Optimizing Material Sourcing and Delivery Operations

Anantaram Balakrishnan and **Xiajun Amy Pan**
McCombs School of Business, University of Texas at Austin, Austin, TX, USA
Email: anantb@mail.utexas.edu, xiajun.pan@phd.mccombs.utexas.edu

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1. Introduction

We address the following Material Sourcing and Delivery Planning problem with time windows (MSDP). Given the material requirements and delivery due dates for a set of geographically-dispersed customers, and the available sources of supply, determine which supplier(s) to use for each customer, and how to sequence the material pickup and delivery operations, using a limited number of available vehicles, in order to supply the required amount of material to each customer before the corresponding due date. This problem is motivated by the material supply planning task for scheduled track maintenance projects at a railway company. In this context, customers correspond to track maintenance jobs at different locations on the nationwide railway network. We are given the scheduled start date for each job, and the total material (e.g., ballast) needed at each job site. Each job requires one or more full-loads (e.g., train loads) of material from alternative suppliers (e.g., quarries). The current planning process to decide the quarterly or annual procurement and delivery plan is manual, and can result in delayed deliveries and high procurement cost. The goal of this work is to develop a decision support model and effective solution approach to minimize the total material procurement, transportation, and delivery cost for the MSDP problem.

The MSDP problem entails three sets of interrelated decisions: sourcing, vehicle assignment, and routing. The sourcing decision requires selecting one or more suppliers from among the candidate suppliers for each customer. Some suppliers are open all the year round, while others are open only in certain time intervals. The prices offered by suppliers vary, as do the supplier-to-customer distances. Moreover, suppliers differ in the amount of material they can supply. These restrictions stem from limited weekly production and loading capacities, as well as limitations on the total amount of material that a supplier can provide over the planning horizon. Transporting the material requires specialized equipment (e.g., trains that can carry ballast, in the railway maintenance context). We consider multiple vehicle types that differ in the time they require to pickup and deliver each load of material. The vehicle assignment decision entails selecting the vehicle type(s) that will service each customer. Finally, the routing and scheduling decision requires sequencing the pickup and delivery activities for each vehicle so as to deliver on time to each customer while meeting the supply capacity restrictions. The goal of the planning exercise is to minimize the total cost of supplying all customers during the planning horizon. This cost includes: (i) material procurement cost, which depends on the amount purchased from each supplier; (ii) transportation cost, which depends on the distance from the supplier to each customer, the return distance from each customer to the next supplier, and the per-mile equipment and consumables cost for the loaded and empty vehicle; and, (iii) delivery cost, which varies by vehicle type and customer. In the rail maintenance context, this cost includes the cost of disrupting regular traffic through the job site when the vehicles deliver the materials. Thus, the MSDP problem seeks the supplier selection, vehicle assignment, and delivery sequencing plan that minimizes the total procurement, transportation, and delivery cost. The main purpose of the model is to support tactical planning of procurement and vehicle deployment over, say, six months to a year. The model can help address questions such as the following: do we have adequate cost-effective sources of supply close to customers, do we have adequate vehicles, can we meet all the deliveries on time, and how do we deploy the vehicles?

To model the material sourcing and delivery planning decisions, we exploit the characteristics of the rail maintenance application context. In this setting, each customer (or job) requires one or more full-load deliveries, and so the vehicle (e.g., train) can only deliver to one customer per trip. The trip consists of going to the supplier location (e.g., quarry), loading the vehicle, traveling to the customer site, unloading the vehicle, and then traveling to the next supplier (assigned to the next customer that the vehicle needs to serve). A significant portion of this time is devoted to the loading and unloading operations, and the associated waiting times. The transportation times do not vary much across supplier-customer pairs because only suppliers within a pre-specified distance from a customer are eligible to supply that customer, and long customer-to-supplier return trips (for empty vehicles) are precluded. So, for tactical planning purposes, planners assume that each trip requires the same amount of time; this time does not depend on the specific supplier-customer pair, but varies by vehicle type. These problem features permit us to model the MSDP problem as a discrete-time model over a time-space network. Each period in the model represents the time to complete one round-trip from supplier to customer and back to a supplier. The nodes of the time-space network correspond to supplier and customer locations in each period for every vehicle type. The network contains supplier-to-customer arcs representing full-load deliveries, customer-to-supplier arcs for empty vehicles, and vehicle entry and exit arcs. In addition to flow conservation requirements on this network, we must impose additional side constraints to ensure that each customer receives the required number of deliveries, and supplier capacity constraints are satisfied.

The MSDP problem differs from the vehicle routing problem with time windows (VRPTW), studied for instance by Kolen, Rinnooy Kan and Trienekens [1], Desrochers, Desrosiers and Solomon [2], Fisher, Jornsten and Madsen [3], Kohl and Madsen [4], Kohl et al. [5] and Kallehauge et al. [6], in several important respects. Typically, VRPTW models consider identical vehicles, with each vehicle starting and ending at the same depot. So, these models do not include sourcing and supplier assignment decisions. On the other hand, unlike the MSDP problem context, each customer's requirement is less than full-load, and so vehicles can serve multiple customers on each trip. Moreover, the trip time depends on the set of customers visited during the trip, and so continuous time models that focus on sequencing the deliveries within each trip are more appropriate. In contrast, the MSDP problem focuses on deciding which supplier to assign to each customer, which period(s) to serve the customer based on due dates and supplier capacities, and how to route the vehicles to satisfy these supplier assignment and timing choices. These decisions must together minimize the total procurement, transportation (full-load and empty), and delivery costs. The MSDP also differs from the vehicle routing and scheduling problem with full truckloads (VRPFL) (e.g., Arunapuram, Mathur and Solow [7]) that focuses on routing trucks to transport full truckloads between various city pairs.

The MSDP problem is NP-hard (since it generalizes, for instance, the traveling salesman problem). We formulate the problem as an integer programming model, and develop several sets of inequalities to strengthen the model's linear programming relaxation. We have successfully applied this approach to an actual sourcing and delivery planning problem to obtain annual cost savings of around 12% compared to a prior heuristic approach. We next formally define the MSDP problem, and present its integer programming formulation. We then list some valid inequalities we have developed to strengthen the model, and present conclusions.

2. Model formulation

Let J be the set of customers, and Q the set of quarries. For each customer $j \in J$, let $QF(j)$ denote the subset of suppliers that can supply customer j, and $QR(j)$ the subset of suppliers to which vehicles can return after a delivery to this customer. These subsets may not include all suppliers because of travel distance restrictions and supplier availability. We index the time periods of the planning horizon from $t = 1$ to n, and define $T = \{1, 2, \ldots, n\}$. Let N_j denote the number of deliveries (of full or nearly full loads) required by customer j, to be delivered in the time window or interval $TF(j) \subseteq T$, with an average quantity of B_j per delivery. Each supplier $q \in Q$ can supply material in the time window $TQ(q) \subseteq T$. Let I_q and G_{qt} represent, respectively, supplier q's initial inventory and production quantity in period t. In addition, this supplier can supply no more than MB_q units during the horizon. The set $J(q) \subseteq J$ represents the subset of customers that supplier q can serve. For expositional and notational convenience, we consider only a single vehicle type. Let NV denote the number of available vehicles.

We consider three types of costs. Let CF_{qj} and CR_{jq} denote the *transportation* cost per delivery from supplier q to customer j and from customer j to supplier q respectively. The *delivery* cost to customer j is CA_j per delivery, and the procurement price from supplier q is P_q per unit. With multiple vehicle types, the transportation and delivery costs can vary by vehicle type.

To model the MSDP problem as an integer program, we define the following decision variables. For each customer $j \in J$, each supplier $q \in QF(j)$, and every period $t \in TF(j) \cap TQ(q)$, the integer variable zf_{qjt} represents the number of vehicles that deliver material from supplier q to customer j in period t. This variable combines both the supplier-to-customer assignment and delivery scheduling decisions. To model the return trips, let zr_{jqt} denote the number of vehicles that go to supplier q after delivering material to customer j in period t, for all $j \in J$, and $q \in QR(j)$. We define y_q as the number of vehicles that enter the system at supplier q in period 1, and x_{jt} as the number of vehicles that leave the system after delivering to customer j in period t. Using these integer decision variables, we can formulate the MSDP problem as the following optimization problem:

$$\text{Minimize} \quad \sum_t \sum_q \sum_j (CF_{qj} + P_q B_j) zf_{qjt} + \sum_t \sum_q \sum_j CR_{jq} zr_{jqt} + \sum_t \sum_q \sum_j CA_j zf_{qjt} \qquad (1)$$

subject to

$$\sum_{t' \le t} \sum_j B_j zf_{qjt'} \le \min\{I_q + \sum_{t' \le t} G_{qt'}, MB_q\} \qquad \text{for all } t \in T, q \in Q, \qquad (2)$$

$$\sum_q y_q \le NV \qquad (3)$$

$$\sum_t \sum_q zf_{qjt} = N_j \qquad \text{for all } j \in J, \qquad (4)$$

$$y_q = \sum_j zf_{qj1} \qquad \text{for all } q \in Q, \qquad (5a)$$

$$\sum_j zr_{jq(t-1)} = \sum_j zf_{qjt} \qquad \text{for all } q \in Q, t \in TQ(q) \setminus \{1\}, \qquad (5b)$$

$$\sum_{q} zf_{qjt} = \sum_{q'} zr_{jq't} + x_{jt} \qquad \text{for all } j \in J,\ t \in TF(j), \tag{5c}$$

$$zf_{qjt} = \text{integer} \qquad \text{for all } j \in J, t \in T, q \in QF(j), \tag{6a}$$

$$zr_{jqt} = \text{integer} \qquad \text{for all } j \in J, t \in T, q \in QR(j), \tag{6b}$$

$$y_q = \text{integer} \qquad \text{for all } q \in Q, \text{ and} \tag{6c}$$

$$x_{jt} = \text{integer} \qquad \text{for all } j \in J, t \in TF(j). \tag{6d}$$

The objective function (1) minimizes the total material purchasing, transportation, and delivery costs. Constraint (2) combines each supplier's production capacity constraint and total supply capacity during the planning horizon. This constraint specifies that, in every period *t*, the cumulative amount of material supplied by this supplier must not exceed the cumulative capacity (plus initial inventory) or the total supply capacity. Constraint (3) ensures that only the available vehicles are used, while constraint (4) specifies that each customer's demand must be satisfied. Constraints (5a) – (5c) are flow conservation constraints at each customer and supplier location in every period during which they are operational. Constraints (5a) and (5b) specify that the inflow of vehicles to a supplier must equal the outflow (number of vehicles delivering material to customers) in every period. In the first time period, the inflow is the number of vehicles entering the system; and for the remaining periods, the inflow is the number of vehicles returning after delivering to customers in the previous period. Constraint (5c) states that, after each delivery, the vehicle must go to the next supplier or leave the system. Constraints (6a) – (6d) are the integrality requirements.

We refer this formulation as a vehicle flow model since it considers the total number of vehicles arriving/leaving customer and supplier locations in each opening period rather than the routing of individual vehicles. From the vehicle flow solutions, we can easily obtain individual vehicle routes using a path decomposition method. Alternatively, we can formulate the problem using a vehicle-indexed formulation that directly models the individual vehicle routes. However, this latter formulation vastly increases the number of variables and constraints. For the basic problem, this disaggregate vehicle-indexed formulation does not provide tighter linear-programming bounds. The larger problem size combined with the symmetry in the vehicle-indexed model can significantly increase computational time.

3. Strengthening the MSDP model formulation

To effectively solve the MSDP problem, we have developed several families of valid inequalities that strengthen the model formulation, thereby increasing the linear programming lower bound. During the past decade, researchers have successfully applied polyhedral approaches to solve many different classes of difficult integer programming problems (see, for instance, Aardal and van Hoesel [8]). This experience has shown that adding valid inequalities not only reduces the enumeration effort for branch-and-bound (due to the tighter lower bounds) and but can also help generate good heuristic solutions. Developing valid inequalities requires understanding the problem and solution structure to identify why and how the linear programming relaxation can reduce cost by selecting fractional values for the decision variables. For the MDSP problem, fractional solutions stem from the LP relaxation's attempt to reduce each of the three cost components – procurement cost, transportation cost, and delivery cost (for problems with multiple vehicle types). Accordingly, we can develop several different inequalities to eliminate these solutions. To illustrate this approach, we next discuss one class of inequalities which we call delivery consolidation inequalities.

Consider the following simple problem with two customers 1 and 2, two suppliers A and B, and one vehicle. Each customer requires one full-load of material that can be delivered in either period 1 or period 2. Customer 1 is close to supplier A, and far from supplier B, while customer 2 is close to supplier B and far from supplier A. So, the transportation cost between customer 1 and supplier B is high, as is the transportation cost between customer 2 and supplier A. To avoid these transportation costs, the linear programming relaxation schedules two "half" deliveries to each of the customers. In particular, it assigns half a vehicle to deliver material from supplier A to customer 1 in each of the two periods, and the other half vehicle to deliver material from supplier B to customer 2 during the two periods. In contrast, the optimal solution requires first picking up material at supplier A and delivering to customer 1 in period 1, and then traveling to supplier B (from customer 1's location), picking up material, and delivering to customer 2 in the second period. To eliminate the fractional LP solution, we can impose the requirement that, in each period, a vehicle can travel from a supplier q to a unit-demand customer j only if the vehicle arrives at q from another customer (other than customer j) during the previous period. Mathematically, for every customer j with unit demand, we can express this requirement using the constraint $zf_{qjt} \le \sum_{j' \neq j} zr_{j'q(t-1)}$ .Adding this constraint eliminates the previous LP solution. We can similarly identify other valid inequalities to eliminate additional fractional solutions.

4. Conclusions

We applied the model to an actual problem with more than 100 jobs, 14 suppliers, and two vehicle types. The solution to the model produced savings of over 12% of annual cost compared to a previous heuristic approach. The model provides useful what-if capabilities to analyze the impact or reducing the number of suppliers, vehicles, and so on. It can also be used on an ongoing basis to adjust the plans as contingencies or new requirements arise. Work is ongoing on reducing the problem size by eliminating variables a priori and developing new cutting planes.

References

[1] Kolen, A.W.J., A.H.G. Rinnooy Kan, and H.W.J.M. Trienekens, "Vehicle Routing with Time Windows," *Operations Research* **35**, pp. 256-273, 1987.

[2] Desrochers, M., J. Desrosiers, and M. Solomon, "A New Optimization Algorithm for the Vehicle Routing Problem with Time Windows," *Operations Research* **40**, pp. 342-354, 1992.

[3] Fisher, M.L., K.O. Jornsten, and O.B.G. Madsen, "Vehicle Routing with Time Windows: Two Optimization Algorithms," *Operations Research* **45**, pp. 488-491, 1997.

[4] Kohl, N., and O.B.G. Madsen, "An Optimization Algorithm for the Vehicle Routing Problem with Time Windows Based on Lagrangian Relaxation," *Operations Research* **45**, pp. 395-406, 1997.

[5] Kohl, N., J. Desrosiers, O.B.G. Madsen, M.M.Solomon, and F.Soumis, "2-Path Cuts for the Vehicle Routing Problem with Time Windows," *Transportation Science* **33**, pp. 101-116, 1999.

[6] Kallehauge, B., J. Larsen, O.B.G. Madsen and M.M.Solomon, "Vehicle Routing Problem with Time Windows," book chapter in "Column Generation," G. Desaulniers, J. Desrosiers, and M.M.Solomon (eds.), pp. 67-98, Springer, New York , 2005.

[7] Arunapuram, S., K. Mathur, and D. Solow, "Vehicle Routing and Scheduling with Full Truckloads," *Transportation Science* **37**, pp. 170-182, 2003.

[8] Aardal, K., and C. P. M. van Hoesel, "Polyhedral Techniques in Combinatorial Optimization Ii: Applications and Computations," *Statistica Neerlandica* **53**, pp. 131-177, 1999.