A Two-Stage Multi-Criteria Stochastic Programming Model for Location of Emergency Response and Distribution Centers *

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1. Introduction

This paper presents a two-stage multi-criteria stochastic programming model for a multi-facility location problem in pre-disaster planning for effective post-disaster emergency logistics. The selection of locations of Emergency Response and Distribution Centers (ERDC) is considered both for pre-disaster storage and post-disaster transportation of commodities such as search and rescue teams, medical teams, food, water, necessity items and machinery. Multiple criteria are identified along with their priority levels. A goal programming formulation is given in which, the facility locations are chosen with their capacities in the first stage and in the second stage, expected values of both the weighted sum of travel times and the maximum time for each commodity to reach a demand point are minimized under different post-disaster scenarios. Other objectives related to minimizing the risk of selected locations, minimizing unsatisfied demand and obeying pre-disaster budget restrictions are also considered at the secondary priority level. The proposed model will be utilized for the site selection of ERDCs in Istanbul with earthquake risk considerations.

The literature on strategic planning for emergency logistics is rather rare but there are considerably many studies on different components of the problem such as the location of emergency services and dispatching of commodities. One of the earliest studies conducted on location of emergency service facilities is by Toregas et al. [8] modelling the problem as a set covering problem and using a linear programming as the solution method. Consignment of goods are typically examined in the literature as a multi-commodity network flow problem, with a multi-period and/or multi-modal setting. Haghani and Oh [5] formulated a multi-commodity, multi-modal network flow model with time windows for disaster response. Two heuristic algorithms are proposed. The flow of goods over an urban transportation network is modelled as a multi-commodity, multi-modal network flow problem by Barbarosoglu et al. [1]. A two-stage stochastic programming framework is formed as the solution approach. Another study on the topic, conducted by Fiedrich et al. [4], model the problem similar to a machine scheduling problem proposing two heuristics, Simulated Annealing and Tabu Search. There are also studies by Tufekci et al. [9] discussing a systems view of emergency management, the role of advanced communications and computing technologies, coupled with analytic procedures and by Bryson et al. [2] emphasizing that the decision makers could benefit from the application of quantitative decision-making techniques. In addition to the above-mentioned studies, there are some applications to real-life cases. Ozdamar et al. [7] analyze the dispatching of commodities to distribution centers as part of emergency logistics planning. The model and the solution methodology is implemented on a scenario based on the 1999 Marmara Earthquake. Similarly, Yi and Ozdamar [10] consider a dynamic and fuzzy logistics coordination model for conducting disaster response activities. The model is illustrated on an earthquake data set from Istanbul.

When the literature is reviewed, most of the models stated deal with predetermined supply nodes. In none of the above, the decision maker tries to optimize the location of supply nodes to maximize the commodity transportation. The models consider arc capacities which may actually be realized at different levels in case of emergency. In this study we cope with the uncertainty by a two-stage stochastic programming model. We represent the functionality of the highway system and the facilities via links that may be non-operational after the disaster with an estimated probability of failure under a most likely disaster scenario with the worst

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estimated potential damage. The demand nodes are also determined according to the most likely disaster scenario and estimated damage in terms of number of deaths and casualties. We do not consider detailed post-disaster dispatching since our main goal is to determine the locations of the Emergency Response and Distribution Centers (ERDC). However, we consider accessibility after the disaster in the two-stage framework, as one of the main criteria for the selection of sites. In fact, our model considers multi-criteria with varying priority levels and we propose a goal programming methodology for the solution.

2. The Model

The problem is formulated as a two-stage multi-criteria stochastic programming model. The first-stage decisions are whether to open a facility at a site and if so, to determine the capacity of the facility and how much to store of the durable commodities. In the second stage, a transshipment problem is solved with two assumptions: 1) The links connecting the regions are uncapacitated but might have failed in a given disaster scenario; 2) The transhipment nodes serve as depots for commodities. The decision of opening such centers are taken with respect to the expected value of several objectives over the possible disaster scenarios with target values determined by the decision maker. The goals are given below with their corresponding priority levels.

(Priority 1)

Goal 1: Total expected weighted time to transport all commodities to the affected areas should not exceed G1, the target level for Goal 1.

Goal 2: The expected maximum time for each commodity to reach a district must be less than G2, the target level for Goal 2.

(Priority 2)

Goal 3: The average risk associated with the locations of open facilities should not exceed G3, the target level for Goal 3.

(Priority 3)

Goal 4: Total expected weighted unsatisfied demand should not exceed G4, the target level for Goal 4.

(Priority 4)

Goal 5: The sum of the fixed opening costs and holding costs over the planning horizon should not exceed G5, the budget.

Four priority levels are determined to reach the affected areas as soon as possible with the necessary commodities. The expectations are taken over all possible disaster scenarios. This first level priority includes two goals. One of these is to minimize the expected weighted time to reach the affected areas. The weights are with respect to both the criticality of a commodity and the criticality of a district. The importance of a commodity is determined by the urgency of requirement. The importance of a district depends on aspects such as the population and the number of industrial centers present, as well as its risk. The second goal in this priority level is the minimization of the expected maximum time for each commodity to reach a district. This is to enforce that all the commodities are delivered to the districts in need as soon as possible. The second level priority goal is minimizing the average risk associated with locations of open facilities. For example, based on most likely earthquake scenarios and the geological condition of the region, a risk index can be assigned to each location. It is crucial that these facilities are functional after an earthquake, therefore the average risk is attempted to be kept below a target value. Since it is very difficult to satisfy all of the demand, a dummy node is created in the model to represent the supply of unsatisfied demand. The third level priority is to minimize total weighted unsatisfied demand. Clearly, some of the commodities such as search and rescue teams, water and medical supplies are critical for the survival of the victims. The fifth goal is in the fourth priority level. It is the total cost of opening the facilities and the inventory cost for holding the durable commodities in these facilities over a specified duration. This goal has the least priority compared to the others. Although budget is an important concern in many cases, in a disaster situation both the government and the people mobilize their resources without hesitation. Still, in pre-disaster planning stage agencies have to operate under a budget limit.

The following notation is used in the model.

Parameters

P	:	Set of potential sites
D	:	Set of districts
S	:	Set of suppliers
W	:	Set of possible scenarios
$p(w)$	:	probability of scenario w occurring
DC	:	Set of durable commodities
NC	:	Set of non-durable commodities
T_{1ijc}	:	time to transport commodity type c from supply i to facility j
T_{2jkc}	:	time to transport commodity type c from facility j to district k
R_j	:	earthquake risk index of potential site j
R	:	average risk that can be tolerated
I_{1k}	:	importance factor of district k
I_{2c}	:	importance factor of commodity c
n_{1i}^w	:	1, if supply node i is operational in scenario w; 0, otherwise
n_{2j}^w	:	1, if potential site node j is operational in scenario w; 0, otherwise
l_{1ij}^w	:	1, if link between i and j is operational in scenario w; 0, otherwise
l_{2jk}^w	:	1, if link between j and k is operational in w; 0, otherwise
S_{ci}	:	supply amount of commodity c present at supply node i
C_j	:	capacity of site j
f_j	:	fixed charge of opening a facility at site j
d_{ck}^w	:	demand for commodity type c at district k in scenario w
h_{cn}	:	holding cost for one unit of commodity c in year n
M	:	a big number
N	:	number of years for which the holding costs are considered
Gi :	:	target level of goal i

Decision Variables

o_j	:	1, if a facility will be open at site j; 0, otherwise
x_{cij}^w	:	amount of commodity type c sent from supply i to facility j in scenario w
y_{cjk}^w	:	amount of commodity type c sent from facility j to district k in scenario w
z_{cj}	:	amount of commodity type c stored in facility j
c_j	:	capacity of the facility to be opened at site j
u_{cij}^w	:	if there is flow of commodity c from supply i to facility j in scenario w
v_{cjk}^w	:	if there is flow of commodity c from facility j to district k in scenario w

m_{ck}^{w}	:	amount of unsatisfied commodity c for district k in scenario w
T_{max}	:	upper limit for the maximum time for any commodity to reach to any district

Goal Formulation 1:

$$\sum_{w\in W} p(w)(\sum_{c\in NC} I_{2c}\sum_{i\in S}\sum_{j\in P} x_{cij}^{w}T_{1ijc} + \sum_{c\in DC,NC} I_{2c}\sum_{j\in P}\sum_{k\in D} I_{1k}y_{cjk}^{w}T_{2jkc}) \leq G1$$

Goal Formulation 2:

$$\sum_{w\in W} p(w)(v_{cjk}^{w}T_{2jkc}) \leq T_{max},\ \forall c\in DC,\ \text{ and } \forall j\in P,\ \forall k\in D$$

$$\sum_{w\in W} p(w)(\sum_{i\in S}\sum_{j\in P} u_{cij}^{w}T_{1ijc} + \sum_{j\in P} v_{cjk}^{w}T_{2jkc}) \leq T_{max},\ \forall c\in NC \text{ and } \forall k\in D$$

$$T_{max} \leq G2$$

Goal Formulation 3:

$$\sum_{j\in P} R_j o_j \leq G3R$$

Goal Formulation 4:

$$\sum_{w\in W} p(w)\sum_{k\in D} I_{1k}\sum_{c\in DC,NC} I_{2c}m_{ck}^{w} \leq G4$$

Goal Formulation 5:

$$\sum_{j\in P} f_j c_j + \sum_{n=1}^{N}(\sum_{j\in P}\sum_{c\in DC,NC} h_{cn}z_{cj}) \leq G5$$

3. Model

$$\min\ \ P1(d_1^+) + P1(d_2^+) + P2(d_3^+) + P3(d_4^+) + P4(d_5^+) \tag{1}$$

subject to

$$\sum_{w\in W} p(w)(\sum_{c\in NC} I_{2c}\sum_{i\in S}\sum_{j\in P} x_{cij}^{w}T_{1ijc} + \sum_{c\in DC,NC} I_{2c}\sum_{j\in P}\sum_{k\in D} I_{1k}y_{cjk}^{w}T_{2jkc}) = G1 + d_1^+ - d_1^- \tag{2}$$

$$T_{max} = G2 + d_2^+ - d_2^-,\ \ \forall w \tag{3}$$

$$\sum_{j\in P} R_j o_j = G3R + d_3^+ - d_3^- \tag{4}$$

$$\sum_{w\in W} p(w)\sum_{k\in D} I_{1k}\sum_{c\in DC,NC} I_{2c}m_{ck}^{w} = G4 + d_4^+ - d_4^- \tag{5}$$

$$\sum_{j\in P} f_j c_j + \sum_{n=1}^{N}(\sum_{j\in P}\sum_{c\in DC,NC} h_{cn}z_{cj}) = G5 + d_5^+ - d_5^- \tag{6}$$

$$\sum_{i\in S} x^w_{cij} + z_{cj} \geq \sum_{k\in D} y^w_{cjk}, \qquad \forall w \in W, \forall j \in P, \forall c \in DC \tag{7}$$

$$\sum_{i\in S} x^w_{cij} \geq \sum_{k\in D} y^w_{cjk}, \qquad \forall w \in W, \forall j \in P, \forall c \in NC \tag{8}$$

$$\sum_{c\in DC} z_{cj} \leq c_j \leq C_j o_j, \qquad \forall j \in P \tag{9}$$

$$y^w_{cjk} \leq M n^w_{2j}, \quad \forall w \in W, \forall j \in P, \forall k \in D, \forall c \in DC, NC \tag{10}$$

$$x^w_{cij} \leq M l^w_{1ij}, \quad \forall w \in W, \forall i \in S, \forall j \in P, \forall c \in DC, NC \tag{11}$$

$$y^w_{cjk} \leq M l^w_{2jk}, \quad \forall w \in W, \forall j \in P, \forall k \in D, \forall c \in DC, NC \tag{12}$$

$$\sum_{j\in P} y^w_{cjk} + m^w_{ck} = d^w_{ck}, \qquad \forall w \in W, \forall k \in D, \forall c \in DC, NC \tag{13}$$

$$\sum_{j\in P} x^w_{cij} \leq S_{ci} n^w_{1i}, \qquad \forall w \in W, \forall i \in S, \forall c \in DC, NC \tag{14}$$

$$\sum_{w\in W} p(w)(\sum_{j\in P} v^w_{cjk} T_{2jkc}) \leq T_{max}, \quad \forall k \in D, \forall c \in DC \tag{15}$$

$$\sum_{w\in W} p(w)(\sum_{i\in S}\sum_{j\in P} u^w_{cij} T_{1ijc} + \sum_{j\in P} v^w_{cjk} T_{2jkc}) \leq T_{max}, \quad \forall k \in D, \forall c \in NC \tag{16}$$

$$o_{ij} \in \{0,1\}, \forall i \in S, \forall j \in P$$

$$u^w_{cij},\, v^w_{cjk} \in \{0,1\},\ \forall w \in W, \forall i \in S, \forall j \in P, \forall k \in D, \forall c \in DC, NC$$

$$x^w_{cjk},\, y^w_{cij},\, m^w_{ck} \geq 0, \forall w \in W, \forall i \in S, \forall j \in P, \forall k \in D, \forall c \in NC$$

$$z_{cj} \geq 0, \forall j \in P, \forall c \in DC$$

$$c_j \geq 0, \forall j \in P$$

$$T_{max} \geq 0$$

$$d^+_i,\, d^-_i \geq 0,\ \ i = 1, ..., 5$$

The objective function (1) is the minimization of deviations from the five goals with four priority levels. The constraints (2), (3), (4), (5) and (6) are the goal equations. In constraint (4) the target value is multiplied with the constant R, which can be set by the decision maker depending on his/her risk attitude. Constraint (7) and (8) are the balance equations at the ERDC for the durable and non-durable commodities, respectively. In constraint (9), the first and the second inequalities guarantee that the amount stored in an ERDC is less than its capacity and the capacity of an ERDC is equal to 0 if that ERDC is not opened, respectively. Constraint (10) represents the fact that the transshipment nodes may not be functional after the disaster. Constraints (11) and (12) force the amount of commodity carried from link i to j and j to k to be 0 if those links have failed in a realization, respectively. Constraint (13) is the demand satisfaction constraint. Constraint(14) is the capacity restriction on the supplier, where capacity is zero if the supplier has failed in a disaster scenario. Lastly constraints (16), (17) force the expected maximum time for each commodity to reach a district to be less than the variable T_{max}.

The two-stage multi-criteria stochastic programming model will be solved with the data that will be obtained from the Local Municipality of Istanbul. The potential sites for the ERDC have been already determined after various studies on the availability of land. In the model both nodes and links are subject to failure meaning that any of the suppliers/facilities or the roads linking them may not be functional after the disaster. These might be due to factors such as failures of bridges/viaducts, natural-gas explosion and consequent fires, as well as building collapses. The failure of the links and the nodes are interrelated because the earthquake affects a region. Therefore the failure probabilities of these are considered to be depending on each other and will be incorporated in the probability of each scenario occurring. Each generated scenario will be distinct in terms of the functionality of the nodes and links, as well as the demand quantities. The travel times are to be calculated with respect to distances because after an earthquake the traffic conditions will not be as usual. The travel behavior of the people under a disaster is still an open issue that needs to be researched.

The uncapacitated fixed-charge facility location problem is known to be an NP-Hard problem [6][3]. In addition, the problem in this paper requires deciding on the capacity of the facilities and the storage quantities for durable commodities. Our model differs from classical location models in that uncertainty due to a disaster has to be considered. We handle this via a two-stage stochastic programming model. In the first stage the facilities to be opened are determined together with their capacities and the storage quantities of each commodity. In the second stage, the distribution of the commodities are optimized. According to the goal programming approach, the model will be solved for each priority level with corresponding goals.

4. Conclusion and Future Work

A pre-disaster facility location problem for disaster relief is formulated in this study. We construct a two-stage stochastic programming model. Multi-criteria objectives are identified to respond to the different needs of the post-disaster stage, and a goal programming model is formulated. An application of the problem is to be considered for Istanbul to provide guidance to the Local Municipality for the location of ERDCs within the scope of planning for post-disaster logistics, especially against an expected major earthquake in the area.

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