A New Heuristic Approach for the
One-Commodity Pickup-and-Delivery Traveling Salesman Problem

Hipólito Hernández-Pérez, *DEIOC, Universidad de La Laguna, 38271 La Laguna, Tenerife, Spain*
Inmaculada Rodríguez-Martín, *DEIOC, Universidad de La Laguna, 38271 La Laguna, Tenerife, Spain*
Juan José Salazar-González, *DEIOC, Universidad de La Laguna, 38271 La Laguna, Tenerife, Spain*

Keywords: pickup and delivery, traveling salesman problem, heuristic.

1. Introduction

The *One-Commodity Pickup-and-Delivery Traveling Salesman Problem* (1-PDTSP) is a generalization of the classical *Traveling Salesman Problem* (TSP), thus a finite set of cities is given and the travel distance between each pair of cities is assumed to be known. In addition, one specific city is considered to be a vehicle depot while the other cities are identified as customers and are divided into two groups according to the type of service required (delivery or pickup). There is a unique product to be transported between customers. Each delivery customer requires a given product amount and each pickup customer provides a given product amount. Any amount of the product collected from a pickup customer can be supplied to a delivery customer. It is assumed that the vehicle has a fixed upper-limit capacity and must start and end the route at the depot. The 1-PDTSP then calls for a minimum distance route for the vehicle to satisfy the customer requirements without ever exceeding the vehicle capacity. The route must be a Hamiltonian tour through all the cities. We will assume that the travel distances are symmetric.

An important assumption in 1-PDTSP is that any amount of product collected from a pickup customer can be supplied to any delivery customer. For example, the product could be milk supplied from cow farms (the pickup customers) to private residences (the delivery customers) with no special requirements on sources or destinations; another example arises when the product is money to be moved between bank branch offices. In other words, there is only one commodity with different sources and destinations. We consider the depot acts as a dummy customer so that total collected product is equal to total supplied product. It also provides the vehicle with any necessary load at the start, since we do not assume that the vehicle must start either empty or full load. That is, we will assume that the depot can also provide the vehicle with the necessary load to guarantee the feasibility of some routes that need a non-empty (or non-full) vehicle load when leaving the depot. Then, the initial load of the vehicle should also be determined in the 1-PDTSP.

The 1-PDTSP has several applications in routing a single commodity through a circular network in a graph connecting different sources and destinations. It was introduced by Hernández-Pérez [7]. Hernández-Pérez and Salazar-González [8] describe a branch-and-cut algorithm able to solve instances with up to 100 customers. The same authors propose in [9] two heuristics to deal with the problem.

Chalasani and Motwani [4] study the special case of the 1-PDTSP where the delivery and pickup quantities are all equal to one unit. This problem is called Q-delivery TSP where Q is the capacity of the vehicle. Anily and Bramel [1] consider the same problem and call it the Capacitated Traveling Salesman Problem with Pickups and Deliveries (CTSPPD). Both articles propose worst-case heuristic algorithms for Euclidian instances.

The 1-PDTSP is closely related to a problem known in literature as the Traveling Salesman Problem with Pickups and Deliveries (TSPPD). In the TSPPD there are pickup and delivery customers and a vehicle with a given capacity originally stationed in the depot. The travel distances are known and the objective is to find a minimum length tour for the vehicle visiting each customer exactly once. The difference between TSPPD and 1-PDTSP is that in the former the product collected from pickup customers is different from the product supplied to delivery customers. Moreover, the total amount of product collected from pickup customers must be delivered only to the depot, and the product collected from the depot must be delivered to the delivery customers. For example, this is the case when empty bottles must be collected from customers and taken to a warehouse and full bottles must be delivered from the warehouse to the customers. The TSPPD was introduced by Mosheiov [10]. Anily and Mosheiov [2] present approximation algorithms for the TSPPD, here renamed TSP with Delivery and Backhauls, and Gendreau, Laporte and Vigo [6] propose several heuristics tested on instances with up to 200 customers. Baldacci, Hadjiconstantinou and Mingozzi [3] deal with the same problem, here named TSP with Delivery and Collection constraints, and present an exact algorithm based on a two-commodity network flow formulation which was able to prove optimality of some TSPPD instances with 200 customers. An important remark is that the TSPPD instances can be transformed in 1-PDTSP instances, hence the algorithms of the 1-PDTSP can be used to solve the TSPPD (see [8] and [9]).

There are many other pickup-and-delivery problems described in the literature. For recent surveys, we refer the reader to Savelsbergh and Sol [13], Courdeau and Laporte [5], and Parragh, Doerner and Hartl [11], [12].

We introduce now the notation used throughout the paper. The depot will be denoted by 1 and each customer by i ($i = 2, \ldots, n$). The set $V := 1, 2, \ldots, n$ is the vertex set and E is the edge set. For each pair of locations $\{i, j\}$, the travel distance (or cost) c_{ij} of traveling between i and j is given. A non-zero demand q_i associated with each customer i is also given, being $q_i < 0$ if i is a delivery customer and $q_i > 0$ if i is a pickup customer. Let $K := max\{\sum_{i \in V: q_i > 0} q_i, -\sum_{i \in V: q_i < 0} q_i\}$. The capacity of the vehicle is represented by Q and is assumed to be a positive number. Note that typically $Q \leq K$ on a 1-PDTSP instance. Clearly, the depot can be considered a customer by defining $q_1 := \sum_{i=2}^{n} q_i$, i.e., a customer absorbing or providing the remaining amount of product to ensure product conservation.

The 1-PDTSP is $\mathcal{NP}$-hard since it coincides with the TSP when the vehicle capacity is large enough. Even more, the problem of checking the existence of a feasible solution is strongly $\mathcal{NP}$-hard (see [7]). In this paper we present a heuristic method that provides good feasible solutions for large instances and is competitive with the methods in [9]. The proposed method is also used to solve the bi-criterion problem of deciding a good vehicle capacity and designing a min-cost route.

2. The heuristic algorithm

Exact methods for solving the 1-PDTSP find difficulties when $n \geq 100$. Hernández-Pérez and Salazar-González [9] proposed a Linear-Programming based heuristic approach able to deal with instances with up to 500 customers. We propose a different heuristic procedure with the advantage of not depending on a Linear-Programming tool. Instead, we combine several operators in a Variable Neighborhood Search scheme. The operators can be grouped into two classes. The first class gathers classical operators for TSP solutions (e.g. 2-opt, Lin-Kernighan, etc). The second class consists of operators exploiting the load evolution through a route. More precisely, the vehicle collects or delivers product through customers, and this creates an up-and-down variation in the load of the vehicle. Since the initial load can be decided, the capacity requirement concerns only the maximum and minimum load through a route. Operators in the second class intend to reduce this gap. To this end, switching movements between customers try to decrease maximum loads and increase minimum loads.

Additionally, our proposal includes perturbation operators to diversify the search. These operators may create infeasible 1-PDTSP routes, and therefore procedures to restore the feasibility are also designed.

The performance of the proposed method will be tested on the benchmark instances described in [9]. This is a collection of difficult problems with up to 500 customers.

3. Acknowledgements

This work has been partially supported by ”Ministerio de Educación y Ciencia”, Spain (research project TIC2003-05982-C05-02).

References

[1] Anily, S., and J. Bramel, “Approximation algorithms for the capacitated traveling salesman problem with pickup and deliveries”, *Naval Research Logistics*, vol. 46, pp. 654–670, 1999.

[2] Anily, S., and G. Mosheiov, “The traveling salesman problem with delivery and backhauls”, *Operations Research Letters*, vol. 16, pp. 11-18, 1994.

[3] Baldacci, R., E. Hadjiconstantinou and A. Mingozzi, “An exact algorithm for the traveling salesman problem with deliveries and collections”, *Networks*, vol. 42, pp. 26–41, 2003.

[4] Chalasani, P., and R. Motwani, “Approximating capacitated routing and delivery problems”, *SIAM Journal on Computing*, vol. 28, pp. 2133–2149, 1999.

[5] Cordeau, J.-F., and G. Laporte “The dial-a-ride problem (DARP): Variants, modeling issues and algorithm”, *4OR - Quarterly Journal of The Belgian, French and Italian Operations Research Societies*, vol. 1, pp. 89–101, 2003.

[6] Gendreau, M., G. Laporte, and D. Vigo, “Heuristics for the traveling salesman problem with pickup and delivery”, *Computers & Operations Research*, vol. 26, pp. 699–714, 1999.

[7] Hernández-Pérez, H., “Traveling salesman problems with pickups and deliveries,” PhD thesis, University of La Laguna, 2004.

[8] Hernández-Pérez, H., and J.J. Salazar González, “A branch-and-cut algorithm for a Traveling Salesman Problem with Pickup and Delivery,” *Discrete Applied Mathematics*, vol. 145, pp. 126–139, 2004.

[9] Hernández-Pérez, H., and J.J. Salazar González, “Heuristics for the One-Commodity Pickup-and-Delivery Traveling Salesman Problem,” *Trasportation Science*, vol. 38, pp. 245–255, 2004.

[10] Mosheiov, G., “The traveling salesman problem with pickup and delivery”, *European Journal of Operational Research*, vol. 79, pp. 299-310, 1994.

[11] Parragh, S.N., K. Doerner, and R.F. Hartl, “A survey on pickup and delivery models Part I: Transportation between customers and depot”, working paper, Chair of Production and Operations Management, University of Vienna, 2006.

[12] Parragh, S.N., K. Doerner, and R.F. Hartl, “A survey on pickup and delivery models Part II: Transportations between pickup and delivey loctioins”, working paper, Chair of Production and Operations Management, University of Vienna, 2006.

[13] Savelsbergh M.W.P. , and M. Sol, “The general pickup and delivery problem”, *Transportation Science*, vol. 29, pp. 17–29, 1995.