On the Minimum-Envy Location Problem

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1 Introduction

The concept of envy has been widely studied in the literature of Decision Theory. In particular, one of the most well-known criteria to judge fairness and satisfaction is envy-freeness, i.e., a solution of a decision process such that every agent of this process likes its own solution at least as much as any other agent's. The envy-free criteria have been used in problems of fair division of indivisible items among people (see e.g. [1, 3, 7, 8, 11]), problems of allocating heterogeneous indivisible objects (see e.g. [6, 9]), queueing problems (see e.g. [2]), auction problems (see e.g. [4, 10]) and implementation of game solution concepts (see e.g. [5]), among others.

To the best of our knowledge, addressing location problems from this perspective has not been previously considered in the literature. The model discussed analyzes the case where demand points have strict preference order on the sites where the plants can be located. The goal is to find the location of the facilities minimizing the *total envy* felt by the entire set of demand points, and we assume that the set of candidate sites for new plants is identical to the set of clients, that we have to locate p plants and that each client will be allocated to its favourite plant.

The definition of the envy of client i_1 for client i_2 that we use is given by

$$e_{i_1 i_2}(X) = \begin{cases} 0 & \text{if } O_{i_1,P_{i_1}(X)} \leq O_{i_2,P_{i_2}(X)}, \\ O_{i_1,P_{i_1}(X)} - O_{i_2,P_{i_2}(X)} & \text{otherwise,} \end{cases}$$

where $\{1, \ldots, M\}$ is the set of sites, $O = (O_{ij})_{i,j=1,\ldots,M}$ is the preferences matrix (each row of matrix O is given by a permutation of $\{1, \ldots, M\}$, in such a way that the smaller O_{ij}, the most preferred site j is for client i), X is the set of located sites (plants), and client i is supplied from plant $P_i(X)$.

We have studied five different formulations for the problem of minimizing the total envy and then we have devised several improvements of the formulations. In particular, we generate valid inequalities for some formulations, develop an *ad-hoc* cut-and-branch algorithm for another one and two variable fixing strategies, and compare formulations and solution methods on a testbed.

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