Linear Relaxations for the Maximum Stable Set Problem: a computational study

M. Giandomenico[1], A. N. Letchford[2], F. Rossi and S. Smriglio[1]

[1] Dipartimento di Informatica, Universitá di L'Aquila, Italy
{giandomenico, rossi, smriglio}@di.univaq.it
[2] Department of Management Science, Lancaster University, UK
A.N.Letchford@lancaster.ac.uk

The *Maximum Stable Set Problem* on a graph $G = (V, E)$ is a well-known combinatorial optimization problem, with a huge number of applications. Although the *Stable Set Polytope* has been extensively researched, the many relevant theoretical results on its structure have not led to significant computational success. In fact, it appears a hard task to strengthen the basic LP relaxation consisting of clique and non-negativity inequalities (see [1], [7] and [8]). On the other hand, Semidefinite Programming (SDP) techniques appear to be much more promising since they provide stronger relaxations (see, for instance, [2], [3] and [4]). Unfortunalely, these may be affected by numerical instabilities and the lack of fast re-optimization algorithms prevents their embedding in a branch-and-bound scheme.
In this talk we present an extended linear relaxation obtained by the application of the Lovász-Schrijver $M(K, K)$ operator [6]. Roughly speaking, the $M(K, K)$ operator "squares" the size of a linear system by multiplying pairs of constraints together. Applying the $M(K, K)$ operator to an LP relaxation consisting of clique and non-negativity inequalities yields the extended linear relaxation investigated in [5]. After a review of the theoretical properties of such formulation, we present a computational experience comparing strength and limitations of the LP and SDP approaches.

References

1. E. Balas, S. Ceria, G. Cornuéjols & G. Pataki (1996) Polyhedral methods for the maximum clique problem. In D.S. Johnson & M.A. Trick (eds.), *Op. cit.*, pp. 11–28.
2. S. Burer & D. Vandenbussche (2006) Solving lift-and-project relaxations of binary integer programs. *SIAM J. on Opt.*, 16, 726–750.
3. I. Dukanovic & F. Rendl (2007) Semidefinite programming relaxations for graph coloring and maximal clique problems. *Math. Program.*, 109, 345–365.
4. G. Gruber & F. Rendl (2003) Computational experience with stable set relaxations. *SIAM J. Opt.*, 13, 1014–1028.
5. M. Giandomenico, A. N. Letchford, F. Rossi, S. Smriglio (2008) An Application of the Lovasz-Schrijver $M(K, K)$ Operator to the Stable Set Problem. *Math. Program.*, to appear.
6. L. Lovász & A.J. Schrijver (1991) Cones of matrices and set-functions and 0-1 optimization. *SIAM J. Opt.*, 1, 166–190.
7. G.L. Nemhauser & G. Sigismondi (1992) A strong cutting plane/branch-and-bound algorithm for node packing. *J. Opl Res. Soc.*, 43, 443–457.
8. F. Rossi & S. Smriglio (2001) A branch-and-cut algorithm for the maximum cardinality stable set problem. *Oper. Res. Lett.*, 28, 63–74.