The Clustered Prize-collecting Arc Routing Problem.

Julian Aráoz[1], Elena Fernández[1], and Carles Franquesa[2]

[1] Departament d'Estadística i Investigació Operativa , Universitat Politècnica de Catalunya, Spain
{julian.araoz, e.fernandez}@upc.edu
[2] Departament de Llenguatges i Sistemes Informàtics, Universitat Politècnica de Catalunya, Spain
carles.franquesa-niubo@upc.edu

1 Introduction

Prize-collecting Arc Routing Problems (PARPs) are arc routing problems where, in addition to the cost function, there is a profit function on the edges that must be taken into account at most once: only when an edge is serviced. PARPs were defined in Aráoz, Fernández and Zoltan [2], and the so called Privatized Rural Postman Problem (PRPP) has also been studied by Aráoz, Férnandez and Meza in [1]. Like in other arc routing problems in the PARPs we assume that the demand of service is placed at the edges of a graph. However, as opposed to typical arc routing problems, there is no specific arc subset to be traversed. Instead, we assume that giving service to an edge will incur not only a cost (associated with displacement), but also a profit (associated with servicing edges). The displacement cost to an edge will certainly account for the cost of all the edges that are traversed in that same route, as many times as they are traversed. Similarly, the profit associated with servicing an edge, should also take into account the profit of the additional edges that are serviced in that same route. However, the profit of each edge serviced in the route will be collected only once, independently of how many times the edge is traversed. In the PRPP we look for traversals that maximize the total servicing profit minus the displacement cost. As shown in [2] the Rural Postman Problem (RPP) is a particular case of PRPP. In fact, PRPPs constitute a generalization of most Edge Routing Problems with one single route and is equivalent to most Traveling Salesman Problems.

To the best of our knowledge, the literature on arc routing problems with profits on the edges is very scarce. Apart from our previous works [1–3], we are aware of only two works, one by Deitch and Ladany [5] and another one by Feillet, Dejax and Gendreau [7] where problems of this type are considered. However, in both papers the setting and the approach are quite different from ours: while our approach addresses the problem as an arc-routing one and exploits explicitly the fact that only one traversal of each edge is beneficial, in [5] the problem is transformed into a node-routing one, whereas in [7] a limit, possibly greater than one, is given on the number of times the traversal of an edge can be beneficial. We are not aware of any work that considers the problem addressed in this paper.

In this work we present the Clustered Prize-collecting Arc Routing Problem (CPARP). CPARP is a PARP where, in addition, we consider the components (clusters) defined by the edges with demand, and for each cluster we require that either all its links are serviced or no link of the cluster is serviced. To some extent, the problem that we address is related to the Clustered Rural Postman Problem (CRPP) studied by Dror and Langevin in [6], where the connected components defined by the arcs with demand must be completely serviced before servicing any other component. However, there are several differences between the CPARP and the CRPP. The first one is that CRPP is an arc routing problem, but not a PARP. The second one is that CRPP is stated on a directed graph, whereas we consider the CPARP on an undirected graph. Finally, in the CRPP of [6] it is required that all components are serviced, and that a component is completely serviced before servicing any other component. This second constraint is not inherent to the problem and is only required for making it possible to transform the problem into a Generalized Traveling Salesman Problem. In the CPARP we release these requirements. On the one hand, we do not require to service all the components. On the other hand, we allow not servicing all the edges of a component consecutively if this results in a better solution. Potential applications of CPARP include, among others, determining the service for garbage collection or street cleaning in districts, neighborhoods in a given area. Indeed, while it is not acceptable that only a part of a given neighborhood is serviced, the whole neighborhood might not be profitable for the servicing company.

As we will see, when we pose the additional requirement that either all the links of a cluster are serviced or no link of the cluster is serviced, we can prove some dominance conditions that otherwise do not hold. These properties allow some transformation of the original graph that results in a stronger formulation of the problem. This formulation has an exponential number of inequalities of two types. Inequalities that guarantee the connectivity of the traversal with the depot, and inequalities that guarantee the even degree of the nodes, which are a particular case of the so-called co-circuit inequalities of Barahona and Grötschel [4].

We propose an exact branch-and-cut algorithm to optimally solve the problem. At the root node we obtain an upper bound by solving the LP relaxation of the model. Due to the exponential number of constraints we use an LP based iterative scheme that starts with a small number of inequalities and at each iteration reinforces the current model with violated inequalities. This is possible since, as we will see, we can solve exactly the separation problem for the two exponential families of inequalities. We also propose a simple heuristic to generate feasible solutions that provides lower bounds at each iteration. For analyzing the performance of the proposed algorithm we have run a series of computational experiments with a set of benchmark instances used for other arc routing problems in the literature.

We present numerical results of our computational experiments. These results assess the efficiency of the exact enumeration algorithm. Over 75% of the considered instances were optimally solved at the root node of the search tree. The remaining instances required, in general, to explore very few nodes of the search tree. All but two instances of the 118 considered ones were optimally solved in less than two minutes of cpu time.

References

1. J. Aráoz, E. Fernández, O. Meza (2005). An LP based algorithm for the Privatized Rural Postman Problem. Report DR-2005/10, EIO Departament, Technical University of Catalonia, Spain.
2. J. Aráoz, E. Fernández and C. Zoltan (2006). The Privatizad Rural Postman Problem. *Computers and Operations Research* 33, 3432-3449.
3. J. Aráoz, E. Fernández, C. Franquesa and O. Meza (2006). Prize-collecting Arc Routing Problems and Extensions. Third International Workshop on Freight Transportation and Logistics (Odysseus 2006), Altea (Spain).
4. F. Barahona and M. Grötschel (1986). On the cycle polytope of a binary matroid.*J. Comb. Theory*, 40:40–62.
5. R. Deitch, S. P. Ladany (2000). The one-period bus routing problem: Solved by an effective heuristic for the orienteering tour problem and improvement algorithm. *Eur. J. Oper. Res.* 127(1):69-77.
6. M. Dror and A. Langevin (1997). A Generalized Traveling Salesman Problem. *Transportation Science* 31(2).
7. D. Feillet, P. Dejax and M. Gendreau (2005). The profitable Arc Tour Problem: Solution with a Branch-and-Price Algorithm. *Transportation Science*, 39(4):539–552.