# Some new attemps to deal with integer multicommodity flow routing problems

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## 1 Introduction

Consider a undirected capacitated graph $G = (V, E, c)$ where $c_e > 0$ is the capacity of edge $e \in E$ and a set of commodities indexed by $K = \{1, \ldots, p\}$, where each commodity $k \in K$ is defined by two terminal nodes $s^k$ and $t^k$ and by an amount of demand $d^k > 0$. Denote by $\mathcal{P}^k$ the set of paths joining $s^k$ and $t^k$ and let $\mathcal{P} = \cup_{k=1}^{p} \mathcal{P}^k$. The multicommodity flow problem consists in defining a feasible way to route each commodity $k$ from $s^k$ to $t^k$. Using flow variables $f_p$ denoting the amount of demand routed on path $p$, the basic set of constraints to satisfy are the following:

$$\sum_{p \in \mathcal{P}^k} f_p \geq d^k, \quad k \in K, \tag{1}$$

$$\sum_{p \in \mathcal{P}: e \in p} f_p \leq c_e, \, e \in E. \tag{2}$$

Constraints (1) impose that the demand associated with each commodity is fully routed and constraints (2) impose that the resulting flow on each edge does not exceed the capacity of that edge.

Multicommodity flow problems have been considerably studied in a wide variety of contexts and with many different approaches. In the continuous case ($f_p \geq 0$), the demand of each commodity can be routed on as many paths as required by a given objective function. Many efforts have been devoted to solve efficiently large-scale instances of the continuous multicommodity flow problem with various types of objective functions [12]: one of the most frequent objective function consists in minimizing an overall routing cost, a unit routing cost $c_e^k$ being associated with each commodity $k$ routed on edge $e$. The well-know non-linear convex *Kleinrock* function is also very often used to model all the situations where the cost of filling an edge become exponentially high when the traffic on that edge approaches its capacity. Note that similar effects can be obtained by piece-wise linear convex cost functions [8]. The *Maximum Concurrent Flow problem* which consists in finding the maximum ratio of common demand that can be multi-routed (change $d^k$ by $\lambda d^k$ in the first set of constraints and maximize $\lambda$), is an example of "difficult" continuous problem [2], [14]. Other types of max-min objectives have also been considered.

Several "integer" multicommodity flow problems have also been investiguated in the past. In the *all-or-nothing multicommodity flow problem*, the routing variables are still continuous, but each commodity must be either completely satisfied or not routed at all and the objective is to maximize the number of commodities completely satisfied [4]. In the so-called *integer multicommodity flow problem*, an integrality constraint is added on the flow variables ($f_p \in \mathbb{N}$) imposing that only integral chunks of demands can be routed in the network [3]. In the *unsplittable flow problems*, only one path can be used for each commodity [1] [7] [11]. Setting each demand to a common integer value $p$ and setting each capacity to 1, this problem reduces to the well-studied *edge-disjoint paths problem* [5] [13].

Many results have been derived for all these kinds of problems. In particular, the extention of the max-flow/min-cut property to multicommodity flows has been deeply investiguated [6]. These weak duality properties have inspired many approximation algorithms [10].

In this paper, we investigate some decomposition approaches for solving integer multicommodity flow problems. The impact of the choice of one among several standard linear programming formulations have already analyzed [9]. However, in some cases, it might be usefull to combine several types of formulations into a single mixed integer model. Another approach consists in using the multi-cut informations provided by the continuous relaxation to drive a partitioning of the graph. The initial integer multicommodity flow problem can then be decomposed into several problems of smaller size, each one being defined in one of the resulting subgraphs. We analyze and apply these two approaches on several integer multicommodity flow problems with a special focus on max-min problems, such as:

$$\max z \tag{3}$$

$$\text{s.t. :} \quad \sum_{p \in \mathcal{P}^k} x_p^k \geq 1, \qquad k \in K, \tag{4}$$

$$\sum_{k \in K} \sum_{p \in \mathcal{P}^k : e \in p} d^k x_p^k + z \leq c_e, \; e \in E, \tag{5}$$

$$x_p^k \in \{0, 1\}. \tag{6}$$

Here, the binary variables $x_p^k$ are used to indicate whether the commodity $k$ is routed on path $p$ or not. Exact and heuristic solution methods will be described and analyzed. Preliminary computational results will be provided.

## References


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