demands. Later, Mulyana and Killat [9] address the case where the polyhedra of demands is defined by simple outbound traffic constraints. More recently Belotti and Pınar [7] consider box and ellipsoidal uncertainty representations. In the present paper, we concentrate on the polyhedral demand uncertainty where the possible traffic matrices are assumed to lie in a polyhedron, defined by any set of linear inequalities.

The problem we study in this paper is a *best-OSPF style routing* problem. We incorporate weight management into our analysis since OSPF might lead to unsatisfactory network performance without a good traffic engineering. Moreover, we extend the previous works on polyhedral demands by using a general definition of the set of feasible traffic matrices in deference to the difficulty of having an exact estimate of the demands in real life. Finally, we apply the Equal Cost Multi-Path (ECMP) rule, which complies with the current forwarding technology. It is worth mentioning that these specifics of our problem make our models practically feasible. Moreover, the added flexibility via weight management and general demand definition improves the performance of the OSPF routing.

To the best of our knowledge, this is the first work where general traffic uncertainty is combined with the oblivious OSPF routing problem. We provide a compact mixed-integer formulation based on flow variables for the best oblivious OSPF routing problem. Furthermore, we present an alternative tree formulation using destination-based multiple shortest paths as well as a solution tool based on a specialized Branch-and-Price algorithm. Also, a relaxation of our flow formulation can be used to model the MPLS routing under general demand uncertainty. Hence we show that optimal oblivious MPLS routing can be found in polynomial time.

*Research supported through grant MISAG-CNR-1 jointly from TUBITAK, The Scientific and Technological Research Institution of Turkey, and CNR, Consiglio Nazionale delle Ricerche, Italy.

†Full version of the paper is available for download from http://www.andrew.cmu.edu/user/belotti/papers/obl-ospf.pdf

Thus, we can discuss the relative performances of the oblivious OSPF routing and the oblivious MPLS routing under a very general setting where any polyhedral definition of traffic demands can be used.

2. Oblivious Routing Under Polyhedral Demand Uncertainty

Consider the undirected graph $G = (V, E)$. We denote edges $\{h, k\} \in E$ as *links* of G. For each link we have the associated directed pairs (h, k) and (k, h), which we call the *arcs* of G. We denote this set of directed node pairs by A. Moreover, we suppose that each link $\{h, k\}$ is assigned c_{hk} units of capacity, which is available for the *total* flow on $\{h, k\}$ in both directions. The estimated traffic flow from the source node $s \in V$ to the sink node $t \in V$ is d_{st} where we define the set of such directed source-sink pairs as $Q = \{(s, t) : s, t \in V, s \neq t\}$ and a traffic matrix (TM) as $d = (d_{st})_{(s,t)\in Q}$. Although d is defined as a vector, the term *traffic matrix* is obiquitous in the Telecommunications literature, and we shall use the term *matrix* throughout to refer to vector d. We denote the fraction of d_{st} routed on the arc (h, k) by f^{st}_{hk}. Then the matrix $f = (f^{st}_{hk})_{(h,k)\in A,(s,t)\in Q}$ defines a *routing* if it satisfies the following conditions:

$$\sum_{k:\{h,k\}\in E} \left(f^{st}_{hk} - f^{st}_{kh}\right) = \begin{cases} 1 & h = s \\ -1 & h = t \\ 0 & \text{otherwise} \end{cases} \qquad \forall h \in V, (s, t) \in Q \tag{1}$$

$$0 \leq f^{st}_{hk} \leq 1 \qquad \forall (h, k) \in A, (s, t) \in Q \tag{2}$$

and we denote the set of all possible routings on G as Λ. The *maximum link utilization* is defined as $\max_{\{h,k\}\in E} \sum_{(s,t)\in Q} d_{st}(f^{st}_{hk} + f^{st}_{kh})/c_{hk}$. Consequently, the most fair routing for a TM d is the one with the smallest maximum link utilization:

$$\min_{f\in\Lambda} \max_{\{h,k\}\in E} \sum_{(s,t)\in Q} d_{st}(f^{st}_{hk} + f^{st}_{kh})/c_{hk}. \tag{3}$$

Consider now a *set* D of traffic matrices, for instance given by the overall traffic in different times of the day. The optimal oblivious routing problem consists in finding a routing for each source-sink pair $(s, t) \in Q$ that minimizes the maximum edge utilization, *independent* of the traffic matrix d. The best routing is required to support any feasible traffic matrix $d \in D$ in the most balanced way. So, the 'goodness' of each routing is measured with its closeness to optimality for any feasible traffic matrix $d \in D$ (Applegate and Cohen [3], Belotti and Pınar [7]). Then the *oblivious ratio* of f on the set D is

$$OR^f_D = \max_{d\in D} \frac{MaxU^f_d}{BEST_d} \tag{4}$$

where $MaxU^f_d$ is the solution of the inner maximization problem in (3) and $BEST_d$ is the smallest maximum link utilization ratio for d, or the solution of the problem (3). As a result, the problem of finding the routing with the minimum OR^f_D for the set D of traffic demands becomes

$$\min_{f\in\Lambda} \max_{d\in D} \max_{\{h,k\}\in E} \frac{\sum_{(s,t)\in Q} d_{st}(f^{st}_{hk} + f^{st}_{kh})/c_{hk}}{BEST_d}. \tag{5}$$

In the sequel, we swap the two $\max$ operators and model (5) as the following mathematical model:

$$\min r \tag{6}$$

$$\text{s.t.} \quad r \geq \max_{d\in D} \frac{\sum_{(s,t)\in Q} d_{st}(f^{st}_{hk} + f^{st}_{kh})}{c_{hk} BEST_d} \quad \forall \{h, k\} \in E \tag{7}$$

$$(1) - (2) \tag{8}$$

where (8) ensures that f is a routing. Notice that the definition of D is important in modeling and solving (6)-(8). Bearing in mind that the set D of feasible traffic matrices can be defined in various ways, we will consider the case of polyhedral demand uncertainty: traffic demand matrices are not known but are supposed to belong to a polyhedron defined by some linear inequalities specifying the capacity of routers or bounds on the traffic flow between some node pairs etc. Consequently, we consider the general traffic uncertainty model

$$D = \{d = (d_{st})_{(s,t)\in Q} : Ad \leq a,\ d \geq 0,\ d \neq 0\} \tag{9}$$

where $A \in \mathbb{R}^{H\times|Q|}$ and $a \in \mathbb{R}^H$ with H being the number of linear inequalities that define D.

An important point that needs some attention here is that unlike the case with fixed traffic demands, although here d is not known it should not be considered as a variable of the optimization model (6)-(8). It is instead a variable of the inner optimization model on the right-hand side of constraint (7). Due to the $\max$ operator in constraint (7), the model (6)-(8) is equivalent to a semi-infinite optimization model with one constraint (7) for each $d \in D$. We can reduce the model to its linear counterpart with the help of LP duality since each maximization problem in (7) is an LP for a given r and routing f.

Proposition 1 *Assuming that the traffic demand set D is subject to polyhedral uncertainty, solving the following LP would yield the optimal oblivious routing on $G = (V, E)$:*

$$\min r \tag{10}$$

$$\text{s.t.} \quad \sum_{k:\{h,k\}\in E} \left(f^{st}_{hk} - f^{st}_{kh}\right) = \begin{cases} 1 & h = s \\ -1 & h = t \\ 0 & \textit{otherwise} \end{cases} \qquad \forall h \in V, (s,t) \in Q \tag{11}$$

$$\chi_{hk} + \sum_{z=1}^{H} a_z \lambda^{hk}_z \leq 0 \qquad \forall \{h,k\} \in E \tag{12}$$

$$\Pi^{st}_{i,hk} - \Pi^{st}_{j,hk} + \eta_{ij,hk} \geq 0 \qquad \forall (i,j) \in A,\ (s,t) \in Q,\ \{h,k\} \in E \tag{13}$$

$$-\pi^{st}_{hk} + \sum_{z=1}^{H} a^{st}_z \lambda^{hk}_z \geq f^{st}_{hk} + f^{st}_{kh} \qquad \forall (s,t) \in Q,\ \{h,k\} \in E \tag{14}$$

$$-\sum_{\{i,j\}\in E} c_{ij}\eta_{ij,hk} + \chi_{hk} \geq -rc_{hk} \qquad \forall \{h,k\} \in E \tag{15}$$

$$0 \leq f^{st}_{hk} \leq 1 \qquad \forall (h,k) \in A, (s,t) \in Q \tag{16}$$

$$\eta_{ij,hk} \geq 0 \qquad \forall \{i,j\}, \{h,k\} \in E \tag{17}$$

$$\chi_{hk} \geq 0 \qquad \forall \{h,k\} \in E \tag{18}$$

$$\lambda^{hk}_z \geq 0 \qquad \forall z = 1,\ldots,H,\ \{h,k\} \in E. \tag{19}$$

where

$$\Pi^{st}_{i,hk} = \begin{cases} \pi^{st}_{hk} & \textit{if } i = s \\ 0 & \textit{if } i = t \\ \sigma^{st}_{i,hk} & \textit{otherwise} \end{cases} \qquad \forall i \in V, (s,t) \in Q,$$

and π^{st}_{hk}, $\sigma^{st}_{i,hk}$, $\eta_{ij,hk}$, χ_{hk}, and λ^{hk}_z are the variables of the dual problem defined for each maximization problem in (7).

Notice that the model (10)-(19) is a Linear Programming problem. Hence we proved that the optimal oblivious ratio for MPLS routing under general traffic uncertainty can be computed in polynomial time.

3. Modeling OSPF routing

The Open Shortest Path First (OSPF) routing protocols route flow between node pairs along the corresponding shortest paths defined with respect to some metric. We consider the so-called *best-OSPF style* routing where the link metric is determined to optimize the oblivious performance ratio. We use the dual shortest path problem and complementary slackness conditions to model this as in Holmberg and Yuan [4]. Moreover, we apply ECMP routing in which the demand d_{st} accumulated at some node i is split evenly among all shortest paths between i and t. To model this problem, we propose two MIP formulations. Weight management is incorporated by the integer variables θ_{ij}, which define the metric used at each arc (i, j) and range between 1 and Θ_{max}[1]. Besides, we define ρ_i^t as the shortest path distance between i and t according to the metric defined by the θ_{ij} variables. In order to impose ECMP constraints, we use a variable φ_i^{st} which gives the fraction of flow that, after entering node i, is split among different outgoing arcs due to the ECMP rules.

For the flow formulation, we also define the binary variable y_{ij}^t to indicate if the arc (i, j) is on some shortest path destined to node t. Note that OSPF is a source invariant routing scheme, and this is the reason why we do not need an index for the source node s in y variables.

Proposition 2 *The optimal oblivious OSPF routing on $G = (V, E)$ with equal load sharing under polyhedral demand uncertainty is found by using the following set of constraints together with (10)-(19):*

$$
\begin{aligned}
& f_{ij}^{st} \leq y_{ij}^t && \forall (i,j) \in A, (s,t) \in Q && (20)\\
& y_{ij}^t + \rho_j^t - \rho_i^t + \theta_{ij} \geq 1 && \forall (i,j) \in A, t \in V && (21)\\
& -y_{ij}^t - \frac{\rho_j^t - \rho_i^t + \theta_{ij}}{2\Theta_{max}} \geq -1 && \forall (i,j) \in A, t \in V && (22)\\
& f_{ij}^{st} \leq \varphi_i^{st} && \forall (i,j) \in A, (s,t) \in Q && (23)\\
& 1 + f_{ij}^{st} - \varphi_i^{st} \geq y_{ij}^t && \forall (i,j) \in A, (s,t) \in Q && (24)\\
& 1 \leq \theta_{ij} \leq \Theta_{max} && integer \;\; \forall (i,j) \in A && (25)\\
& y_{ij}^t \in \{0,1\} && \forall (i,j) \in A, t \in V && (26)\\
& 0 \leq \varphi_i^{st} \leq 1 && \forall i \in V, (s,t) \in Q && (27)
\end{aligned}
$$

where (23)-(24) are the ECMP constraints.

Even for medium sized networks, the size of our flow formulation can get very large and the time required to solve these problems using MIP solvers is quite long. Therefore, we propose an alternative tree formulation, whose linear relaxation can be solved by column generation. In this model each tree variable corresponds to a Shortest Paths Tree (*SP tree*), which is an acyclic graph such that, for at least one metric, all and only paths within the *SP tree* are the shortest ones. In other words, each *SP tree T* defines a routing configuration, where multipath routing is allowed, for its root node. We define the binary τ_T^t variable, which indicates whether the implicitly defined *SP tree* T is used to route all traffic flow ending at t or not. The tree formulation is obtained by simply replacing each y_{ij}^t with $\sum_{T \in \Omega_t \cap \Omega_{ij}} \tau_T^t$ in (20)-(27) and including the constraint $\sum_{T \in \Omega_t} \tau_T^t = 1$ where Ω_t is the set of *SP* trees with destination t and Ω_{ij} is the set of *SP* trees containing arc (i, j).

Notice that the number of *SP trees* in a network can be huge and hence we develop a branch-and-price algorithm that is strengthened by the inclusion of cutting planes as an exact solution tool for the tree formulation. B&P is a column generation integrated branch-and-bound technique which starts with a restricted LP relaxation with fewer variables than the original problem and applies column generation at each node of the B&B tree. In the pricing step we solve a shortest path problem, which ensures to yield *SP trees* of the desired structure, on auxiliary graphs created by considering the current dual solution.

4. Computational Experiments

In order to test our models as well as the B&P algorithm, we have considered two well known demand uncertainty definitions. We let $W \subseteq V$ be the set of demand and/or supply nodes, which we call *terminal*

[1] This is a parameter of value 65535 and is the common constant used in the literature when integer link weights are required.

nodes. Moreover, $Q = \{(s,t) : s,t \in W, s \neq t\}$ is the set of directed demand pairs with flow demands d_{st}. The first case is the Hose Model of Duffield et al. [6] that defines the polyhedra of feasible demands as

$$D = \{d \in \mathbb{R}^{|Q|} : \sum_{t \in W \setminus \{s\}} d_{st} \leq b_s^+; \quad \sum_{t \in W \setminus \{s\}} d_{ts} \leq b_s^-; \quad d_{st} \geq 0 \quad \forall (s,t) \in Q\} \tag{28}$$

where b_s^- and b_s^+ are the ingress and egress capacities of the terminal node $s \in W$, respectively. In addition, the second uncertainty model is the Bertsimas-Sim model with the definition

$$D = \{d \in \mathbb{R}^{|Q|} : d'_{st} \leq d_{st} \leq d'_{st} + \hat{d}_{st} \quad \forall (s,t) \in Q; \ \textstyle\sum_{(s,t) \in Q} \frac{d_{st} - d'_{st}}{\hat{d}_{st}} \leq \Gamma\} \tag{29}$$

where Γ is a measure of the trade off between the level of conservatism of the model and its robustness.

We have implemented the B&P method within the MINTO framework, and used AMPL/Cplex 9.1 to solve the other two formulations. We have set a time limit of 2 hours for all models. A sample of our test results for the two models are given in Table 1 and Table 2. In both tables, the columns N, E, and W show the number of nodes, edges, and terminals in each instance, respectively. Columns z contain the oblivious ratios obtained by solving the tree and flow formulations as well as the MPLS routing problem. Columns t show the solution times in CPU seconds. Column p in Table 2 is a measure of the level of uncertainty defined in terms of the gaps between the lower and upper bounds on pairwise demands in (29).

In these tables, a number with a * or in brackets means that we could find an upper or a lower bound for that instance, respectively. Besides, MA or *2 hrs* in the t columns indicate termination due to insufficient memory or time limit.

Instance	N	E	W	z_{tree}	t_{tree}	z_{flow}	t_{flow}	z_{mpls}	t_{mpls}
Exodus	7	12	7	1	0.04	1	0.05	1	0.031
nsf	8	20	5	4*	MA	2	2839.41	1.517	0.403
VNSL	9	22	3	1.0655	0.85	1.0655	0.214	1	0.16
example	10	30	4	2.7*	MA	2.25*	2 hrs	1.079	0.767
bhvac	19	44	11	(2.853)	2 hrs	(1.524)	2 hrs	(1.515)	2 hrs
Abovenet	19	68	5	(1.116)	2 hrs	(1.116)	2 hrs	1.045	326.125
Telstra	44	88	7	1.925	420	1.925	1.231	1.283	0.084

Table 1: Results for the hose uncertainty model

5. Conclusion

We have studied a network routing problem that combines two well-known problems: oblivious routing under polyhedral demand uncertainty and OSPF routing. These problems have been studied separately in the past, but the joint problem has appeared to be of interest only recently, and to the best of our knowledge this is the first paper in that direction.

Mulyana and Killat [9] considers inclusion of polyhedral demands with OSPF routing in a very specific form of uncertainty definition. Applegate and Cohen [3] initiates the works on oblivious routing with demand uncertainty, which has been extended by Belotti and Pınar [7] recently.

We have proposed two Mixed Integer Linear Programming models, namely the flow and tree formulations. We have tested our models and the B&P algorithm on two traffic uncertainty definitions. We have presented a comparison of the two formulations in terms of the solution quality and computation times. We have observed that it pays to create a specialized B&P algorithm especially for the BS uncertainty case. Additionally, we have compared the OSPF style routing and the MPLS style routing for these two traffic polyhedra. First, we have realized that for the BS case the optimal oblivious ratios for both routing styles increase as the level

Instance	N	E	W	p	z_{tree}	t_{tree}	z_{flow}	t_{flow}	z_{mpls}	t_{mpls}
Exodus	7	12	7	1.1	1	0.06	1	0.052	1	0.052
				2	1	0.05	1	0.051	1	0.048
nsf	8	20	5	1.1	1.168*	2 hrs	1.05*	2 hrs	1.013	0.368
				2	2.045*	2 hrs	1.556	3821.53	1.44	0.752
VNSL	9	22	3	1.1	1.066	4.02	1.066	0.19	1	0.016
				2	1.066	3.61	1.066	0.14	1	0.024
example	10	30	4	1.1	1	0.10	(1)	2 hrs	1	0.332
				2	1	0.16	1	1900.19	1.014	0.432
bhvac	19	44	11	1.1	1	109.63	(1)	2 hrs	1	81.177
				2	1	120.03	(1.0004)	2 hrs	1	23
Abovenet	19	68	5	1.1	1	12.78	1	60.78	1	9.917
				2	1	13.58	2.243*	2 hrs	1	42.073
Telstra	44	88	7	1.1	1	1.75	1	5.24	1	0.156
				2	1	1.79	1	4.62	1	0.158

Table 2: Results for the BS uncertainty model

of demand variability increases. Another important observation is that the performance of OSPF routing degrades more than the MPLS routing as the demand uncertainty increases. We plan to extend our investigation to other polyhedral sets of demands that have practical appliations in today's networks.

References

[1] A. Tomaszewski, M. Pióro, M. Dzida, and M. Zagożdżon, Optimization of administrative weights in IP networks using the branch-and-cut approach. Proceedings of *INOC 2005*, B2, pp. 393–400, 2005.

[2] B. Fortz and M. Thorup, Internet traffic engineering by optimizing OSPF weights. Proceedings of *IEEE INFOCOM 2000*, pp. 519–528, 2000.

[3] D. Applegate and E. Cohen, Making intra-domain routing robust to changing and uncertain traffic demands:Understanding fundamental tradeoffs. Proceedings of *SIGCOMM '03*, Karlsruhe, Germany, pp. 313–324, 2003.

[4] K. Holmberg and D. Yuan, Optimization of internet protocol network design and routing. *NETWORKS*, 43(1), pp. 39–53, 2004.

[5] L. De Giovanni, B. Fortz, and M. Labbé, A lower bound for the internet protocol network design problem. Proceedings of *INOC 2005*, B2, pp. 401–408, 2005.

[6] N. Duffield, P. Goyal, A. Greenberg, P. Mishra, K. Ramakrishnan, J.E. van der Merive, A flexible model for resource management in virtual private networks. *Proceedings, SIGCOMM*, pp. 95–108, 1999.

[7] P. Belotti and M.Ç. Pınar, Optimal oblivious routing under linear and nonlinear uncertainty, submitted to *Optimization and Engineering*, 2005.

[8] P. Broström and K. Holmberg, Design of IP/OSPF networks using a lagrangean heuristic on an in-graph based model. Proceedings of *INOC 2005*, B3, pp.702.

[9] E. Mulyana and U. Killat, Optimizing IP networks for uncertain demands using outbound traffic constraints. Proceedings of *INOC 2005*, pp. 695–701, 2005.