On the design of Wireless Mesh Networks

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1. Wireless Mesh Network design

As the demand for wireless service provisioning keeps increasing, new wireless technologies are required to extend the coverage capabilities of classical wireless access networks like WLANs, WMANs and cellular systems. Wireless mesh networking seems to be one of the most promising solution for the provision of wireless connectivity in a flexible and cost effective way, see e.g. [1].

A *Wireless Mesh Network* (WMN) is composed of a mix of fixed and mobile nodes interconnected via wireless links to form a multi-hop ad hoc network. There are three types of network devices: *Mesh Routers* (MRs), *Mesh Access Points* (MAPs) and *Mesh Clients* (MCs). The functionality of both the MRs and the MAPs is twofold: they act as classical access points towards the MCs, whereas they have the capability to set up a *Wireless Distribution System* (WDS) by connecting to other mesh routers or access points through point to point wireless links. Both MRs and MAPs are often fixed and electrically powered devices. Furthermore, the MAPs are geared with some kind of broadband wired connectivity (ADSL, fiber, etc ...) and act as gateways towards the wired backbone. The structure of a WMN is illustrated in Figure 1.

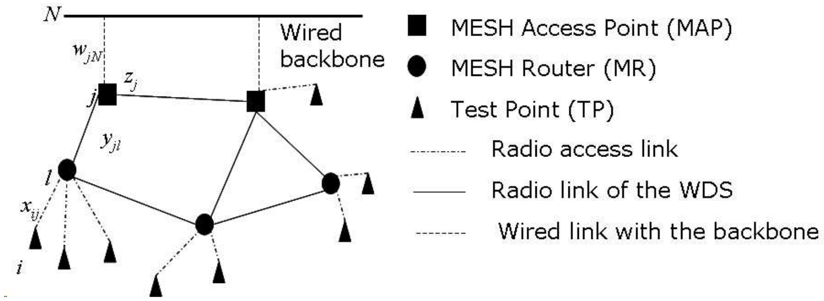


Figure 1: Wireless Mesh Network structure.

The problem of planning WMNs differs from that of planning other wireless access networks, such as second generation cellular systems or WLANs. In the latter cases, network planning involves selecting the locations in which to install the base stations or access points, setting their configuration parameters (emission power, antenna height, tilt, azimuth, etc.), and assigning channels so as to cover the service area and to guarantee enough capacity to each cell [2].

In the case of WMNs, each candidate site can host either MAPs or MRs, which have different installation costs. Roughly speaking, MAPs are more expensive than MRs since they must be directly connected to the wired backbone and might be more powerful than MRs in terms of both processing and transmission capabilities. The traffic to/from each MC must be routed to the wired backbone along one or multiple paths, which may contain intermediate MRs and which lead to a MAP. In this context, capacity limits of radio links among MRs and between MRs and MAPs play a key role since the traffic routed on a link must not exceed its capacity.

Thus WMN design involves deciding where to install the network nodes (out of a set of candidate sites), which type of node to select (MAP or MR) and which channel to assign them, while taking into account users coverage, wireless connectivity between MRs and MAPs, and traffic flows. In the resulting network design problem we must simultaneously consider radio coverage, like in classical radio planning for wireless access networks, and traffic routing, like in the design of wired networks.

Due to the lack of space, we just mention that most of the previous work deals with protocol optimization (see e.g. [3]) or with routing and channel assignment for a fixed topology (see e.g. [4, 5]). To the best of our knowledge, the main attempt to address the problem of locating WMN devices appears in [6]. But the models do not consider the coverage part of the problem and focus only on the optimization of routing and connectivity among mesh nodes considering an approximated interference model.

2. Mixed integer programming models

Let $S = \{1, \dots, m\}$ denote the set of *Candidate Sites* (CSs) where network devices can be installed, and $I = \{1, \dots, n\}$ the set of *Test Points* (TPs) representing the users in the service area. A special node N represents the wired backbone network. The cost for installing a MR in CS j is denoted by c_j, while the additional cost required to install a MAP in CS j is denoted by p_j, $j \in S$. The total installation cost for a MAP in CS j is thus equal to $c_j + p_j$.

The traffic generated by TP i is given by the parameter d_i, $i \in I$. The capacity of the wireless link between CSs j and l is denoted by u_{jl}, with $j, l \in S$, and the capacity of the radio access interface of CS j is denoted by v_j, $j \in S$. For each TP $i \in I$, let $S_i = \{j_1^{(i)}, j_2^{(i)}, \dots, j_{L_i}^{(i)}\}$ denote the ordered sequence of indices of the CSs from which TP i could be covered, where the CSs are ordered according to non-increasing received signal strength.

The connectivity parameters can be derived from the TPs and CSs location and propagation information. In particular, we define the coverage parameter:

$$a_{ij} = \begin{cases} 1 & \text{if a MAP or MR in CS } j \text{ covers TP } i \\ 0 & \text{otherwise,} \end{cases}$$

for each pair $i \in I, j \in S$, and the set L of all possible radio links, where L contains all ordered pairs (j, l), with $j, l \in S$, for which a wireless link can be established between CSs j and l.

As to the decision variables, we consider the TP assignment variables

$$x_{ij} = \begin{cases} 1 & \text{if TP } i \text{ is assigned to CS } j \\ 0 & \text{otherwise,} \end{cases}$$

for each pair $i \in I, j \in S$, the installation variables

$$z_j = \begin{cases} 1 & \text{if a MAP or a MR is installed in CS } j \\ 0 & \text{otherwise,} \end{cases}$$

for each $j \in S$, the wired backbone connection variables

$$w_{jN} = \begin{cases} 1 & \text{if a MAP is installed in CS } j \\ 0 & \text{otherwise,} \end{cases}$$

for each $j \in S$ (if $z_j = 1$, w_{jN} indicates whether j is connected to the wired network N, i.e., whether it is a MAP or a MR), and the wireless connection variables

$$y_{jl} = \begin{cases} 1 & \text{if there is a wireless link between CS } j \text{ and } l \\ 0 & \text{otherwise,} \end{cases}$$

for each pair $j, l \in S$.

We also consider the continuous variables f_{jl}, for $(j,l) \in L$, to represent the traffic flow routed on link (j,l). The special variable f_{jN} denotes the traffic flow on the wired link between MAP j and the backbone network.

The WMN design problem can then be formulated as follows:

$$
\begin{array}{lcll}
\min & \sum_{j\in S}(c_j z_j + p_j w_{jN}) & & (1)\\
s.t. & \sum_{j\in S} x_{ij} = 1 & \forall i \in I & (2)\\
 & x_{ij} \le a_{ij} z_j & \forall i \in I, \forall j \in S & (3)\\
 & \sum_{i\in I} d_i x_{ij} + \sum_{l\in S}(f_{lj} - f_{jl}) - f_{jN} = 0 & \forall j \in S & (4)\\
 & f_{lj} + f_{jl} \le u_{jl} y_{jl} & \forall (j,l) \in L & (5)\\
 & \sum_{i\in I} d_i x_{ij} \le v_j & \forall j \in S & (6)\\
 & f_{jN} \le M w_{jN} & \forall j \in S & (7)\\
 & y_{jl} \le z_j & \forall (j,l) \in L & (8)\\
 & y_{jl} \le z_l & \forall (j,l) \in L & (9)\\
 & z_{j_\ell^{(i)}} + \sum_{h=\ell+1}^{L_i} x_{ij_h^{(i)}} \le 1 & \forall \ell = 1,\ldots,L_i - 1, \forall i \in I & (10)\\
 & x_{ij},\ z_j,\ y_{jl},\ w_{jN} \in \{0,1\} & \forall i \in I, \forall j,l \in S &
\end{array}
$$

The objective function (1) accounts for the total network cost, including installation costs c_j and the costs related to the connection of MAPs to the wired backbone p_j. If for any practical reason only a MR can be installed in a given CS j, the corresponding variable w_{jN} is set to zero.

Constraints (2) assign each TP to exactly a single MR or MAP. Constraints (3) make sure that a TP i is assigned to CS j only if a device (MAP or MR) is installed in j and if i belongs to the coverage set of j. Constraints (4) define the flow balance in node j: the term $\sum_{i\in I} d_i x_{ij}$ is the total traffic of the TPs assigned to node i, $\sum_{l\in S} f_{lj}$ is the total traffic received by j from neighboring nodes, $\sum_{l\in S} f_{jl}$ is the total traffic transmitted by j to neighboring nodes, and f_{jN} is the traffic transmitted to the wired backbone. Constraints (5) impose that the total flow on the link between device j and l does not exceed its capacity u_{jl}, provided that the wireless link does exist ($(j,l) \in L$). Constraints (6) ensure that all the traffic serviced by any network device (MAP or MR) does not exceed the capacity of the wireless link used for the access. Constraints (7) force the flow between device j and the wired backbone to zero if no MAP is installed in j; the parameter M is the MAP capacity limit. Constraints (8) and (9) force the decision variables y_{jl} (defining the existence of a wireless link between CS j and l) to zero if no device is installed either in CS j or in CS l. Finally, constraints (10) make sure that each TP i is assigned to the "best" CS in which a MAP or MR is installed, namely a CS providing the highest received signal strength in i. Notice that in this simplified model the y variables can be clearly ommitted by deleting constraints (8) and (9), and by replacing constraints (5) with

$$f_{lj} + f_{jl} \le u_{jl} z_j \qquad f_{lj} + f_{jl} \le u_{jl} z_l \qquad \forall (j,l) \in L, \tag{11}$$

but we keep them because there are needed in the more realistic extensions that are described next.

The above mixed integer programming (MIP) model, which considers fixed transmission rates for both the wireless access interface and the wireless distribution system, is referred to as *Fixed Rate Model* (FRM). Since in real wireless systems the capacity of a given wireless link depends on the distance between transmitter and receiver, we extend the FRM to account for transmission rate adaptation.

As to the wireless distribution system, it suffices to let the capacities u_{jl} depend on the propagation conditions between CSs j and l. To account for rate adaptation in the wireless access network, we must slightly modify constraints (6). For any given CS j, we define a set of nested coverage regions R_j^k centered in j, with $k = 1, ..., K$, and the sets $I_j^k \subset I$ containing all the TPs falling into the kth region around CS j. Such sets can be determined by using the following incidence variables:

$$a_{ij}^k = \begin{cases} 1 & \text{if a TP } i \text{ falls within region } k \text{ of the CS } j \\ 0 & \text{otherwise.} \end{cases}$$

If each region R_j^k is assigned a maximum capacity defined by a variable v_j^k, rate adaptation in the wireless access part can be accounted for by substituting constraints (6) with:

$$\sum_{k \in R_j} \frac{\sum_{i \in I_j^k} d_i x_{ij}}{v_j^k} \leq 1 \quad \forall j \in S. \tag{12}$$

The resulting MIP model is referred to as *Rate Adaptation Model* (RAM).

So far we have neglected the effect of interference on the access capacity and on the capacity of wireless links connecting mesh nodes. In wireless technologies like IEEE 802.16 mesh mode and IEEE 802.11 multi-radio mesh networks, the impact of interference is limited by assigning appropriate frequencies (or sub-carriers) to the various wireless links. We focus here on the case of IEEE 802.11 and assume that all MAPs and MRs share the same radio channel for the access part and use another shared channel for the backbone links.

The access capacity being shared by all mesh nodes, interference can be easily accounted for by modifying constraints (12) and by considering not only the TPs assigned to the device in CS j but all TPs in the coverage region:

$$z_j \sum_{k \in R_j} \frac{\sum_{i \in I_j^k} d_i}{v_j^k} \leq 1 \quad \forall j \in S. \tag{13}$$

In fact, since transmissions between MAPs or MRs and MCs occur on the same channel and the CSMA/CA (Carrier Sense Multiple Access Collision Avoidance) protocol is adopted to regulate channel access, a single transmission at a time is allowed in the coverage range [7]. Since a single decision variable z_j appears in each constraint (13), z_j can be set to zero when the constraint is not satisfied by installing a MAP or MR in CS j. Thus constraints (13) just amount to reducing, during a pre-processing step, the number of available CSs.

The interference limiting effect on the wireless link capacities is more difficult to account for, since it depends on the network topology and on the multiple access protocol. By considering the *protocol interference model* proposed in [8], we define sets of links that cannot be simultaneously active. These sets depend on the specific multiple access protocol. In the case of CSMA/CA, adopted by IEEE 802.11, the set C_{jl} associated with each link (j, l) includes all links that are one and two hops away in the mesh-network graph (links connecting j and l to their neighbors and their neighbors to the neighbors of their neighbors). By adopting a fluidic version of the protocol interference model [9], we impose for each such subset the following constraint on the flows across those links:

$$y_{jl} \sum_{(k,h) \in C_{jl}} \frac{f_{kh}}{u_{kh}} \leq 1 \quad \forall j, l \in S. \tag{14}$$

Replacing constraints (5) with (14) and (6) with (13) we have the so-called *Interference Aware Model* (IAM). Note that the nonlinear constraints (14) can be easily linearized.

By introducing additional binary variables z_j^q for each CS j and channel q, we obtain an overall model which takes into account also the assignment of multiple channels. The reader is referred to the full version of the paper for the details.

3. Some computational results

Interestingly, in spite of the considerable size of the three above formulations, medium-to-large-scale instances can be solved to optimality with a state-of-the-art MIP code. Due to lack of space, we just report a few typical computational results obtained with Cplex 10.0 on a workstation with 1.2GHz AMD Athlon processor and 1024Mb of RAM.

For the sake of testing, we have developed a generator of synthetic WMN topologies. Besides the numbers m of CSs and n of TPs (MCs), the input parameters include: the dimension D ([m]) of the square area to be covered, the coverage range R_A ([m]) of the wireless access part of the network (MC-MR), the coverage range R_B ([m]) of the wireless backbone links (MRs, MRs), the uniform traffic demand of the MCs $d_i = d$, for all $i \in I$, and the ratio β between the installation cost of a MR and of a MAP $(c_j/(c_j + p_j))$.

Our standard settings are: $n = 100$, $D = 1000$, $R_A = 100$, $R_B = 250$, $\beta = 1/10$, $v_j = 54$ Mb/s for all $j \in S$, $u_{jl} = 54$ Mb/s for all $j, l \in S$, and $M = 128$ Mb/s. The connectivity in the wireless access part of the network (between MRs and MCs) is assumed to be a circular coverage region with radius R_A, while the connectivity among MRs and between MRs and MAPs is assumed to be a circular region with radius R_B. The positions of the m CSs and n TPs are randomly generated according to the above parameters, and the network topology is guaranteed to be feasible (all TPs can be covered).

Table 1: Solutions of the FRM for different numbers of CSs m.

	$d = 600Kb/s$				
	MAP	**MR**	**Links**	**# Hops**	**Time (s)**
m=30	2.25	23.75	21.50	3.14	040
m=40	1.45	24.00	22.55	3.43	1.43
m=50	1.25	24.15	22.90	3.50	4.69

	$d = 2Mb/s$				
	MAP	**MR**	**Links**	**# Hops**	**Time (s)**
m=30	3.20	23.75	20.85	2.34	0.79
m=40	2.45	24.00	22.05	2.55	2.57
m=50	2.25	24.15	22.35	2.60	7.69

	$d = 3Mb/s$				
	MAP	**MR**	**Links**	**# Hops**	**Time (s)**
m=30	4.00	23.65	20.20	2.09	0.63
m=40	3.40	23.75	21.00	2.15	10.93
m=50	3.25	23.95	21.55	2.19	32.88

Table 2: Solutions of the FRM for different installation cost ratios β. Number of CSs $m = 30$ and MAP capacity $M = 128Mb/s$.

	$d = 600Kb/s$			
β	**MAP**	**MR**	**Links**	**# Hops**
1/10	2.10	23.40	21.30	3.22
1/7	2.40	24.00	21.60	3.22
1/5	2.40	24.30	22.70	3.22
1/3	2.40	24.30	21.60	3.22
1/2	2.80	23.60	20.80	2.95

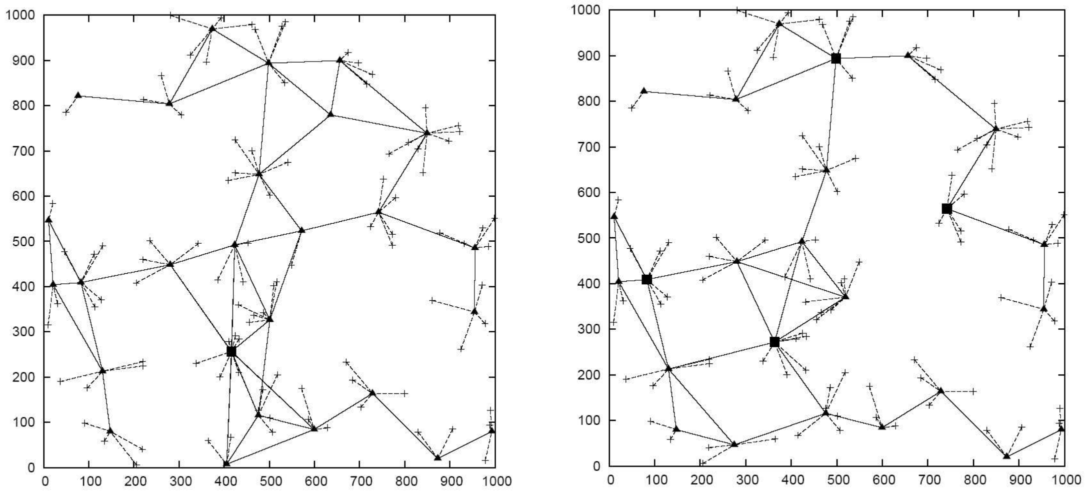
	$d = 2Mb/s$			
β	**MAP**	**MR**	**Links**	**# Hops**
1/10	3.00	23.40	20.60	2.39
1/7	3.30	24.00	20.90	2.30
1/5	3.30	24.00	20.90	2.30
1/3	3.30	24.00	20.90	2.30
1/2	3.70	23.60	20.00	2.20

Table 1 summarizes the solution characteristics of the FRM when the number of CSs varies. The values correspond to averages on 20 instances. For each couple (m, d), we report the number of installed MAPs and MRs, the number of wireless links of the WDS, the average number of wireless hops between a generic TP and the nearest MAP, and the computing time. Table 2 reports the same information of for instances with $m = 30$ CSs positions when the installation cost ratio β varies.

An increase in the traffic demand leads to an increase in the dimension of the WDS, which conveys the traffic, in terms of the number of MAPs. For a given traffic level, increasing the number of CSs to 50 increases the probability that a MC is connected to a MAP through a multi-hop wireless path, so that less MAPs and more MRs tend to be installed. Conversely, if the number of CSs is decreased to 30, more MAPs are installed since not all the MCs can be connected to those MAPs through multi-hop wireless paths. In other words, for a higher number of CSs m the solution space is larger and the model favors solutions providing connectivity with a lower impact on the network cost, i.e., networks containing more MRs than MAPs.

The number of installed MAPs and MRs clearly depends on the installation cost ratio β between them. Figure 2 illustrates this effect by showing the network layout in the case of traffic demand $d = 3$ Mb/s and very large MAP capacity when varying the installation costs. Notice that an increase in the value of β leads to a network with multiple MAPs. On the other hand, if the installation cost of MAPs is twice that of MRs the incidence of the MAPs installation cost on the overall network cost becomes relevant and consequently a lower number of MAPs are installed, resorting to paths with multiple hops to service the MCs' traffic. The same trend is observed in the averaged results of Table 2 in the case of finite MAP capacity.

Due to lack of space, we cannot compare here the computational results obtained with the RAM and IAM models and report those of the overall model with channel assignment. Let us just mention that more than



(a) $\beta = 1/2$

(b) $\beta = 10/11$

Figure 2: Example of WMNs obtained with the FRM for varying installation cost ratio β and very large MAPs capacity.

75% of the instances with $m = 50$ CSs, $n = 200$ TPs and three channels we considered could be solved to optimality in a few hours of computing time. For larger instances we are developing efficient heuristics.

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