A Hybrid Monte Carlo Local Branching Algorithm for the Single Vehicle Routing Problem with Stochastic Demands

M. Gendreau[1], W. Rei[2] and P. Soriano[3]

1. CIRRELT and DIRO, Université de Montréal, Canada
michel.gendreau@cirrelt.ca

2. CIRRELT and Département de management et technologie, Université du Québec à Montréal, Canada
walter@crt.umontreal.ca

3. CIRRELT and Service de l'enseignement des méthodes quantitatives de gestion, HEC Montréal Canada
patrick.soriano@cirrelt.ca

Keywords: Local branching, Monte Carlo sampling, Stochastic vehicle routing problems.

1 Introduction

For many years, a lot of attention has been devoted to vehicle routing problems (VRP), most of it dealing with deterministic cases. In practice, however, one has seldom access to perfect information concerning the parameters of a problem. Therefore, in recent years, stochastic versions have been considered where certain parameters of the VRP are modeled by random variables. By solving stochastic routing problems, one can obtain significantly better solutions whenever there is uncertainty in the situation being considered. In this presentation, we present a new algorithm that uses both local branching and Monte Carlo sampling in a multi-descent search strategy for solving 0-1 integer stochastic programming problems. This procedure is applied to the single vehicle routing problem with stochastic demands and computational results are reported.

2 The single vehicle routing problem with stochastic demands

The single vehicle routing problem with stochastic demands (SVRPSD) is defined as follows: let *G(V, E)* be an undirected graph, where $V = \{v_1, \ldots, v_N\}$ is a set of vertices and $E = \{(v_i, v_j): v_i, v_j \text{ in } V, i < j\}$ is a set of edges. Defined on E is a symmetric matrix $C = [c_{ij}]$ that corresponds to the travel costs between vertices. Vertex v_1 represents a depot from which the vehicle must start and finish its route. If one searches for a route that visits all vertices once and minimizes the total travel cost, then one is in fact solving the well known travelling salesman problem (TSP). To obtain the SVRPSD, let us suppose that the vehicle has a limited capacity D and that each vertex j in $V\backslash\{v_1\}$ corresponds to a customer that has a nonnegative demand χ_j that is stochastic. We also assume that this demand χ_j only becomes known when the vehicle arrives at v_j.. In this case, whenever a customer is visited, the residual capacity of the vehicle may not be sufficient to fulfill the observed demand. If this situation occurs, one says that there is a *route failure* and a *recourse action* must be undertaken to correct it. In this talk, we focus on the classical replenishment strategy proposed by Dror et al. [1], which requires the vehicle to return to the depot to restock before resuming its route at the point where the failure occurred. This modus operandi can easily be modelled as a two-stage stochastic programming problem in which the first stage consists in constructing an a priori route visiting all customers and the second stage amounts to simply following the route, taking recourse actions as required by failures, once demands are known. The objective of the problem then becomes the minimization of the sum of the cost of the a priori tour plus the expected course f recourse actions.

3 Monte Carlo local branching hybrid algorithm

The solution approach that we propose relies on some key insights. In “reasonable” instances of the SVRPSD, the expected number of failures, while significant, is still low. This implies that the cost of the a priori route will in general amount for a large fraction of the overall objective. Thus, our optimal first-stage solution has to be a pretty good solution of the TSP. In fact, we can interpret this solution as an optimal TSP solution “adjusted” to account for possible failures.

Our algorithm is a multi-descent scheme where each descent starts from the solution of a “modified” TSP problem, in which additional constraints are used to account for the portion of the solution space explored by previous descents. Each descent is a fixed-depth one and is performed in accordance with the *Local Branching* scheme proposed by Fischetti and Lodi [2], which consists in exploring exhaustively small areas of the solution space around the current solution.

To overcome the stochastic complexity of the SVRPSD, we use Monte Carlo sampling of the demand scenarios to estimate the expected cost of recourse actions.

4 Computational experiments

The proposed method was tested on two sets of randomly-generated instances. The first set consisted of instances having between 60 and 90 customers that had been shown to be hard to solve for an exact solution approach to the problem [3], while the second was made up of 20 larger 150-customer instances. The results obtained show that our heuristic was able to find optimal or near-optimal results for all the instances that could be solved by the exact approach. As for problems that could not be solved to optimality by the exact algorithm, in most cases, the multi-descent heuristic finds better solutions. Moreover, in several cases, these can be significantly better than the solutions produced by the exact method.

References

[1] M. Dror, G. Laporte, and P. Trudeau.Vehicle routing with stochastic demands: properties and solution frameworks. *TransportationScience*, 23(3), 166–176.

[2] M. Fischetti and A. Lodi (2003). Local branching. *Mathematical Programming*, 98, 23-47.

[3] W. Rei, M. Gendreau, and P. Soriano (2007). *Local branching cuts for the 0-1 integer L-shaped algorithm*, Working paper CIRRELT-2007-23, CIRRELT, Université de Montréal, Montréal, Canada.