Polyhedral Analysis for the Two Item Uncapacitated Lot Sizing Problem with One Way Substitution

H. Yaman

Bilkent University, Department of Industrial Engineering, Turkey
hyaman@bilkent.edu.tr

Keywords : lot-sizing, product substitution, polyhedral analysis, facet defining inequalities, convex hull.

1 Introduction

We consider a production planning problem for two items where the high quality item can substitute the demand for the low quality item. Given the number of periods, the demands, the production, inventory holding, setup and substitution costs, the problem is to find a minimum cost production and substitution plan. We call this problem the *Two Item Uncapacitated Lot Sizing Problem with One Way Substitution*, *2ULS* for short. This problem generalizes the well-known uncapacitated lot-sizing problem (*ULS*).

Balakrishnan and Geunes [1] model the production planning problem with substitutable components and an arbitrary substitution structure. They derive properties of optimal solutions and propose a dynamic programming algorithm. The proposed algorithm finds a shortest path in a graph with $O(n^m)$ nodes and $O(n^{m+1})$ arcs where n is the number of periods and m is the number of components. Hence the method's worst case running time is exponential in the general case. When applied to the two-item problem, the algorithm runs in polynomial time and hence our problem is polynomially solvable. Geunes[3] models the same problem as an *Uncapacitated Facility Location (UFL) Problem*, solves using a dual-ascent heuristic and presents computational results where the performance of the heuristic approach is tested in comparison with the one of the exact shortest path approach. Hsu, Li and Xiao [4] consider two versions of the production planning problem with one-way substitution, propose dynamic programming algorithms as well as a heuristic. Li, Chen and Cai [5,6] consider product substitution together with remanufacturing. In [5], the authors propose a dynamic programming approach by extending the one of [1] to handle remanufacturing. They also propose a heuristic algorithm and present computational results. In [6], a genetic algorithm is proposed for a capacitated version of the problem with batch processing. We have not encountered any study on the polyhedral analysis of the production planning problem with substitution.

2 Polyhedral Results

We propose a formulation for *2ULS* in the space of production and setup variables for the case where the substitution costs are zero and inventory holding costs for the two items are equal.

We investigate the dimension, trivial facets and properties of facet defining inequalities of the convex hull of feasible solutions.

We also derive an *UFL* formulation and project it on the space of production and setup variables. We characterize the nondominated projection inequalities. These inequalities, called the $(l_1, l_2,$

S^1, S^2)-inequalities, generalize the well-known *(l,S)*-inequalities for the *ULS* polytope [2]. We present necessary and sufficient conditions for these inequalities to be facet defining for the polytope associated with *2ULS*. We prove that these inequalities can be separated in polynomial time.

The (l_1, l_2, S^1, S^2)-inequalities together with some of the constraints describe the 2*ULS* polytope if the number of periods is two. This is not true in general for larger number of periods.

3 Computational Results

We report the results of a computational study to see the quality of the bounds given by the linear programming relaxation of the model strengthened with the (l_1, l_2, S^1, S^2)-inequalities. The results suggest that the resulting formulation is quite strong. 3600 random instances with varying number of periods, setup costs, ratio of production and inventory holding costs, and demand variability are solved. The largest percentage gap between the optimal value and the lower bound turned out to be less than 0.6 %. The overall average percentage gap is 0.009 % and the gap is zero for 2474 instances over 3600.

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