The m-Capacitated Peripatetic Salesman Problem

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1 Introduction

The undirected *m-Capacitated Peripatetic Salesman Problem* (m-CPSP) is defined on a complete simple graph $G = (V, E)$, where $V = \{1, \ldots, n\}$ is the vertex set and $E = \{e = (i, j) : i, j \in V, i < j\}$ is the edge set. With each edge e is associated a cost c_e and a capacity C_e. The m-CPSP consists of determining m Hamiltonian cycles of least total cost on G such that all edges $e \in E$ are used less than C_e times. The m-CPSP is NP-hard and reduces to the *m-Peripatetic Salesman Problem* (m-PSP) when $C_e = 1$ for all $e \in E$. To avoid trivial or infeasible cases we assume that $n \geq 6$ and $m \leq \lfloor (n-1)/2 \rfloor$. To our best knowledge, this is the first study on the m-CPSP. In his seminal paper on the m-PSP, Krarup [5] presented a simple procedure to determine an upper bound on the optimal m-PSP solution value. This procedure can easily be generalized in order to find an upper bound $\overline{z}$ for the m-CPSP. Initially $F = \emptyset$ and $\overline{z} = 0$. The procedure executes the following iteration m times: solve a TSP on $E \backslash F$ and let z be its optimal value; set $\overline{z} := \overline{z} + z$, $C_e = C_e - 1$ for all e belonging to the TSP solution and $F := F \cup \{e \in E : C_e = 0\}$.

Applications of the m-CPSP arise naturally in the design of patrol routes, of automated guided vehicle (AGV) loops, and of hazardous material (hazmat) transportation routes. An example of the first application for the m-PSP is provided by Wolfler Calvo and Cordone [8] who have analyzed the design of patrol routes for security agents in charge of checking several locations on consecutive nights. In order to provide enhanced security to the patrolmen, a set of several partially edge disjoint routes are used. A similar example is the design of military patrol routes. In the second application the aim is to design several AGV loops serving manufacturing cells in a factory. Several configurations are available for such loops (Asef-Vaziri and Laporte [1]). Creating several partially edge disjoint loops with some cells (vertices) in common avoids congestion and accidents engendered by track sharing, while providing communication between the loops (Blazewicz et al. [2], Venkataramanan and Wilson [7]). The m-CPSP corresponds to the case where all loops go through all vertices. In the hazmat application the aim is to create m Hamiltonian paths through several locations. In order to provide a fair distribution of risk, these paths should ideally have very few edges in common (Gopalan, Batta, and Karwan [4], Lindner-Dutton, Batta, and Karwan [6]). The path version of the m-CPSP applies to the case of m edge disjoint paths.

Duchenne, Laporte and Semet [3] have formulated the m-PSP by means of a 3-index formulation which can be extended to the m-CPSP. In the presentation we will describe this 3-index model as well as a new model using edge-edge variables. We will also present some valid inequalities for these models and two algorithms based on these formulations followed by computational results.

References

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