Online optimization: Some recent results

Patrick Jaillet

Department of civil and Environemental Engineering
School of Engineering, MIT
jaillet@mit.edu

An *online problem* is one that must be "solved" without knowing the future or without having complete information. One common approach to evaluating strategies for such problems is to assume a specific stochastic model of the unknown and use various probabilistic tools. Another approach, sometimes called competitive analysis, and related to min-max strategies in game theory, evaluates strategies under worst-case scenarios.

We first quickly review past and current methodological works dealing with online problems, emphasizing mathematical programming applications.

We then present updated results we have obtained on online routing. In particular, we consider first online routing optimization problems where the objective is to minimize the time needed to visit a set of locations under various constraints; the problems are online because the set of locations are revealed incrementally over time. We consider several problems such as (1) the online Traveling Salesman Problem (TSP) with precedence and capacity constraints and (2) the online TSP with *m* salesmen.

For both problems we propose online algorithms, each with a competitive ratio of 2; for the *m*-salesmen problem, we show our result is best-possible. We also consider polynomial-time online algorithms. We then consider resource augmentation, where we give the online servers additional resources to offset the powerful offline adversary advantage. We derive improved competitive ratios. Finally, we study online algorithms from an asymptotic point of view. We show that, under general stochastic structures for the problem data, *unknown and unused by the online player*, the online algorithms are almost surely asymptotically optimal.

We conclude with open research questions.